Modelling the critical states of coupled superposed superconducting films

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***Abstract*—We investigate the magnetic field and current distributions in structures made of superposed superconducting films. A recent experimental study of a planar bilayer structure (Tamegai 2017) has revealed unusual patterns in the critical state, with an unexpected discontinuity line arising in the region where the layers overlap. We use a finite element model to investigate the critical states in such 3D structures with several layers and different overlapping configurations. We show that the experimentally observed discontinuity line arises as a result of a magnetic field dependence of the critical current density. Additionally, we show that the network of discontinuity lines not only depends on the geometry of the system but also evolves with the applied magnetic field as the structures are magnetized in a zero-field-cooled mode. The strength of the magnetic coupling is shown to depend on the number of layers, their overlap and separation, with a direct effect on the length and the shape of the discontinuity lines.**

Keywords—FEM, critical state, superconductivity, coupled superconductors, thin films.

I. INTRODUCTION

Critical states in planar superconducting structures are characterized by discontinuity lines (d-lines) where the current density changes abruptly its direction. D-lines typically appear along symmetry axes and form a pattern intimately related to the geometry of the film and its current-carrying properties. For instance, in square films, d-lines appear along diagonals (thus forming an X pattern) and delimit a $\pi/2$ angular change in the direction of the current density, as shown by magneto-optical imaging in Fig. 1a. In rectangular films, d-lines run along the bisector of each corner angle and merge into a straight segment extending in the middle of the film, forming a double-Y shape, as represented in Fig. 1b. Recently, a new type of pattern was experimentally identified by the group of Tamegai [1] in a 3D structure made of a superconducting strip on top of a square superconducting film (the strip and the square were separated by an electrically insulating SiO_2 layer, as schematically depicted in Fig. 1d). As observed in the magneto-optical image of the sample in the remanent state (Fig. 1c), the magnetic field pattern shows an unexpected d-line extending around the center of the structure, which is perpendicular to the long edge of the strip, and is absent from the critical state of the square film alone or that of the strip alone. In order to elucidate the formation of such d-lines, we perform numerical simulations for the penetration of magnetic flux in the square+strip assembly under a uniform applied magnetic field.

II. NUMERICAL MODEL

In order to model the penetration of magnetic flux inside the assembly of Fig. 1d, we use a finite-element method with a three dimensional $H-\phi$ formulation [2]. Faraday's law is solved in the weak form,

$$\int_\Omega \mu_0\, \dot{\vec{H}}_a \,.\, \vec{\psi}\, d\Omega + \int_\Omega \mu_0\, \dot{\vec{h}} \,.\, \vec{\psi}\, d\Omega + \int_{\Omega_c} \rho(|\vec{\nabla}\times\vec{h}|)\,(\vec{\nabla}\times\vec{h})\,.\,(\vec{\nabla}\times\vec{\psi})\, d\Omega = 0, \qquad (1)$$

where μ_0 is the vacuum permeability, ρ is the electrical resistivity in the superconducting regions, $\vec{H}_a$ is the applied field, $\vec{h}$ is the reaction field and $\vec{\psi}$ are the finite element test functions. Ω refers to the total computation domain, while Ω_c contains the superconducting materials. In Ω_c, $\vec{\psi}$ are first-order curl-conforming elements. If Ω/Ω_c is a simply connected set, the test functions $\vec{\psi}$ can be written as $\vec{\psi} = -\vec{\nabla}\varphi$, where φ is represented with nodal functions.

Boundary conditions at infinite distance from the superconducting films are applied by means of a unidirectional shell-transformation, so that calculations are carried out in a

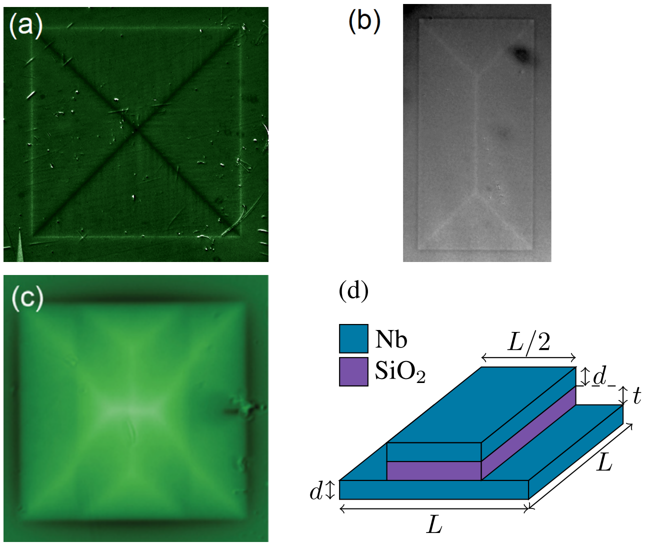

Fig. 1. Magneto-optical image of the critical state in a completely penetrated square superconducting film (a), in a rectangular superconducting strip in the remanent state (b), and in the superposition of a rectangular superconducting strip over a square superconducting film in the remanent state (c). The geometry of the assembly corresponding to the image in panel (c) is depicted in (d). For every magneto-optical image, the brighter the contrast, the higher the magnetic field.

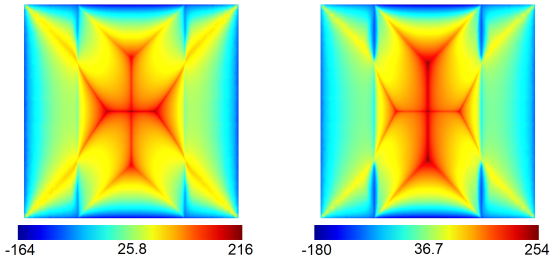

Fig. 2. Out-of-plane magnetic field, H_z, in the square film in a bilayer assembly (left panel) and a trilayer assembly (right panel) in the remanent state. The geometry of the bilayer assembly corresponds to the one drawn in Fig. 1d. The geometry of the trilayer assembly is made of a square film, which is included in between two identical centred rectangular strips. In both simulations, $j_{c0} = 12$ MA/cm^2, $B_0 = 5$ mT. Results are in Oersted.

finite domain without any truncation approximation. Moreover, using the divergence-free property of $\vec{H}_a$, it is possible to rewrite (1) so that a uniform magnetic field, perpendicular to the film, is set inside an arbitrary box, Ω_b, which encloses Ω_c, without calculating its far-field decay. If the boundary of Ω_b is denoted by Γ_b, (1) becomes:

$$\begin{aligned} &\int_{\Omega} \mu_0\, \dot{\vec{h}}\,.\,\vec{\psi}\, d\Omega \\ &+ \int_{\Omega_b} \mu_0\, \dot{\vec{H}}_a\,.\,\vec{\psi}\, d\Omega + \int_{\Gamma_b} (\mu_0\, \dot{\vec{H}}_a\, \varphi)\,.\,\vec{n}_b\, d\Gamma \\ &+ \int_{\Omega_c} \rho(|\vec{\nabla}\times\vec{h}|)\,(\vec{\nabla}\times\vec{h})\,.\,(\vec{\nabla}\times\vec{\psi})\, d\Omega = 0. \end{aligned} \tag{2}$$

Equation (2) is solved with a power-law constitutive relation between the electric field, $\vec{E}$, and the current density, $\vec{j} = \vec{\nabla}\times\vec{h}$, which yields the expression of the electrical resistivity, $\rho(|\vec{j}|) = E_c/j_c(|\vec{j}|/j_c)^{n-1}$. Here, $E_c = 10^{-4}$ V/m is the critical electric field, n is a dimensionless exponent, and j_c is the critical current density. j_c is either assumed to be constant, or to follow the extended Kim's law

$$j_c(|\vec{B}|) = \frac{j_{c0}}{(1 + |\vec{B}|/B_0)^{\alpha}}, \tag{3}$$

where $\vec{B} = \mu_0(\vec{H}_a + \vec{h})$, and α is a dimensionless parameter.

III. RESULTS

Numerical simulations are carried out to ascertain the crucial role played by the magnetic field dependence of j_c in the apparition of the additional horizontal d-line. Values of α, B_0 and j_{c0} are chosen to fit as accurately as possible experimental $j_c(|\vec{B}|)$ curves. The numerical results are shown to faithfully reproduce the magneto-optical images of the magnetic field, both qualitatively and quantitatively. It is shown experimentally and checked numerically that, when $\vec{H}_a$ is ramped up to a maximal value, a vertical d-line appears at the center of the films. When $\vec{H}_a$ is ramped back to zero, the vertical d-line fades out and a horizontal d-line progressively develops instead.

The d-lines are shown to depend not only on the geometry of the sample, but also on the inhomogeneous distribution of critical current densities in the films. A common magnetic field threads the superposed films, so that the levels of current densities are different in the overlap region from those in the peripheral regions. A simplified analytical model exploiting these observations is shown to reproduce the salient features of the critical states which were observed experimentally.

Numerical modelling is also used to shed light on the characteristics of these new critical states. The length of the horizontal d-line in the remanent state, ℓ_h, is studied for different sets of parameters j_{c0} and B_0. The effects of the spacing between the films and the width of the rectangular film are also investigated.

The results are based on [3] and are extended to asymmetric systems and to multilayer systems, as illustrated in Fig. 2, which compares the distribution of the out-of-plane component of the magnetic field, H_z, in a bilayer and in a trilayer system, in the remanent state. It can be observed that, for the same set of parameters, the horizontal d-line extends further in the trilayer than in the bilayer system due to the stronger coupling.

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