$\mathbf{h}$-ϕ Finite-Element Formulation for Modelling Thin Superconducting Layers

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***Abstract*—A nonlinear time-domain approach is proposed to model thin superconducting layers using an $\mathbf{h}$-ϕ formulation to solve the diffusion of the magnetic field tangential components inside the layers. The model is computationally inexpensive thanks to i) the use of the magnetic scalar potential ϕ in the non-conducting regions, and ii) the reduction of the number of degrees of freedom resulting from the thin layer approximation.**

***Keywords*—HTS tapes, nonlinear FE analysis, thin layer approximation, time-domain simulations.**

I. INTRODUCTION

Problems involving High Temperature Superconductors (HTS) in the form of thin-film tapes bring new challenges in numerical analysis. Indeed, these tapes are used in most HTS applications involving high currents and/or high magnetic fields. Second generation (2G) HTS tapes are composed of a stack of thin films with a high aspect ratio. The superconducting layer is approximately 1 μm thick, which makes it difficult to mesh in a Finite Element (FE) environment. Furthermore, the highly nonlinear E-J macroscopic model of HTS generates sharp current fronts that severely impede convergence.

In conventional (ohmic) conductors, the meshing of thin layers is avoided through the use of the "Thin-Shell" (TS) FE approach. With this approach, the volume (surface) of the thin region is collapsed on a surface (line) situated halfway between its wide faces. This reduction in dimensionality avoids completely the need for meshing the thin region [1]. Suitable interface boundary conditions (IBCs) are however required in order to consider the right physics in the thin regions. A nonlinear time-domain extension of the TS approach has been proposed in [2], where the *tangential components* of the current density and magnetic field are represented by mathematics series in terms of Legendre polynomials. Both the magnetic field ($\mathbf{h}$) and magnetic vector potential ($\mathbf{a}$) formulations are studied in this paper, but only in the context of nonlinear ferromagnetic materials.

The modeling of HTS tapes as thin regions has recently been proposed in [3], based on a $\mathbf{t}$-$\mathbf{a}$-formulation. The approach is different from the generalized TS model since it considers solving an auxiliary 1-D FE problem in the thin region in terms of the current vector potential $\mathbf{t}$, related to the *normal component* of $\mathbf{h}$ across the thin region. In this model, the tangential components of the magnetic field are neglected. This is valid as long as the normal component dominates the dynamics of the problem (this is often the case, but not when tapes are closely packed in a solenoidal coil).

In an attempt to achieve a more general version of nonlinear thin region models, we present in this paper an $\mathbf{h}$-ϕ formulation that solves current and field distribution is HTS tapes using the tangential component of $\mathbf{h}$, similarly as in the generalized TS approach. The IBCs are written in terms of small auxiliary 1-D FE problems across the tape thickness, allowing to compute the penetration of the tangential components of the field into the HTS layer in a time-dependent simulation. Results show good agreement with the $\mathbf{t}$-$\mathbf{a}$-formulation, as well as with a full discretized model, used as a reference solution. We also show that this approach is efficient at reducing the number of degrees of freedom (DoFs).

II. MAGNETODYNAMIC $\mathbf{h}$-FORMULATION

The well-known $\mathbf{h}$-formulation is obtained from the weak form of Faraday's law ($\mathrm{curl}\mathbf{e} = \partial_t\mathbf{b}$). We consider the magnetodynamic problem in a domain $\Omega = \Omega_c \cup \Omega_c^C \in \mathbb{R}^3$ with boundary Γ, where Ω_c and Ω_c^C denotes the conductive and non-conductive parts of Ω, respectively. Thus, the weak form, for all test-function $\mathbf{w}$, is

$$\partial_t(\mu\mathbf{h},\mathbf{w})_\Omega + (\rho\mathrm{curl}\mathbf{h},\mathrm{curl}\mathbf{w})_{\Omega_c} + \langle\mathbf{n}\times\mathbf{e},\mathbf{w}\rangle_{\Gamma_e\cup\Gamma_s} = 0, \quad (1)$$

where $\mathbf{h}$ and $\mathbf{e}$ are the magnetic and electric field vectors, μ is the permeability, ρ is the resistivity, $\mathbf{n}$ is the outward unit normal vector and $(\cdot,\cdot)_\Omega$ and $\langle\cdot,\cdot\rangle_\Gamma$ denote, respectively, the volume integral over Ω and the surface integral over Γ of the scalar product of their two arguments. The IBCs are expressed by the integral over Γ_s.

The domain Ω and the weak form (1) can be discretized by edge elements in Ω_c. However, it is known that expressing $\mathbf{h}$ by a magnetic scalar potential ($\mathbf{h} = -\nabla\phi$) provides advantages in terms of reduction of the total number of DoFs. A net current constraint on Ω_c^C can be imposed by edge-based cohomology basis functions (ψ) on Ω_c^C [5]. This procedure provides computation time gains, especially in highly nonlinear cases such as HTS modelling.

A. 1-D Eddy Current Problem

The behavior of electromagnetic quantities inside the thin region ($-d/2 \leq y \leq d/2$) is locally determined by a 1-D eddy current problem in terms of the tangential components of the magnetic field (h_t). A local coordinate system xyz is adopted with the y axis normal to the thin surface and with $y = 0$ at its center. Thus, the equation to be solved is

$$\partial_y(\rho \partial_y h_t(y,t)) = \sigma\mu\partial_t h_t(y,t), \tag{2}$$

with the nonlinear electrical resistivity of the HTS material given by the E-J power-law

$$\rho = \frac{e_c}{j_c}\left(\frac{|\mathbf{j}|}{j_c}\right)^{n-1} \overset{\text{1-D}}{=} \frac{e_c}{j_c}\left(\frac{|-\partial_y h_t(y)|}{j_c}\right)^{n-1}, \tag{3}$$

and the boundary conditions associated with the upper and lower surfaces of the thin region given by

$$h_t^+ = h_t(d/2,t), \qquad h_t^- = h_t(-d/2,t). \tag{4}$$

The 1-D FE problem is solved locally in each pair of edges that belong to the thin layer in the 2-D or 3-D FE model. Classical first-order Lagrange basis functions are considered.

B. *Proposed TS Approach and Implementation*

In the classical TS approach, the thin region volume (Ω_s) is excluded from the calculus domain and is represented by interface conditions at its boundaries (Γ_s) via surface integral terms [1]. In a linear case, (2) is solved analytically in terms of the tangential fields. Instead, here we propose to solve a local 1-D FE (Fig. 1). The surface integral terms on Γ_s are derived from the volume integrals in Ω_s and reduced to surface integrals in terms of the tangential components of the magnetic field in the week form of the $\mathbf{h}$-formulation (1), i.e.,

$$\langle \mathbf{n}\times\mathbf{e}, \mathbf{w}\rangle_{\Gamma_s} = -\partial_t(\mu\mathbf{h},\mathbf{w})_{\Omega_s} - (\rho\,\mathrm{curl}\mathbf{h}, \mathrm{curl}\mathbf{w})_{\Omega_s}, \tag{5}$$

where the magnetic field in Ω_s is rewritten as $h_t(t)p(y)$, with h_t tangential to Γ_s and $p(y)$ differentiable with respect to y ($-d/2 \le y \le d/2$). The volume integrals terms in (5) are then reduced to the following surface integrals

$$\partial_t(\mu\mathbf{h},\mathbf{w})_{\Omega_s} = \partial_t\langle\mu h_t, w_t\rangle_{\Gamma_s}\int_{-d/2}^{d/2} p(y)p'(y)\mathrm{d}y, \tag{6}$$

$$(\rho\,\mathrm{curl}\mathbf{h},\mathrm{curl}\mathbf{w})_{\Omega_s} = \langle\rho h_t, w_t\rangle_{\Gamma_s}\int_{-d/2}^{d/2}\partial_y p(y)\partial_y p'(y)\mathrm{d}y. \tag{7}$$

Finally, (1), (6) and (7) are described only through the tangential components of the magnetic field on Ω_c and along Γ_s, and the scalar potential is expressed in the cohomology basis of Ω_c^C, thereby leading to a nonlinear system of algebraic equations that can be solved by the Newton-Raphson Method.

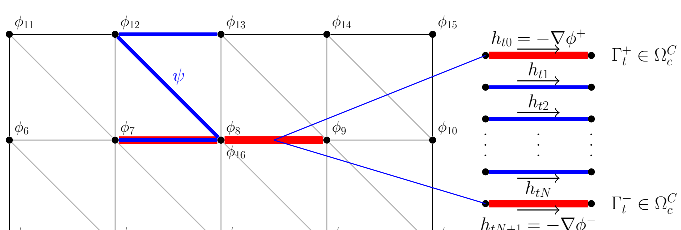

Fig. 1: Proposed approach, geometry and mesh example

III. Simple Application Example

We apply the proposed approach to a single thin superconducting layer embedded in a non-conducting region (free space). The layer is 10 μm thick and 4 mm wide. In the E-J power-law, $e_c = 10^{-4}$ V/m, $j_c = 5\times10^8$ A/m^2 and $n = 21$ are constants. A sinusoidal current of $j_c S/2$ is imposed, where S is the cross-section of the thin layer. In terms of discretization, the same number of elements across the width of the tape is used in the TS and in the 2-D models. The local tangential magnetic field distribution near the upper boundary of the layer is presented on the left side of Fig. 2. The normal component of the magnetic field is computed by average their values on each side of the thin HTS layer (computed from the ϕ representation), and presented on the right side of Fig. 2. The number of DoFs in this particular case is reduced by up to 50%, depending on the selected 1-D FE discretization.

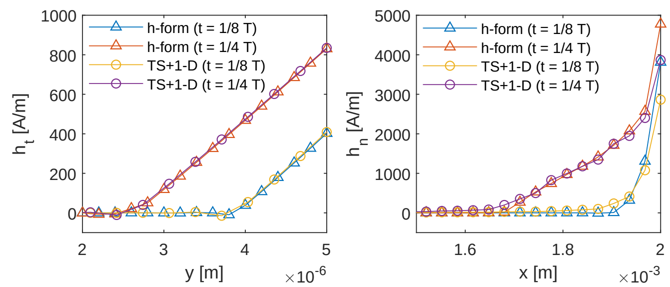

Fig. 2: Local magnetic field components distribution.

IV. Discussion

The proposed approach shows good agreement with the reference model in terms of both the normal and the tangential magnetic field components local distribution, even if only the tangential components are considered in the IBCs. Only a small difference is observed in the normal component of the magnetic field near the extremity of the layer when compared to the fully discretized reference model. This formulation is thus the dual of the $\mathbf{t}$-$\mathbf{a}$-formulation introduced in [3], which solves only for the normal component of $\mathbf{h}$ and ignore its tangential component. Both methods can reduce the number of DoFs substantially compared to a fully discretized model. Our latest progress towards a general formulation that takes into account both components will be presented.

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