The Campbell Model as a Tool to Introduce Students to AC Loss Calculation in Superconductors

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Abstract—**A finite-element model implemented in the open-source FreeFem++ program is presented, with the aim of introducing students to the calculation of AC losses in superconductors. With this simple approach, students can learn about the critical state model used to describe the macroscopic electromagnetic behavior of superconductors and the importance of several factors influencing the AC losses of superconductors.**

Keywords—AC losses, critical state model, finite-element method, Maxwell's equations, superconductors.

I. INTRODUCTION

In 2007, Prof. A. M. Campbell presented a numerical model for the solution of the critical state in superconductors [1]. The physics behind the model is the force-displacement curve of the flux lines. In practice, the model solves Faraday's equation with a current density **J** which can take values in the interval $[-J_c, J_c]$ and whose sign depends on that of the magnetic vector potential.

For simple problems, a single-step calculation can be used for evaluating the cyclic losses of the superconductors, when this is subjected to a transport AC current or an external AC magnetic field. Despite the limitation of the scenarios that can be simulated, students can gain insights in the problem of AC loss calculations of superconductors by means of this simple approach.

Here, we present the implementation of the model in the open-source finite-element software FreeFem++ [2], which can be easily installed as a stand-alone application on different platforms. This enables students to run the scripts on their computers also outside the lecture and review the problems at their own pace.

II. NUMERICAL MODEL

The model aims at calculating the current density and magnetic field distribution in the transverse cross-section of an infinitely long superconductor. The simulated domain can therefore be limited to two dimensions. Fig. 1 shows the geometry and the mesh of a typical problem: a superconductor of elliptical cross-section surrounded by an air domain. The magnetic flux density has only two components B_x, B_y, the
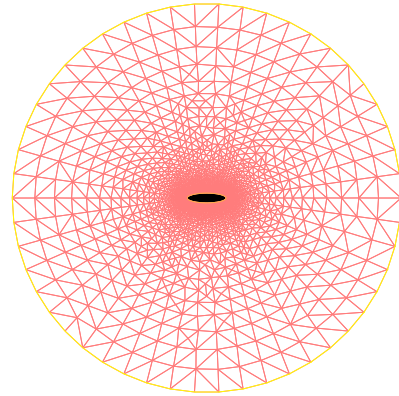

Fig. 1. Geometry and mesh for the 2D simulation of an elliptical superconductor surrounded by an air domain.

current density and magnetic vector potential have only the z component and are therefore scalars. The current density distribution in the superconductor is calculated by solving Faraday's equation

$$\nabla^2 A = -\mu_0 J \tag{1}$$

where J takes values approximating the transition between $+J_c$ and $-J_c$ or zero:

$$J = J_c \text{erf}(-A/A_r). \tag{2}$$

In (2), erf is the error function [3] and A_r is a parameter defining the steepness of the transition. In this work, we used $A_r = 1 \times 10^{-7}\,\text{Wb m}^{-1}$. Instead of erf, other functions based on exponential [1] or hyperbolic arctangent [4] can be used. The sign of J is given by the opposite of that of the magnetic vector potential.

An external uniform magnetic field is applied by setting the boundary conditions for A on the outer air boundary. In 2D Cartesian coordinates, $\mathbf{B} = \nabla \times \mathbf{A}$ becomes $B_x = \partial A/\partial y$, $B_y = -\partial A/\partial y$. For example, in order obtain a magnetic field B_0 along y, it is sufficient to impose $A = -B_0 x$ as boundary condition.

In a similar way, a transport current is imposed by setting a constant value for the magnetic vector potential on the
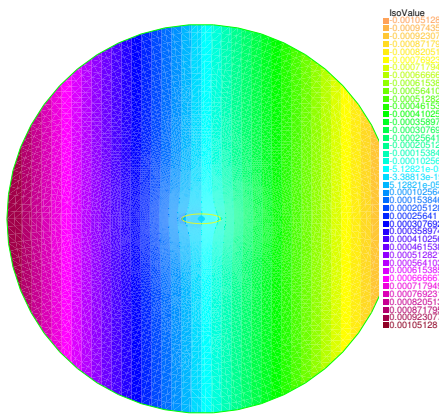
Fig. 2. Magnetic vector potential distribution for a vertical magnetic field of $30\,\mathrm{mT}$, obtained with the condition $A = -0.03x$ applied on the outer boundary.
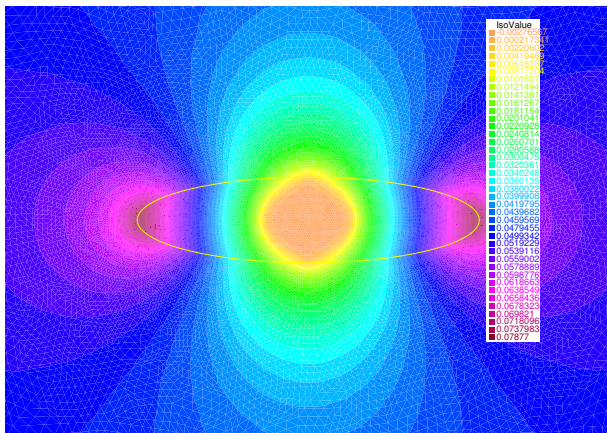

Fig. 4. Current density distribution in the superconductor for the conditions of Fig. 2.

Fig. 3. Magnetic flux density distribution in and around the superconductor for the conditions of Fig. 2.

- Vary the amplitude of the magnetic field and plot the losses as a function of the amplitude. Observe the non-linear dependence and the change of slope (from $Q \approx B^3$ to $Q \approx B^1$), occurring when the field fully penetrates the superconductor.
- Change the orientation of the field by $90°$ and observe the large change in the losses.
- Vary the aspect ratio of the ellipse (without changing its area and J_c) and observe its influence on the losses.

IV. Conclusion

The presented finite-element model, implemented in the open-source FreeFem++ program, allows introducing students to calculation of AC losses in superconductors. The students can learn about the influence of the conductor's geometry and of field orientation on the AC losses, which is an important topic for the design of efficient superconducting applications.

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boundary. The actual value of the current can be calculated in the post-processing.

The cyclic losses can be computed by knowing the current density and the magnetic vector potential at the peak of the excitation A_p as (see equation (9) in [1] and equation (17) in [5] for details).

III. Example and Possible Exercises

As an example, we show the case of a superconducting tape of elliptical cross-section subjected to a magnetic field of $30\,\mathrm{mT}$ perpendicular to the major axis of the ellipse. Such magnetic field is obtained by applying the condition $A = -0.03x$ applied on the outer boundary. Fig. 2 shows the distribution of magnetic vector potential, which bends around the superconductor. As a result of the demagnetization effect, the magnetic field in the superconductor is locally higher than the applied one, reaching a maximum value of $78\,\mathrm{mT}$, as shown in Fig. 3. A typical current density distribution, with values equal to $\pm J_\mathrm{c}$ in the regions penetrated by the magnetic field, is shown in Fig. 4. Possible exercises consist in asking the students to run 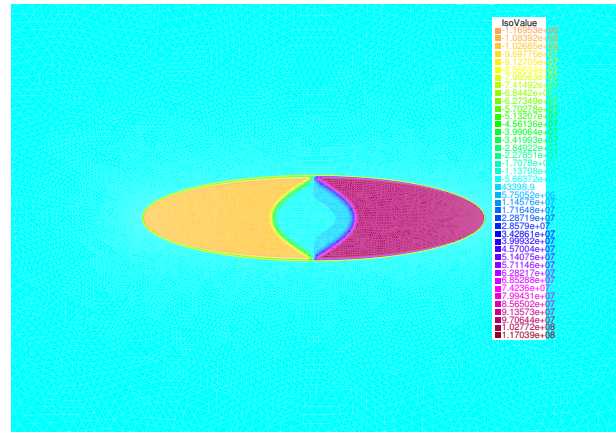simulations for different cases, as follows: