Dynamic modelling of the high temperature superconducting maglev system using different *E*-*J* relationships

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***Abstract*—Three *E-J* relationships describing the electromagnetic behavior of the high-temperature superconducting (HTS) bulk over a permanent magnet guideway (PMG) are discussed in this paper. They are the power law model (PLM), the flux flow and creep model (FFCM), and the flux flow model (FFM). With the aid of the finite element software COMSOL Multiphysics, these models were successfully established combining with the AC/DC module and the Heat Transfer module. The irregularity of the surface of the real PMG is considered in the modeling by applying a small-amplitude vertical vibration function to the permanent magnet guideway which is built as a geometric entity. The movement of the guideway is obtained by setting moving mesh. In view of the application of high-speed HTS magnetic levitation (Maglev) system, the dynamic response of the levitation force, the temperature distribution and the current density distribution of the HTS bulk under different vibration frequencies was analyzed. This work can provide a reference for the modeling of the dynamic response of the electromagnetic-thermo-force characteristics of the HTS Maglev system.**

Keywords—high temperature superconducting bulk, power law mode, flux flow model, flux flow and creep model, levitation force, high-speed

I. INTRODUCTION

Since the discovery of high temperature superconductors (HTSCs) with higher critical temperature than 77 K (liquid nitrogen temperature zone in open air) in 1986, high temperature superconducting (HTS) applications have being extensive in the field of magnetic levitation (Maglev) transports due to its advantages as self-stabilization, free of active control and energy conservation, etc. For the researches on the HTS Maglev systems, the electromagnetic field problem in the applied permanent magnet guideway (PMG) field is the key issue and the superconducting ***E-J*** relationship is the origin of the research. There have been several popular ***E-J*** relationships: the critical state model (CSM), the power law model (PLM), the flux flow and creep model (FFCM) and the flux flow model (FFM). Thereinto, the critical state model (CSM) was always selected for studying magnetic hysteresis and AC losses of superconducting materials in the past time. The power law model (PLM) is currently widely used due to its continuity and simplicity. While the flux flow and creep model (FFCM)fully simulates the flux motion inside the superconductors, accordingly in which many parameters are included which makes it complicated but exact.

In previous works, there were some literatures like Ref. [1] investigating the influence of thermal effect and anisotropy of material resistivity on ac losses considering different HTSC's constitutive laws. More recently, F. Sirois et al. [2] have compared the magnetization and losses of slab or strip superconductor with the CSM and PLM. For the HTSC/PM system, Yoshida et al. compared the magnetic force of the superconductor based on the FFCM and the CSM [3]. Despite the popularity of these models, there is no substantial investigations discussing the adaptability of different HTSC's ***E-J*** relationships for the HTS Maglev systems which is featured by the designated PMG field, especially under alternating magnetic field with high frequency.

In this paper, the PLM, the FFCM as well as the FFM are used for calculating the electromagnetic characteristics of the HTS bulk under dynamic operation. Models have been established in the COMSOL Multiphysics software, with the Magnetic Field Formulation (mfh) interface in the AC/DC module and the Heat Transfer module selected. This research indicates that the magnetic-thermo-force coupling method based on these three ***E-J*** relationships can be successfully stablished.

II. THEORETICAL MODEL

A. Modelling

The geometry and the displacement of the permanent magnet guideway are shown as Fig.1.

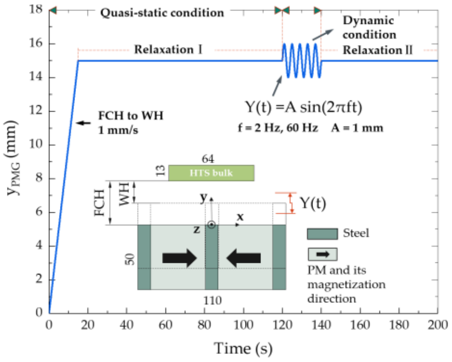

Fig. 1. Geometrical configuration of the high temperature superconducting maglev system and the vertical displacement function of the guideway [4].

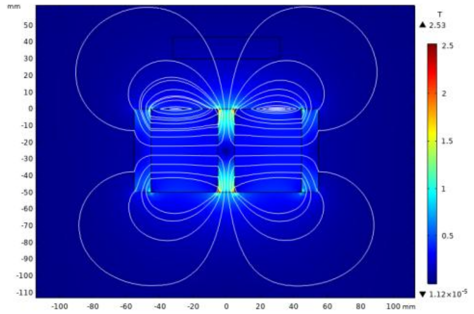

Fig. 2. Distribution of the magnetic field of the HTS maglev system at the begining.

In the modeling, the external magnetic field fluctuation of the HTS bulk is simulated by applying a sine function to the guideway. More details about the conversion between the simulated vibration frequency of the guideway and the actual linear velocity of the vehicle can be found in [4].

B. *Equations*

Maxwell equations:

$$\mu\frac{\partial \boldsymbol{H}}{\partial t}+\nabla\times\boldsymbol{E}=0 \tag{1}$$

where $\boldsymbol{E}=\rho\boldsymbol{J}$, and $\boldsymbol{J}=\nabla\times\boldsymbol{H}$. For the high temperature superconducting material, the strong nonlinear ***E-J*** relationships are chosen as the power law model (2), the flux flow model (3), and the flux flow and creep model (4):

$$\boldsymbol{E}=E_c\left(\frac{\boldsymbol{J}}{J_c}\right)^n, \tag{2}$$

$$\boldsymbol{E}=\begin{cases}0 & 0\le|\boldsymbol{J}|<J_c\\ \rho_f J_c\left(\frac{|\boldsymbol{J}|}{J_c}-1\right) & |\boldsymbol{J}|\ge J_c\end{cases}, \tag{3}$$

$$\boldsymbol{E}=\begin{cases}2\rho_c J_c\sinh\left(\frac{U_0}{kT}\frac{|\boldsymbol{J}|}{J_c}\right)\exp\left(-\frac{U_0}{kT}\right)\frac{\boldsymbol{J}}{|\boldsymbol{J}|} & 0\le|\boldsymbol{J}|\le J_c\\ \left(\rho_c J_c+\rho_f J_c\left(\frac{|\boldsymbol{J}|}{J_c}-1\right)\right)\frac{\boldsymbol{J}}{|\boldsymbol{J}|} & |\boldsymbol{J}|>J_c\end{cases}, \tag{4}$$

where $\boldsymbol{J}_\mathrm{c}$ is the critical current density, which is the dependence of temperature and magnetic field:

$$J_\mathrm{c}=J_{\mathrm{c}0}\left(1-\left(\frac{T}{T_\mathrm{c}}\right)^2\right)^2\frac{B_0}{|B|+B_0} \tag{5}$$

The thermal equilibrium equation is expressed as:

$$C_\mathrm{p}\frac{\partial T}{\partial t}-\nabla\cdot\left(\lambda\nabla T\right)=EJ \tag{6}$$

The convective boundary condition is applied:

$$\lambda\frac{\partial T}{\partial n}+h(T-T_0) \tag{7}$$

The levitation force is obtained by the Lorenz force formula:

$$F_y\,(t)=\int_S B\times J\;\mathrm{d}S \tag{8}$$

III. RESULTS

From Fig. 3-5, we can see that the three ***E-J*** relationships can successfully simulate the electromagnetic characteristics of the HTS maglev system by COMSOL software.

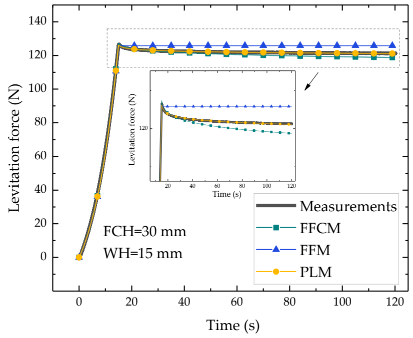

Fig. 3. Comparison of the quasi-static levitation force versus time by the measurement and the simulations by the FFCM, the FFM, and the PLM.

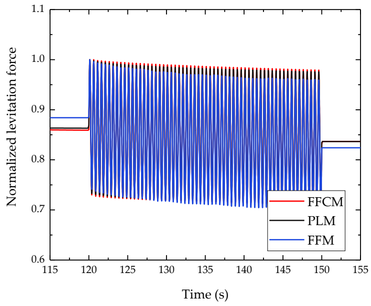

Fig. 4. Normalized dynamic levitation force calculated by different ***E-J*** relationships under vibration frequency of 2 Hz.

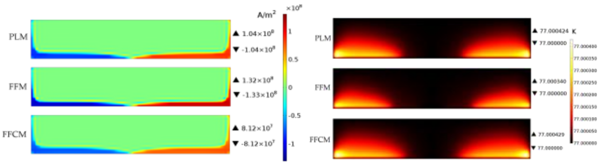

Fig. 5. Current density distribution (left) and temperature distribution (right) of the HTS bulk at 15 s (the 15 mm working height) under vibration frequency of 20 Hz using different ***E-J*** relationships

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