A volume integral equation based equivalent circuit for 3D calculation of the levitation force

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***Abstract*— The volume integral equation method based on the current vector potential approximated by means of edge element basis function is a well-established approach for 3D eddy currents computation. Within this approach the unknowns of the problem are expressed in terms of projection of the current vector potential along the edges of the discretization and a solving system is obtained based on tree/cotree decomposition of the primal graph. Due to its integral nature, an interpretation of the method in terms of circuits can be made. In this paper, the link between the volume integral equation method and the circuit is investigated and the numerical model is formulated in terms of a distributed parameter equivalent RL circuit. The numerical equivalent circuit is applied for investigating the lateral levitation force between a PM and a bulk superconductor with axial symmetry. The comparison between calculated and measured values is presented.**

Keywords—flux pump, HTS dynamo, numerical modelling, HTS modelling, benchmarking

I. Introduction

The volume integral equation method based on the current vector potential approximated by means edge element basis function is a well-established approach for 3D eddy currents computation [1]-[3]. Within this approach the unknowns of the problem are expressed in terms of projection of the current vector potential along the edges of the discretization and a solving system is obtained based on tree/cotree decomposition of the primal graph. Due to its integral nature, an interpretation of the method in terms of circuits can be made. In this paper, the link between the volume integral equation method and the circuit is investigated and the numerical model is formulated in terms of a distributed parameter equivalent RL circuit. It is shown that the circuit view can be used as a guide for dealing with multiply connected geometries and/or external generators.

The numerical equivalent circuit is applied for investigating the lateral levitation force between a cylindrical PM and a cylindrical bulk superconductor. Due to the lateral movement of the PM the problem is fully 3D. The comparison between calculated and measured values is discussed in the paper. It is also shown that the volume integral equation based equivalent circuit approach can be used for analysing a wide variety of HTS problems of practical interest.

II. Mathematical Model – The Distributed Equivalent Circuit

The A−φ formulation is used for expressing the electric field E at any point of the superconductor bulk by means of the Faraday's law. The current density J of each element is expressed based on the current through the faces by means of facet element shape functions. This is equivalent to say that J is expressed via the current vector potential approximated by means of edge element basis functions. A maximal set of independent cordless-loops is found on the dual graph. Since in the considered case the superconductor domain is simply connected the maximal set can be obtained via tree-cotree decomposition of the primal graph. Each of the cordless-loops is a closed chain of elements sharing one co-tree edge lying at the interior of the domain. A cordless-loop shape function is introduced by envisioning that a current circulates all over the chain. As a result, the current density at any point is expressed as a linear combination of loop shape functions. A weak solution of the numerical problem is finally obtained by applying the weighted residual approach to Faraday's law expressed in terms of A−φ potentials. In particular, by assuming as independent weighting functions the maximal set of cordless-loop shape function, the following matrix equation is obtained:

$$\begin{aligned}
&\mathbf{M}\frac{d}{dt}\mathbf{I} = -\mathbf{R}\mathbf{I} + \mathbf{V}^{\text{ext}} \\
&\textit{with} \\
&M_{ij} = \int_{\tau_i}\int_{\tau_j} \frac{\mathbf{U}_i(\mathbf{r})\cdot\mathbf{U}_j(\mathbf{r}')}{\mathbf{r}-\mathbf{r}'}\, d^3r'\, d^3r \\
&R_{ij} = \int_{\tau_i} \rho\, \mathbf{U}_i(\mathbf{r})\cdot\mathbf{U}_j(\mathbf{r})\, d^3r \\
&V_i = \int_{\tau_i} \mathbf{U}_i(\mathbf{r})\cdot\mathbf{v}\times\mathbf{B}^{PM}(\mathbf{r})\, d^3r
\end{aligned} \tag{1}$$

where τ_i and τ_j are the volumes of loop i and loop j respectively, **I** is the vector of currents of the loops, $\mathbf{U}_i$ and $\mathbf{U}_j$ are the loop shape functions, **v** is the velocity of the PM and

$\mathbf{B}^{PM}$is the field produced. The electric scalar potential gives no contribution to (1) since any weighting function is divergence-free and has no component. The model depicted so far is essentially the vector field correspondent of the mesh (cordless-loop) currents method for solving electric circuits. Any of the equations can be seen as the voltage balance of loop i where M_{ij} is the self/mutual inductance of loops i and j considered as massive circuits, R_{ij} is the resistance shared and V_i^{ext} is the electromotive force produced on loop i by the moving magnet. Once that equation (1) is solved the current density at any point of the domain can be reconstructed and the levitation force $\mathbf{F}$ can be finally calculated as

$$\mathbf{F} = \int_{\tau_i} \mathbf{J} \times \mathbf{B}^{PM} \, d^3r \tag{2}$$

III. Numerical and Experimental Results

In case of time changing stationary (not moving) external magnetic field source the 3D equivalent circuit model described in section I has been already validated against experimental data and/or numerical benchmark. The 2D axisymmetric version of the model has also been validated against experimental data as shown in Fig. 1. In particular, the figure shows the comparison of the experimental and the numerical levitation force obtained via the 2D axisymmetric version. The validation of the 3D equivalent circuit model in the case of PM moving in the lateral direction is in progress and will be discussed in the paper. In particular, the numerical values of the lateral levitation force obtained via the 3D model will be compared with the measured data shown in Fig. 2.

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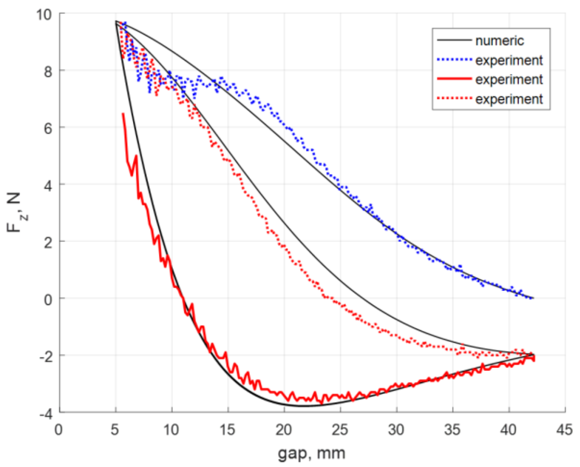

Fig. 1. Comparison of numeric and experimental axial levitation force in case of axial movement of the PM. Numerical results are obtained via the 2D axisymmetric version of the equivalent circuit model.

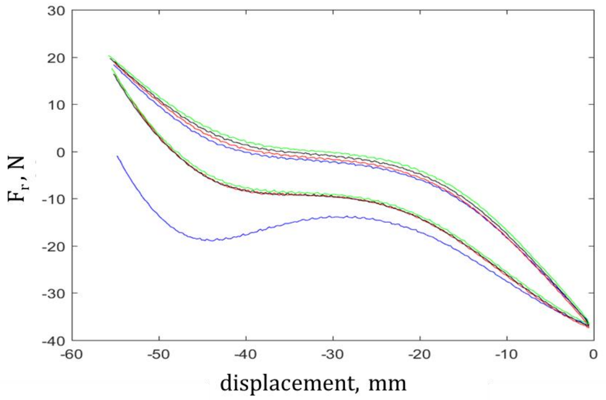

Fig. 2. Experimental lateral levitation force in case of lateral movement of the PM.

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