A New Benchmark Numerical Model: The High-T_c Superconducting Dynamo

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***Abstract*—The high-T_c superconducting (HTS) dynamo is a promising device that can inject large DC supercurrents into a closed superconducting circuit. It could be used, for example, to energise rotor windings in superconducting rotating machines without the need for connection to a power supply via current leads. A number of different numerical models have now been developed as useful and cost-efficient tools to further examine and explain experimental results, as well as optimise and improve flux pump design. To adequately compare the different modelling tools available, we propose a new benchmark numerical model for the HTS modelling community: the HTS dynamo. In this work, this benchmark problem is implemented using several different methods: (1) coupled *H-A* formulation, (2) *H*-formulation + shell current, (3) segregated *H*-formulation, (4) Minimum Electromagnetic Entropy Production (MEMEP), (5) coupled *T-A* formulation, (6) integral equation and (7) volume integral equation-based equivalent circuit. These different techniques are used to solve the benchmark problem and compared in terms of computational requirements, ease of use and the solutions obtained with reference to each other, as well as experimental measurements.**

Keywords—HTS dynamo, flux pump, coated conductor, numerical modelling, HTS modelling, benchmarking

I. INTRODUCTION

The high-T_c superconducting (HTS) dynamo is a promising device to inject large DC supercurrents into a closed superconducting circuit. It could be used, for example, to energise rotor windings in superconducting rotating machines without the need for connection to a power supply via current leads [1]. Despite the extensive experimental work carried out to date, comprehensively understanding the underlying physical mechanism of such dynamo-type flux pumps has proved challenging. A number of different explanations have been proposed to explain this mechanism, but quantitatively-accurate, predictive calculations have been difficult. It was shown recently in Mataira *et al.* [2] that the open-circuit voltage can be explained well – most importantly, with good quantitative agreement – using classical electromagnetic theory. The gap dependence of the open-circuit voltage computed by Ghabeli and Pardo [3] also agrees with experiments. In [3], it is also shown that this voltage is independent of the critical current density, J_c, when the superconductor is fully saturated. The time-averaged DC output voltage obtained from an HTS dynamo arises naturally from a local rectification effect caused by overcritical eddy currents flowing within the HTS stator tape: a classical effect that has been observed in HTS materials as far back as Vysotsky *et al.* [4].

A number of different numerical models have now been developed as useful and cost-efficient tools to further examine and explain the experimental results, as well as optimise and improve flux pump design. To adequately compare the different modelling tools available, we propose a new benchmark numerical model for the HTS modelling community [5]: the HTS dynamo. In the following section, the geometry of this benchmark problem is described, as well as the relevant assumptions made to simplify the problem, including implementing the model in 2D and assuming a constant J_c for the superconducting wire.

In this work, this benchmark problem is implemented using several different methods:

- Coupled *H-A* formulation [6];
- *H*-formulation + shell current [2];
- Segregated *H*-formulation [7];
- Coupled *T-A* formulation [8, 9];
- Minimum Electromagnetic Entropy Production (MEMEP) [10, 11];
- Integral equation [12]; and
- Volume integral equation-based equivalent circuit [13].

These models are compared in terms of computational requirements, ease of use and the solutions obtained with reference to each other, as well as experimental measurements.

II. BENCHMARK PROBLEM

The geometry of the HTS dynamo benchmark problem is shown in Figure 1, assuming for simplicity the 2D (infinitely long) case. A permanent magnet (PM), of width *a* and height *b*, rotates anticlockwise past the stationary HTS stator wire at the top, and the face of the PM is located at a radius, R_{rotor}. The initial position of the PM is such that the centre of its face is at (0, $-R_{\mathrm{rotor}}$). The HTS wire has a width *e* and thickness *f* and is positioned such that its inner face is located at (0, R_{rotor} + *airgap*).

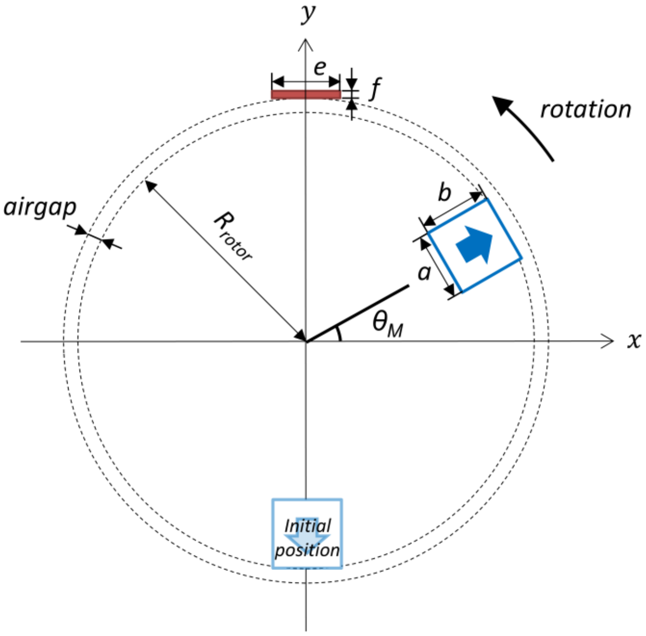

Fig. 1. Geometry of the HTS dynamo benchmark problem. A permanent magnet rotates anticlockwise past an HTS wire.

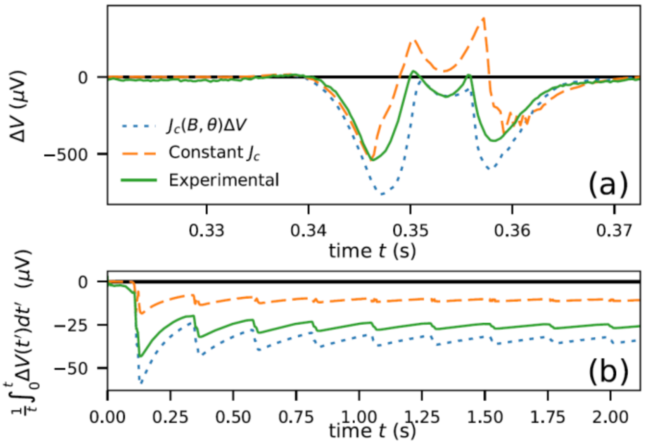

Fig. 2. (a) Open-circuit voltage waveforms for the HTS dynamo presented in [2] for the *H*-formulation + shell current model using measured in-field $J_c(B, \theta)$ data for the HTS wire or a constant J_c assumption, which are compared with experimental results. (b) Cumulative time-average for each waveform, which converge to V_{DC} in each case at $t \rightarrow \infty$.

Table I lists the assumed parameters for the model, which are based on the model presented in [2] and correspond to the experimental setup in [14]. For simplicity, J_c is assumed to be constant, since it was shown in [2] that this assumption does not impact the essential dynamics to deliver a DC voltage, i.e., a non-linear resistivity. The open-circuit and cumulative time-average voltage waveforms shown in Fig. 2 for the constant J_c case should be obtained by implementing the benchmark model.

TABLE I. HTS DYNAMO BENCHMARK ASSUMED PARAMETERS

Permanent magnet (PM)	***Width, a***	6 mm
	Height, b	12 mm
	Active length/depth, L	12.7 mm
	Remanent flux density, B_r	1.25 T
HTS stator wire	***Width, e***	12 mm
	Thickness, f	1 μm
	Critical current, I_c [self-field, 77 K]	283 A
	n value	20
Rotor radius, R_{rotor}		35 mm
Distance between PM face & HTS surface, airgap		3.7 mm
Frequency of rotation		4.25 Hz
Number of cycles		10

REFERENCES

[1] C. W. Bumby et al., “Development of a brushless HTS exciter for a 10 kW HTS synchronous generator,” Supercond. Sci. Technol., vol. 29, no. 2, Art. no. 024008, Feb. 2016.

[2] R. C. Mataira et al., “Origin of the DC output voltage from a high-T_c superconducting dynamo,” Appl. Phys. Lett., vol. 114, no. 16, Art. no. 162601, Apr. 2019.

[3] V. S. Vysotsky et al., “The possibility of using high-Tc superconducting films as elements of a rectifier,” Supercond. Sci. Technol., vol. 3, no. 5, pp. 259-262, May 1990.

[4] A. Ghabeli and E Pardo, “Modeling of Airgap Influence on DC Voltage Generation in a Dynamo-Type Flux Pump”, Supercond. Sci. Technol., at press [DOI: 10.1088/1361-6668/ab6958], Jan. 2020.

[5] HTS Modelling Workgroup: Benchmarks [http://www.htsmodelling.com/?page_id=2]

[6] R. Brambilla et al., “A Finite-Element Method Framework for Modeling Rotating Machines With Superconducting Windings,” IEEE Trans. Appl. Supercond., vol. 28, no. 5, Art. no. 5207511, Aug. 2018.

[7] L. Quéval et al., “Superconducting magnetic bearings simulation using an H-formulation finite element model,” Supercond. Sci. Technol., vol. 31, no. 8, Art no. 084001, Aug. 2018.

[8] E. Pardo, J. Ŝouc and L. Frolek, “Electromagnetic modelling of superconductors with a smooth current-voltage relation: variational principle and coils from a few turns to large magnets,” Supercond. Sci. Technol., vol. 28, no. 4, Art. no. 044003, Apr. 2015.

[9] E. Pardo and M. Kapolka, “3D computation of non-linear eddy currents: Variational method and superconducting cubic bulk,” J. Comput. Phys., vol. 344, pp. 339-363, Sep. 2017.

[10] H. Zhang et al., “An efficient 3D finite element method model based on the *T-A* formulation for superconducting coated conductors,” Supercond. Sci. Technol., vol. 30, no. 2, Art. no. 024005, Feb. 2017.

[11] T. Benkel et al., “T-A Formulation to Model Electrical Machines with HTS Coated Conductor Coils,” IEEE Trans. Appl. Supercond., vol. 30, no. 6, Art. no. 5205807, Sep. 2020.

[12] R. Brambilla et al., “Integral equations for the current density in thin conductors and their solution by the finite-element method,” Supercond. Sci. Technol., vol. 31, no. 10, Art. no. 105008, Oct. 2008.

[13] A. Morandi and M. Fabbri, “A unified approach to the power law and critical state modeling of superconductors in 2D,” Supercond. Sci. Technol., vol. 28, no. 2, Art. no. 024004, Feb. 2015.

[14] R. A. Badcock et al., “Impact of Magnet Geometry on Output of a Dynamo-Type HTS Flux Pump,” IEEE Trans. Appl. Supercond., vol. 27, no. 4, Art. no. 5200905, June 2017.