Numerical modelling of the ferromagnetic-superconductor interaction in 3D geometry using A–formulation.

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***Abstract*—A special kind of magnetic shield, called the "magnetic cloak" consists of two parts: superconducting insert and ferromagnetic shell. In the numerical modelling utilized e.g. for the design and optimization purposes, electromagnetic behaviour of both these materials should be properly included. A similar situation exists also in case of conventional superconducting power devices like transformers and motors. The implementation of A-formulation in COMSOL Multiphysics allows building a numerical model that considers in the same physics the superconducting material and the ferromagnetic one. An important feature is that the Jiles-Atherton model of magnetic hysteresis could be easily implemented. In this way, the interaction between superconductors and ferromagnetic material can be introduced directly in the computation. We present numerical and experimental results for the magnetic cloak, where the superconducting part is made from six layers of coated conductor (CC) tapes and the ferromagnetic one is from an epoxy/ferrite powder composite. The current distribution in superconducting tapes and the magnetization of the ferromagnetic shell are solved within the same physics using the A-formulation.**

Keywords—superconductors, ferromagnetic, FEM modelling, A-formulation

I. INTRODUCTION

The combination of superconducting and ferromagnetic materials in the same device is a common situation for superconducting power applications like motors, generators and power transformers. The conventional magnetic shields usually do not deal with such a situation, unless we speak about the magnetic cloak [1]. This special kind of the magnetic shielding not only reduces the magnetic field in the shielded space but do it in such way that outer magnetic field distribution remains unchanged. Such properties are possible thanks to the combination of the superconducting inner part of the shield and the ferromagnetic shell.

Proper functionality of the magnetic cloak required certain geometry of used components in the right combination of its electromagnetic properties. The situation becomes to be more complicated when we use the coated conductors (CC) for the construction of the superconducting part. From the other hand, the application of CC tapes in the magnetic cloak gives a lot of advantages in comparison to bulk materials, like better in-field performance, lower energy dissipation at the AC magnetic field, better mechanical properties, price, etc. By these reasons the increased computation costs of using CC tapes seems to be justified.

II. SUPERCONDUCTORS IN A-FORMULATION

A. 2D numerical modelling

We would like to note, that formulation in terms of magnetic vector potential (*A*), nowadays, is the more common implementation for solving electromagnetic problems using the final element method (FEM) in the commercial software. Nevertheless, the superconducting materials are still not natively introduced in this formulation in the commonly utilized Multiphysics products like Ansys and Comsol. We used our adaptation of A-formulation for numerical computation in Comsol Multiphysics for 2D geometry for a long period [2]. The basic idea is introducing the current density (*J*) into the superconducting regions of the model geometry with respect to local time derivative of the magnetic vector potential. For the 2D geometry, it was done by combination the Maxwell's equation (1) and the current-voltage relation (2).

$$E_z = -\frac{\partial A_z}{\partial t} - V_z \tag{1}$$

$$J_z = J_c \tanh\left(\frac{E_z}{E_c}\right) \tag{2}$$

For the 2D approximation in Cartesian coordinates the problem is investigated in the *x*-*y* plane and all the used vector quantities, namely: electric field, magnetic vector potential, electrostatic potential, and current density are represented by only one, *z*-component: E_z, A_z, V_z and J_z, respectively. The superconducting critical current density is taken by the J_c, that could be field-dependent or may have a constant value. The extent of transition region between positive and negative current density in superconductor is controlled by E_c . This constant has the same meaning as the critical field criterion for critical current definition, but, due to the using tangent hyperbolical function as an alternative to *n*-power, may have slightly different effect than commonly used 1 μV/cm for the electrical measurements. Replacing the power function (*E*-*J* relation) or signum function (Bean model [3]) by the hyperbolical tangent is motivated by numerical requirements of function continuity and differentiability for the full range of input arguments. Introduced smoothed transition, usually, also significantly improves the FEM solver convergence.

B. *3D numerical modelling*

The equation (2) is valid for the simple 2D case only. Extending this approach to three-dimensional problem requires additional control on critical current density anisotropy and fulfilling the condition of local collinearity between the vectors $\vec{J}$ and $\vec{E}$. Implementing these features [4] produces the final definition of all components of the current density in the superconductor:

$$\vec{J} = \frac{J_{c0}}{|E|}\left(|E_x| \tanh\left(\frac{E_x}{E_c}\right)\hat{\imath} + |E_y| \tanh\left(\frac{E_y}{E_c}\right)\hat{\jmath} + \right.$$

$$\left. +|E_z| \tanh\left(\frac{E_z}{E_c}\right)\hat{k}\right) \tag{3}$$

Presented division by the modulus of $\vec{E}$ in (3), may cause a numerical error at the very initial moment of time when the electric field usually equals to zero. The additional condition that for zero electric fields also pushed the current density to zero (that is physical) allows avoiding such inconvenience. It was found that the model produces the correct results for the frequencies about tens of Hz but may show different results in comparison to the FEM model based on H-formulation for lower frequencies [5].

III. SUPERCONDUCTOR-FERROMAGNETIC INTERACTION

For our numerical modelling, we are using the Comsol Multiphysics software, which proposed an additional AC/DC module with an extended option for electromagnetic computation. More advanced representation of the ferromagnetic materials, implemented in this module, is the Jiles-Atherton model of magnetic hysteresis. It allows to combine ferromagnetic materials together with the superconductors implementation described above into the complex electromagnetic model within the same physics and avoid the vector field transition through the boundaries from one domain to another one. Two configurations of the magnetic cloak with a different arrangement of 12 mm wide SF12050AP SuperPower® tape, as shown on the Fig. 1, were modelled using this method. In both cases Fig.1 a) and b), the former diameter was 45.1 mm and the overall length was 145 mm. For the straight tape arrangement, it was possible to simplify the geometry to 2D representation. Otherwise, the helicoidal winding requires full 3D numerical model. The ferromagnetic shell in both cases was the same - the cylinder with the inner diameter 50 mm and 150 mm length, with the wall thickness 7.5 mm. It was manufactured from the $Li_{0.575}Zn_{0.4}Ti_{0.55}Fe_{1.475}O_4$ ferrite powder mixed with the epoxy resin. The 10 μm thickness of the superconducting layer was assumed for both 2D and 3D numerical models. The applied magnetic field in the 2D model has reproduced the non-uniform field generated by copper magnet used in experiments. For simplicity the 3D model uses a uniform magnetic field. Computed magnetic field distribution for the 2D model is shown on the Fig.2, the 3D model is presented by Fig.3. On both figures is shown the snapshot of computation of the AC cycle with 8 mT amplitude of the applied magnetic field, at the moment of time corresponded to ¼ period.

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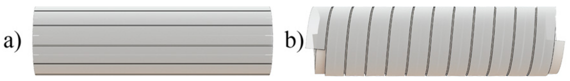

Fig. 1. Superconducting tape arrangement on the cylindrical former: a) straight tapes placed along with the former; b) tapes are wound around the former in a helicoidal way.

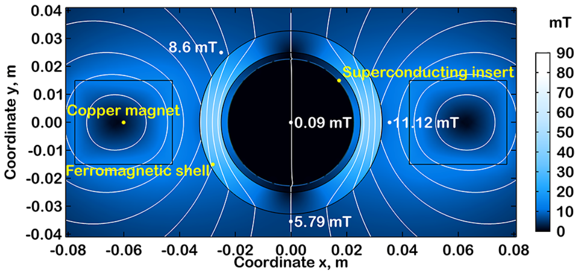

Fig. 2. Magnetic cloak with the 6 layers of straight CC tapes, placed into the copper racetrack magnet. The normal component of the magnetic flux is marked by colour.

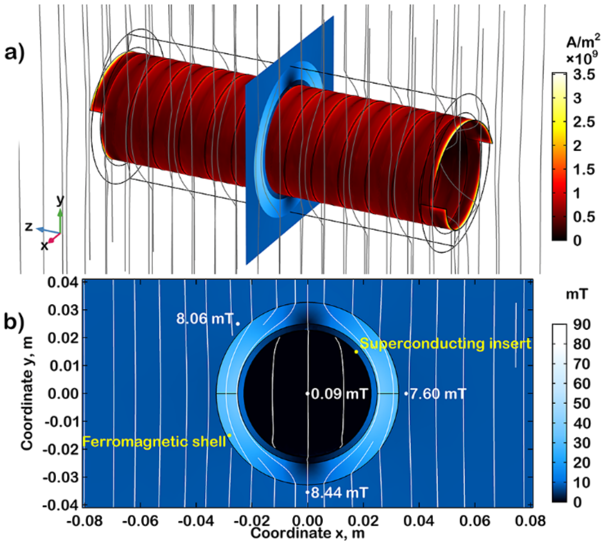

Fig. 3. Magnetic cloak with the 6 layers of helicoidally wound CC tapes. a) 3D model, where colour scale represents a normal component of current density. b) The cross-section of the magnetic cloak, normal component of the magnetic flux density is marked by colour.

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