3D Stack Magnetization Problems: Solution by the Fast Fourier Transform-Based Method

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***Abstract*—A stack of coated conductors is a perspective configuration for various applications of high temperature superconductors. We solve magnetization problems for a stack of flat films of an arbitrary shape using a fast Fourier transform-based numerical method. A properly rescaled solution for a stack of only several films is employed to obtain an accurate approximation to the solution for stacks containing a large number of densely packed films. For an infinite stack the problem simplifies and becomes similar to that for a single film.**

Keywords—*superconducting film stacks, magnetization problems, numerical modeling, fast Fourier transform.*

I. INTRODUCTION

The fast Fourier transform (FFT) based numerical method for 2D thin film magnetization problems was proposed by Vestgården et al. [1,2] and used by several authors, mainly to simulate thermal instabilities and flux avalanches in superconducting films. The method was modified and extended to 3D magnetization problems for bulk superconductors in our works [3,4]. Here and in [5] we present another extension of the FFT-based method and solve the magnetization problems for stacks of flat superconducting films of the same but arbitrary shape. Previously, such problems were solved only in the 2D case, for stacks of infinitely long strips.

To model magnetization of a large number of densely packed films, the stack can be replaced by a stack of only a few films with a proper rescaling of parameters. We show that employing then the FFT-based method can be at least as efficient as transition to the fully homogenized anisotropic bulk problem and using a finite element method.

Finally, a general and convenient formulation is derived for magnetization of an infinite stack of arbitrary shaped flat films. Such problems are also efficiently solved by the FFT-based method.

II. MAGNETIZATION PROBLEM

We consider a stack of N thin superconducting films, $\{(x,y,z_m)\,|\,(x,y)\in\Omega,\ z_m=md\}$, $m=1,\dots,N$, and assume, for simplicity, that the domain $\Omega\subset R^2$ is simply connected, the normal to films component of the applied magnetic field is uniform, $h_{e,z}=h_{e,z}(t)$, and in all films the same nonlinear relation, $\boldsymbol{e}_m=\rho\left(\left|\boldsymbol{j}_m\right|\right)\boldsymbol{j}_m$, holds between the sheet current density $\boldsymbol{j}_m$ and the parallel-to-film electric field component $\boldsymbol{e}_m$. Since $\nabla\cdot\boldsymbol{j}_m=0$ and the normal to $\Gamma=\partial\Omega$ component of $\boldsymbol{j}_m$ is zero, there exist stream functions g_m such that $j_{m,x}=\partial_y g_m$, $j_{m,y}=-\partial_x g_m$ and $g_m\big|_\Gamma=0$. Extending the functions g_m by zero into $\Omega_{\text{out}}=R^2\setminus(\Omega\cup\Gamma)$, differentiating the Biot-Savart law in time, and applying the Fourier transform

$$F[f]=\int_{R^2}f(x,y)\exp(-\mathrm{i}[k_x x+k_y y])\mathrm{d}x\,\mathrm{d}y\,,$$

we obtain

$$\dot{\boldsymbol{g}}=F^{-1}\left[\Psi(k)F\left[\dot{\boldsymbol{h}}_z-\dot{h}_{e,z}(t)\boldsymbol{1}\right]\right]-\boldsymbol{C}(t)\,, \qquad (1)$$

where $\dot{u}$ means $\partial_t u$, bold $\boldsymbol{u}$ denotes the vector or vector function $(u_1,\dots,u_N)^T$, $\boldsymbol{1}=(1,\dots,1)^T$, k is the modulus of the wave vector (k_x,k_y), the matrix

$$\Psi(k)=\frac{2}{k\left(1-q^2\right)}\begin{pmatrix}1 & -q & 0 & \dots & & 0\\ -q & q^2+1 & -q & 0 & \dots & 0\\ 0 & -q & q^2+1 & -q & \dots & 0\\ & & \dots & & & \\ 0 & \dots & 0 & -q & q^2+1 & -q\\ 0 & \dots & & 0 & -q & 1\end{pmatrix}$$

with $q=\exp(-kd)$ should be replaced by a zero matrix for $k=0$, and components $C_m(t)$ of vector $\boldsymbol{C}$ are chosen to satisfy the conditions $\int_{\Omega_{\text{out}}}\dot{g}_m=0$ for any t.

Suppose the functions g_m are known at a time moment t. Then one can find the sheet current densities $\boldsymbol{j}_m$ and also $\boldsymbol{e}_m=\rho\left(\left|\boldsymbol{j}_m\right|\right)\boldsymbol{j}_m$ in Ω. The Faraday law determines

$$\dot{h}_{m,z}=-\mu_0^{-1}\nabla\times\boldsymbol{e}_m=\mu_0^{-1}\nabla\cdot\left[\rho(|\nabla g_m|)\nabla g_m\right] \qquad (2)$$

but only in the films, since the electric field remains unknown in the non-conducting domain Ω_{out}. Equation (2) is not sufficient yet for using (1) as an evolutionary equation for $\boldsymbol{g}$: in (1) $\dot{\boldsymbol{h}}_z$ should be known in the whole plane. However, $\dot{\boldsymbol{h}}_z$ in Ω_{out} should be such that (1) holds with

$$\dot{\boldsymbol{g}}\big|_{\Omega_{\text{out}}}=0\,. \qquad (3)$$

The condition (3) can be satisfied iteratively (see [5]) and determines $\dot{h}_z$ in Ω_{out}. This enables us to regard (1) as an evolutionary equation and integrate it in time.

For a computer implementation of this method, a regular $N_x \times N_y$ grid should be defined in a rectangle containing and several times larger than Ω, the continuous Fourier transform and its inverse in (1) should be replaced by their discrete analogues and computed using the FFT algorithm. The derivatives in (2) should be also computed in the Fourier space with an appropriate smoothing. Finally, an ordinary differential equations solver should be employed for integration in time.

III. HOMOGENIZATION

Desirable for applications are usually stacks of hundreds of densely packed coated conductor films. An efficient and accurate solution to magnetization problems can then be obtained by homogenization which leads to the anisotropic bulk model [6]. It is also possible to replace the stack by a stack containing only a few films with a properly scaled parameters [7], and we used such approach to compare our FFT-based method to the finite element solutions of the anisotropic bulk problem with a power current-voltage relation [8,9]. Our solution of this benchmark problem (Fig. 1) is very similar to those in [8,9], the calculated AC losses coincide within 1-2%, and our computation on a usual PC was faster.

IV. INFINITELY HIGH STACKS

If a stack of many films has a height greater than the film sizes, the sheet current density distributions are usually similar in all except the several films closest to the stack top and bottom. Because of this the sheet current distribution in the films of an infinite stack can be of interest. An analytical solution to the infinite stack problem has been obtained for the stacks of infinitely long strips under the Bean model assumption [10]. A formulation for films of an arbitrary shape and a general current-voltage relation has been obtained in [5] as follows.

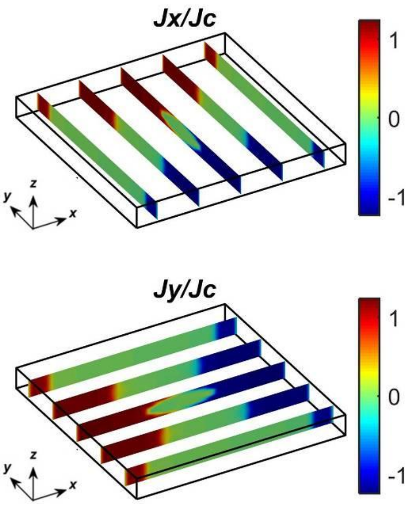

Fig. 1. Current density at a peak of a sinusoidal applied field: solution of the anisotropic bulk benchmark problem computed using a 6-film stack approximation.

Since all films in an infinite stack are under the same conditions, $g_m = g, h_{m,z} = h_z, \boldsymbol{e}_m = \boldsymbol{e}$ for $m = 0, \pm 1, \pm 2, \ldots$, and (1) is reduced to the single equation

$$\dot{g} = F^{-1}\left[\psi(k)F\left[\dot{h}_z - \dot{h}_{e,z}(t)\right]\right] - C(t),$$

where

$$\psi(k) = \frac{2(1-q)}{k(1+q)}$$

should be replaced by zero for $k = 0$ and $C(t)$ chosen using the condition $\int_{\Omega_{\text{out}}} \dot{g} = 0$. Numerical solution of the obtained problem by the FFT-based method is similar to that for a single film, see [1-3]; the only difference is that now we have $\psi(k)$ instead of $2/k$. As an example, we studied magnetization of an infinite stack of thin superconducting disks characterized by the power current-voltage relation.

V. CONCLUSION

The FFT-based method has been adapted to stacks of superconducting films, a perspective replacement of bulk superconductors in many practical applications. We showed that, to simulate magnetization of a densely packed stack of a large number of films, employment of the anisotropic bulk model is not the only possible approach: an accurate solution can be obtained using a stack with only a few films with the properly chosen characteristics. The developed approach was compared to the recently proposed finite element methods; this comparison showed that the FFT-based method can be regarded a simpler but, nevertheless, efficient and competitive alternative to the finite element methods.

[1] J. I. Vestgården and T. H. Johansen, "Modeling non-local electrodynamics in superconducting films: the case of a right angle corner" Supercond. Sci. Technol. vol. 25, 104001, 2012.

[2] J. I. Vestgården, P. Mikheenko, Y. M. Galperin, and T. H. Johansen, "Nonlocal electrodynamics of normal and superconducting films" New J. Phys. vol. 15, 093001, 2013.

[3] L. Prigozhin and V. Sokolovsky, "Fast Fourier transform-based solution of 2D and 3D magnetization problems in type-II superconductivity" Supercond. Sci. Technol. vol. 31, 055018, 2018.

[4] L. Prigozhin and V. Sokolovsky, "3D simulation of superconducting magnetic shields and lenses using the fast Fourier transform" J. Appl. Phys. vol. 123, 233901, 2018.

[5] L. Prigozhin and V. Sokolovsky, "Solution of 3D magnetization problems for superconducting film stacks" Supercond. Sci. Technol. vol. 31, 125001, 2018.

[6] J. R. Clem, J. H. Claassen, and Y. Mawatari "AC losses in a finite Z stack using anisotropic homogeneous-medium approximation" Supercond. Sci. Technol. vol. 20, 1130, 2007.

[7] L. Prigozhin and V. Sokolovsky, "Computing AC losses in stacks of high-temperature superconducting tapes" Supercond. Sci. Technol. vol. 24, 075012, 2011.

[8] M. Kapolka, V. M. R. Zermeno, S. Zou, A. Morandi, P. L. Ribani, E. Pardo, and F. Grilli, "Three-dimensional modeling of the magnetization of superconducting rectangular-based bulks and tape stacks" IEEE Trans. Appl. Supercond. vol. 28, 6801405, 2017.

[9] M. Olm, S. Badia, and A. F. Martin "Simulation of high temperature superconductors and experimental variation" Computer Phys. Comm. vol. 237, pp. 154-167, 2019.

[10] Y. Mawatari "Critical state of periodically arranged superconducting strip lines in perpendicular magnetic field" Phys. Rev. B vol. 54, pp. 13215-21, 1996.