The Campbell penetration depth in type-II superconductors

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***Abstract*— The flux-line-lattice response to AC magnetic fields in type-II superconductors is determined by the shape of the flux pinning potential in the material. This response is normally well described within the Bean critical state model, provided the amplitude of the AC magnetic field is sufficient to unpin and move the vortices from their pinning sites, leading to hysteresis. Conversely, if the vortices remain within their potential wells, their movement remains reversible, and cannot be described by the Bean critical state model. Instead, the mixed state can be viewed as an ensemble of coupled linear harmonic oscillators, each oscillating within its potential well. This is explained by the Campbell model, which extends upon the Bean model to include the finite size of the pinning potential, and a corresponding linear and elastic pinning force. Here, the dynamic equations for flux movement – given a linear pinning force – are described, linearized, and solved analytically. Subsequently, the solution is used to extract the value of the Campbell penetration depth of an applied AC magnetic field from experimental data obtained with a pick-up method.**

Keywords*— *Bean model, Campbell model, penetration depth, AC magnetic fields

I. INTRODUCTION

The flux pinning force in the Bean critical state model [1] is assumed equal and opposite the Lorentz force, regardless of the vortex position with respect to its pinning centre. Essentially, the pinning force is treated as a constant frictional force opposing vortex movement. This assumption is sufficient to lead to the prediction of the hysteretic behaviour, as observed in type-II superconductors.

In reality, the pinning force will depend on the vortex position with respect to its pinning centre – close to the equilibrium position within the pinning centre the force will be linearly proportional to the displacement from equilibrium. This is true because the minimum of the pinning potential can always be approximated locally by a harmonic potential, which is quadratic in displacement (thus, the force is linear). Conversely, for large displacements the pinning force becomes frictional, as in the Bean model, hence its value approaches a constant independent of displacement. This was first observed and explained by Campbell [2] in pick-up measurements of magnetic flux due to an applied AC magnetic field.

A candidate form of the pinning force, proposed by Campbell, taking into account the above assumptions, can be written as

$$F_P(y)=B_0J_C(1-exp(-y/d)), \tag{1}$$

and is shown in Figure 1. Here, y denotes the vortex displacement from its equilibrium in the pinning potential well. B_0 and J_C are the background DC magnetic field and critical current density, respectively. The parameter d is the effective size of the pinning potential and marks the displacement, at which the pinning force, F_P, transitions from the linear (Campbell) to the constant (Bean) regime. For instance, for small displacements $y<d$ the pinning force can be written as

$$F_P(y)\approx(B_0J_C/d)y, \tag{2}$$

whereas for $y>d$ the force becomes

$$F_P(y)\approx B_0J_C. \tag{3}$$

Thus, the pinning force, as it is defined in equation (1), will describe the transition from the Campbell regime to the Bean regime, depending on the magnitude of the vortex displacement y.

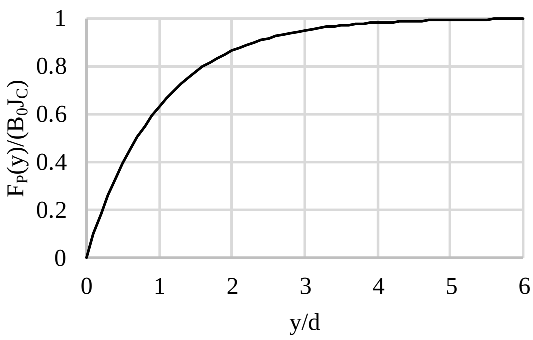

Figure 1. The pinning force transition from the linear Campbell regime to the constant Bean regime.

In this work, the above pinning force is combined with the general flux conservation equation, leading to a system of two partial differential equations

$$dy(x)/dx=-b(x)/B_0, \tag{4}$$

and

$$db(x)/dx=\mu_0 J_C(1-exp(-y(x)/d), \tag{5}$$

where μ_0 is the permeability of free space. The solutions of equations (4) and (5) are $y(x)$, the vortex displacement as a function of position in the superconductor x, and $b(x)$, the local magnetic field change due to $y(x)$. In general, the system has no analytical solutions and must be solved by numerical integration. However, it is possible to linearize the pinning force expression at a given value of x simply by substituting its linear approximation (i. e. tangent) into the equations (4) and (5). This leads, subsequently, to an analytical solution for vortex displacement, y(x), which can be shown to decay exponentially with distance into the superconductor,

$$y(x) \propto exp(-x/\lambda_C), \tag{6}$$

where λ_C is the Campbell penetration depth, and is defined as

$$\lambda_C{}^2 = (B_0 d)/(\mu_0 J_C). \tag{7}$$

Since its value directly determines the vortex displacement, $y(x)$, and, in turn, the local magnetic field change, $b(x)$, it can be determined experimentally via magnetic flux measurements using pick-up methods. Hence, the effective size of the pinning potential, the parameter d, which is typically on the nanometre scale, can be determined. In this talk, we present such measurements in a bulk $GdBa_2Cu_3O_{7-\delta}$ superconductor.

II. Results

By measuring the changing magnetic flux in a 5 mm wide and 10 mm long $GdBa_2Cu_3O_{7-\delta}$ sample due to an applied AC magnetic field, the shape of $b(x)$ in the sample can be deduced, since its integral over the cross-section of the sample will correspond to the magnetic flux. In turn, the shape of $y(x)$ can be deduced from $b(x)$ by using equation (4), from which the value of the Campbell penetration can be extracted. Here, this is done by measuring the pick-up voltage in a coil, wound round the superconducting sample, exposed to and AC magnetic field of amplitude 1 mT at a frequency of 300 Hz. The sample temperature is 70 K, with helium gas used as coolant. The resulting values of Campbell penetration depth, at different values of background DC magnetic field, B_0, are shown in Figure 2 (date presented originally in [3]).

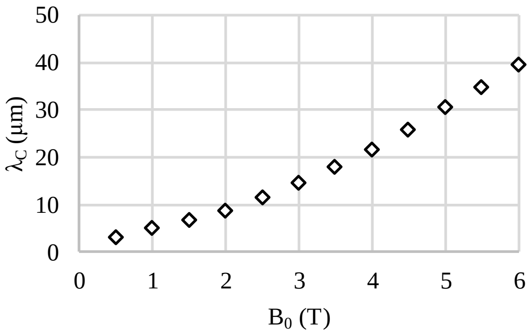

Figure 2. The Campbell penetration depth for background DC magnetic fields up to B_0 = 6 T.

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