Coupled multiphysics modelling of the thermal-magnetic-mechanical instability in bulk superconductors during magnetization

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***Abstract*—Magnetization is one of the most significant challenges for the application of bulk superconductors. During the pulsed field magnetization process, large numbers of quantized magnetic flux vortices can rush in or out of the superconducting sample, which generates a significant amount of heat and can lead to the thermomagnetic instability of the bulk superconductor. The large Lorentz force and the sharp temperature rise can result in mechanical failure (or instability), which also hinders the application of bulk superconductor. In addition, thermomagnetic instability and mechanical failure may couple with each other. Thus, it is challenging to simulate the magnetization of bulk superconductors used as high field magnets. In this paper, a numerical simulation framework based on the coupled H-formulation for the electromagnetic behavior of superconductors, the heat diffusion equation, and the phase-field model for the failure of solids is proposed and implemented to simulate the thermal-magnetic-mechanical instability behavior of bulk superconductors during the magnetization process. The thermal-magnetic-mechanical instability-induced dendritic flux patterns are obtained and the corresponding material damage are evaluated through these numerical simulations. The magnetic field, current density, temperature, stress/strain field distribution, and the material failure within the bulk sample in the high magnetic field magnetization process will also be presented.**

Keywords—multiphysical modelling, bulk superconductor, magnetization, mechanical failure, pulsed field magnetization

I. Introduction

Large, single-grain $REBa_2Cu_3O_{7-\delta}$ (REBCO) bulk high temperature superconductors, which can trap magnetic fields up to 17.6 T at 26 K [1], have demonstrated significant potential for use in a variety of engineering applications [2]. However, REBCO is a ceramic material, brittle in nature with poor mechanical properties [3]. Mechanical stress is inevitable for their practical use, which needs special attention especially for high magnetic field applications. The range of applications of bulk superconductors is limited by their poor mechanical stability. Fracture and microscopic damage have been found extensively in REBCO bulk superconductor magnets during the high magnetic field magnetization process. Moreover, the motion of magnetic flux during the magnetization process dissipates a lot of energy and can lead to thermomagnetic instability. Thermomagnetic instability induced flux jumps are frequently observed during pulsed field magnetization.

Giant magnetostriction (up to 10^{-4}) has been measured by Ikuta et al [4]. Cracking of a mini-magnet composed of four YBCO discs under an applied field of 14 T at 49 K has been observed by Ren et al [5]. Takahashi et al [6] observed fracture behavior of an EuBaCuO ring bulk during field-cooled magnetization and numerically simulated the mechanical stress. Ainslie et al developed a modelling framework that couples electromagnetic, thermal and structural mechanics, and analyzed the mechanical stresses and flux jumps of bulk superconductor during magnetization [7, 8]. Research group lead by Zhou has made significant contributions to modelling the mechanical behavior of bulk superconductors [9], and has numerically simulated the fracture [10] and flux jump [11] behavior of the bulk superconductor.

In this paper, a theoretical modelling framework which couples the *H*-formulation, the heat diffusion equation, and the phase-field fracture model is proposed to simulate the coupled electromagnetic-thermal-mechanical behavior of bulk superconductors during the magnetization process.

II. Theoretical Modeling and Numerical Methods

A. Electromagnetic modelling

The electromagnetic behavior of the bulk superconductor is governed by the following Maxwell's equations:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B} / \partial t \tag{2}$$

where $\boldsymbol{H}$ is the magnetic field strength, $\boldsymbol{J}$ represents the current density, $\boldsymbol{E}$ is the electric field. The magnetic flux density $\boldsymbol{B}$ is related to $\boldsymbol{H}$ by $\boldsymbol{B}=\mu_0\boldsymbol{H}$, where μ_0 is the vacuum permeability. The relationship between the electric field and the current density is given by the $\boldsymbol{E}$-$\boldsymbol{J}$ power law,

$$\boldsymbol{E} = E_0 \left| \boldsymbol{J} / J_c \right|^{n-1} \boldsymbol{J} / J_c \tag{3}$$

where E_0 is the electric field constant (E_0=1×10^{-4} V/m), J_c is the critical current density and defined as a function of the temperature T, the magnetic flux density $\boldsymbol{B}$ and the phase field variable ϕ by:

$$J_c(B,T,\phi) = (1-\phi)^2 \left(J_{c1} \exp\left(-B/B_L\right) + J_{c2} B / B_{\max} \exp\left[\frac{1}{y}\left(1-\left(B/B_{\max}\right)^y\right) \right] \right) \tag{4}$$

in which J_{c1}, J_{c2}, B_L, B_{max} and y are temperature dependent parameters and can be found in [7] and references therein.

B. Heat diffusion equation

The thermal behavior is described by

$$c \partial T / \partial t = \nabla \cdot (\kappa \nabla T) - h(T - T_0) + \boldsymbol{J} \cdot \boldsymbol{E} \tag{5}$$

where κ is the thermal conductivity, c is the specific heat, T_0 is the ambient temperature and h is the heat transfer coefficient.

C. Phase-field modelling

In the phase-field fracture model, the coupled governing equations for displacement and the phase field are:

$$\nabla\cdot\boldsymbol{\sigma}+\boldsymbol{b}=\rho\partial^2\boldsymbol{u}/\partial t^2 \;\; in\;\Omega \tag{6}$$

$$(G_c/2l_0+\mathcal{H})\phi-G_cl_0/2\Delta\phi=\mathcal{H} \;\;\; in\;\Omega \tag{7}$$

The boundary conditions along the domain are given by:

$$\boldsymbol{\sigma}\cdot\boldsymbol{n}=\boldsymbol{t} \;\; on\;\partial\Omega_t \tag{8}$$

$$\boldsymbol{u}=\bar{\boldsymbol{u}} \;\; on\;\partial\Omega_u \tag{9}$$

$$\nabla\phi\cdot\boldsymbol{n}=0 \;\; on\;\Gamma \tag{10}$$

where $\boldsymbol{n}$ denotes the outward normal vector to the boundary, and $\bar{\boldsymbol{u}}$ and $\boldsymbol{t}$ are the prescribed displacement and traction force on the boundary, respectively.

In our numerical simulations, a multiphysical field coupling scheme based on the finite element method is proposed and applied in the commercial software COMSOL Multiphysics. The electromagnetic equations are solved using a general form PDE, the phase field fracture model is adapted into the Solid Mechanics module and the heat equation is applied in the Heat Transfer module.

III. RESULTS AND DISCUSSIONS

Here, we present the numerical simulation results for the thermal-magnetic-mechanical instability behavior of a cylindrical bulk superconductors with reinforcement during pulsed field magnetization. The variations of electromagnetic fields in the thickness direction is neglected, and a 2D plane-strain problem is considered in our simulation. The radius of the bulk sample is 10 mm, and thickness of the stainless-steel reinforcement ring is 2 mm. The practical experimental waveform for the applied external magnetic field B_{app} observed during the PFM is taken as: $B_{app}=B_{max}\frac{t}{\tau}\exp\left(1-\frac{t}{\tau}\right)$. We take τ = 12 ms in the simulation, typical of experiments.

Fig. 1 shows the time evolution of magnetic flux (a_1)- (d_1), the temperature (a_2)- (d_2), the phase field (a_3)- (d_3), and the critical current density (a_4)- (d_4) during a typical pulsed field magnetization process with B_{max} = 7 T at 40 K. It is clearly shown that dendritic flux avalanches (similar to that found in thin disks [12]) develop within the bulk sample from the periphery as the applied field rises to about 4 T at 40 K.

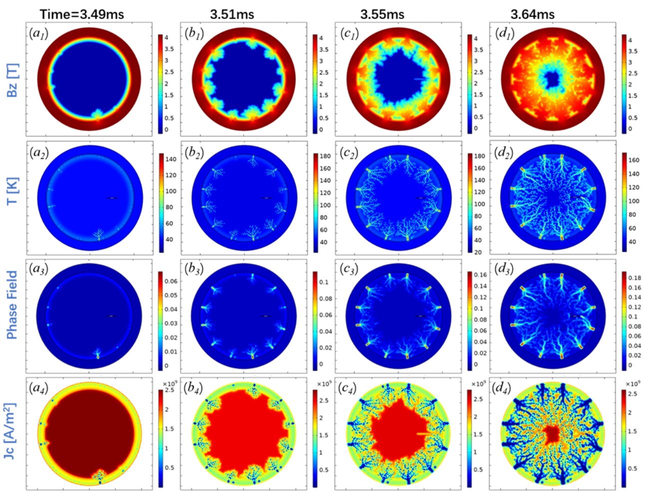

Fig. 1. Magnetic flux, temperature, phase field and the critical current density distribution within the bulk superconductor with reinforcement under pulsed field magnetization (B_{max} = 7 T at 40 K).

In Fig. 2, we present the magnetization jump curve and the maximum temperature of the bulk superconductor during pulsed field magnetization. It is shown that the magnetization changes sharply during the avalanche process, and the temperature can even rise up to 180 K, which may induce mechanical damage in the bulk (as shown in Fig.1 (a_3)-(d_3)).

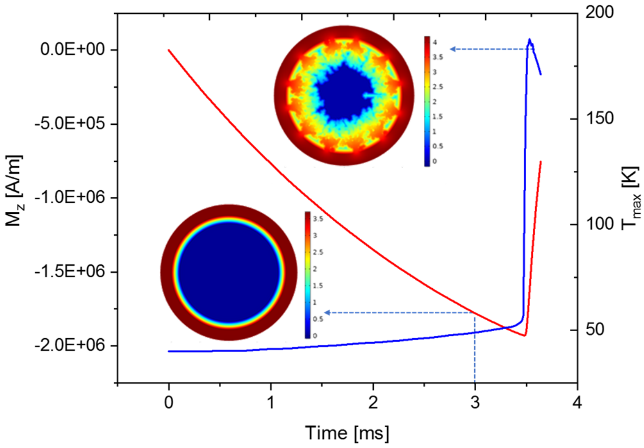

Fig. 2. Magnetization jump and the maximum temperature of the bulk superconductor under pulsed field magnetization with maximum magnetic field B_{max} = 7 T at 40 K.

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