Contribution to the modeling of eddy current losses in HTS tapes

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***Abstract*—This paper presents a simple semi-analytical modeling to evaluate the eddy current losses development in multifilamentary high temperature superconductors (HTS) tapes submitted to external time varying magnetic fields.**

***Keywords*—Analytical modeling, eddy current losses , HTS tapes.**

I. INTRODUCTION

AC losses evaluation in high-temperature superconductors (HTS) is essential for the development of HTS devices. Their experimental characterization is a delicate operation, which is generally limited to relatively low frequencies [1]. Modeling is thus generally unavoidable. Due to the geometrical complexity of the HTS structures, and to the nonlinearity and anisotropy of their electromagnetic properties, 3D time domain numerical modeling approaches are often necessary [2]. These approaches are however time consuming, and present numerical instabilities, in particular when the frequency increases.

In this work, we developed a simple analytical approach to study the eddy current losses in multifilamentary HTS tapes submitted to external time varying magnetic fields. The tape is represented by a homogenized anisotropic and nonlinear conductivity tensor, where the power law is used as the E(J) characteristic of the HTS tape, and the critical current density dependence on the magnetic field is taken into account by using the Kim's model [3]. The Biot-Savart formula is used to evaluate the magnetic reaction of the tape. The model is validated by comparison to the Campbell's model [4].

II. MODEL DEVELOPMENT

Consider a HTS tape submitted to an external applied magnetic field (B^a), oriented normal to its large surface (Fig.1).

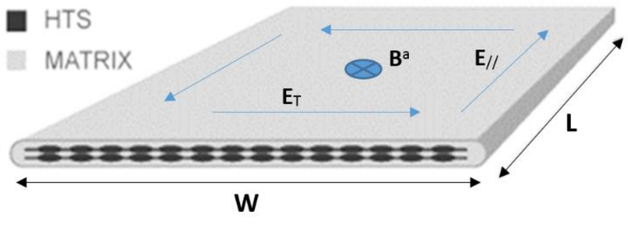

Fig. 1. The modelled system.

The applied field vary sinusoidally in time with a variable angular frequency ω, and the tape length is considered to be smaller than the critical length. The homogenized conductivity tensor of the tape is given by (1), where $E_{//}$ and $J_{//}$ are the electric field and electric current density parallel to the HTS filaments direction, η_s is the HTS volume fraction in the tape, and σ_m is the matrix conductivity [5]. The power law, given by (2),is used to express the $E_{//}(J_{//})$ characteristic, where J_{c0} is the critical current density at zero magnetic field, B is the magnetic flux density, and B_0 is a characteristic constant of the material. The relation between the transverse components of the electric current density and electric field, denoted J_T and E_T respectively, is considered linear.

$$\overline{\overline{\sigma}}(J)=\begin{bmatrix} \eta_s\dfrac{J_{//}}{E_{//}}+(1-\eta_s)\sigma_m & 0 & 0 \\ 0 & \dfrac{(1+\eta_s)}{(1-\eta_s)}\sigma_m & 0 \\ 0 & 0 & \dfrac{(1+\eta_s)}{(1-\eta_s)}\sigma_m \end{bmatrix} \quad (1)$$

$$E_{//}=E_c\left(\frac{J_{//}}{J_{c0}(1+\frac{|B|}{B_0})^{-\beta}}\right)^n \quad (2)$$

The current induced in the tape, as well as the circulation of the electric field are evaluated on an average contour as shown in figure 2. The applied magnetic field and the magnetic field of reaction of the tape are calculated in the center of the latter. The magnetic reaction is also assumed to be normal to the tape surface.

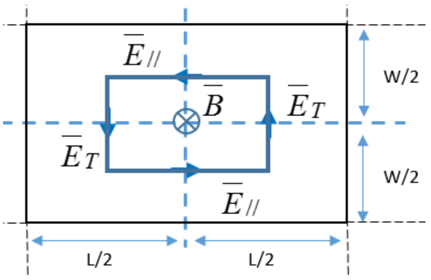

Fig. 2. Circulation of the electric field on an average contour in the tape.

The Biot-Savat law is used to compute the magnetic reaction (B^r) due to the eddy currents (I) in the tape, leading to the relation (3) when the latter is calculated at the center of the tape. In (3), μ_0 is the free space permeability.

$$B^r = \frac{4\mu_0 I}{\pi}\frac{L+W}{LW} \tag{3}$$

The computation is achieved iteratively according to the following algorithm:

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Input: L, W, T, $B^a$, $\sigma_m$, $\eta_s$, $B_0$, n, $\beta$, $\omega$
$\sigma_T = (1+\eta_s)(1-\eta_s)^{-1}\sigma_m$
Initialization: $E_{//}^0 = 0$ , $E_T^0 = -\omega B^a \frac{L}{4}$ ,
$J_c^0 = \frac{J_{c0}}{\left(1+B^a B_0^{-1}\right)^{\beta}}$ , $I^0 = \frac{TL}{2}\sigma_T E_T^0$

Do for i=1, 2 …
    $E_{//}^i = E_c\left[\frac{2\left\|I^{i-1}\right\|}{WTJ_c^{i-1}}\right]^n$
    $B^i = B^a + \frac{4\mu_0}{\pi}\frac{L+W}{LW}I^{i-1}$
    $J_c^i = \frac{J_{c0}}{\left(1+B^i B_0^{-1}\right)^{\beta}}$
    $E_T^i = -\omega B^i \frac{L}{4} - \frac{L}{W}E_{//}^i$
    $I^i = \frac{TL}{2}\sigma_T E_T^i$
Until convergence (test on the current)

$P = 0.5 \times I(E_{//}L + E_T W)$
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Fig. 3. The itterative solving algorithm.

III. RESULTS AND DISCUSSIONS

Table I gives the simulation parameters. Figure 4 present a comparison with the Campbell's model on the evolution of the eddy current losses with the frequency (B^a=1mT). A good agreement is found between the two results.

TABLE I. SIMULATION PARAMETERS

Parameter	*Description*	*Value*
L | W | T	Tape dimensions	5mm | 3.3mm | 0.25mm
σ_T	Transverse conductivity	5.8×10^7 S/m
J_{c0}	Zero field critical current	7.52×10^7 A/m^2
n	Creep exponent	11
Ec	Critical electric field	10^{-4} V/m
B_0 | β	Kim's model parameters	0.14T | 2.28
B^a	Magnitude of the applied field	10^{-3}T

As expected, the increase of the applied magnetic field increases the gap between the two models, as shown in figure 5. Indeed, the increase of the applied magnetic field decreases the critical current density, leading to an increase of the losses, which is not taken into account in the Campbell's model.

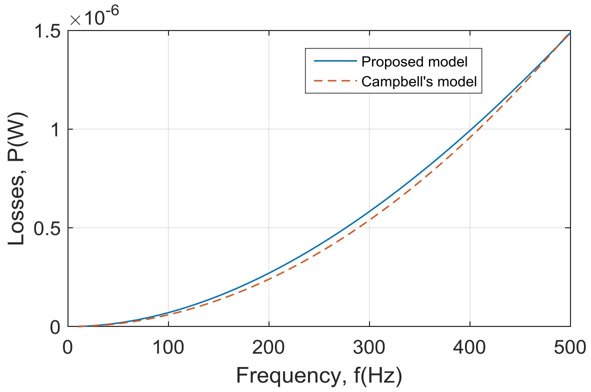

Fig. 4. Eddy current losses evolution with the frequency (B^a=1mT).

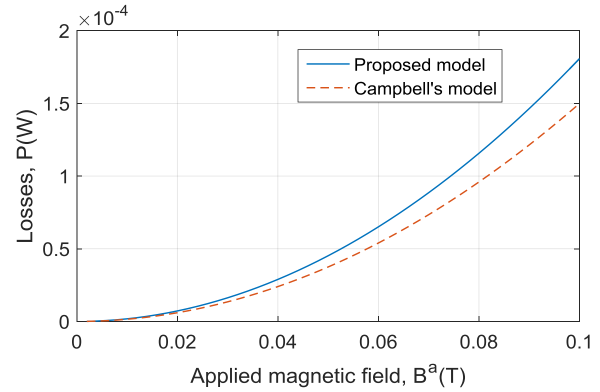

Fig. 5. Eddy current losses evolution with the applied magnetic field (f=50Hz).

IV. CONCLUSION

A very simple and rapid semi-analytical model is developed for eddy current losses evaluation in HTS tapes. As formulated, the model is limited to small lengths, but it can be extended to model larges lengths by adapting the discretization.

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