2D and 3D validation of a hybrid method based on **A** and **H** formulations for Pulsed Field Magnetization

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***Abstract*—This paper presents a validation of the hybrid method using the H and A formulations for Pulsed Field Magnetization (PFM). This hybrid method can help for simulating more complex problems involving superconductors, electric circuit and/or ferromagnetic materials. In some cases, the hybrid method can even speed up the simulation compare to the model that is based on H formulation only. A 2D axisymmetric model of PFM coupled with electrical circuit is first considered. To validate our concept, we simulated the same model governed by the hybrid method and the H formulation. The comparison of the final trapped magnetic flux density is then performed. The hybrid method was faster around 46 %. In the final paper, the validation and comparison will also be made on a 3D problem.**

***Keywords*—3D modelling, coupled methods, H formulation, A formulation, pulsed magnetic field, HTS**

I. Introduction

It is widely known that superconductors can be used not only as a medium for transporting huge amounts of current but also as strong magnets. A suitable process of Pulsed Field Magnetization (PFM) of High Temperature Superconductors (HTS) can result in a superconducting magnet with more than 5 T of the trapped magnetic field [1]. With such a trapped magnetic field B_t, the superconductor can be easily used in various applications like electrical machines. However, to trap this amount a magnetic field a complex structure of the system is required [1], [2]. In practical solutions, B_t is still less than the present record of 5 T [3]-[5].

The main limitation in the numerical modelling of the magnetization process is the strong nonlinearity of superconducting material. As a result, it is common to use **H** formulation for solving Maxwell's equations with superconducting material [6], [7]. This solution is well known for providing good convergence in such problems. On the other hand, many applications involving superconductors also include materials with a nonlinear magnetic permeability μ. For example, it is common to use iron yokes or rings to enhance B_t when the PFM is performed with a cryocooling system [2], [7]-[8]. Unlike to the superconducting material, it is preferable to use the **A** formulation for material with nonlinear μ, which provides better convergence. The coupling of both formulations will allow to solve more easily complex models during a magnetization process or for electromagnetics problems where superconductors are used.

The 2D hybrid model based on the **A** and **H** formulations is well presented in [9], where the electric motor is taken under consideration. The positive feature of the hybrid solution in 2D problems is that the **A** formulation usually consists of a single varible, which can reduce the number of Degrees of Freedom (DoF).

In this paper, a universal method for coupling both formulations not only in 2D but also in 3D problems is proposed. We will compare the 2D axisymmetric model based on **H** formulation with the hybrid solution, i.e. **A** and **H** formulation, in terms of accuracy and computing time. Moreover, the final paper will consider a 3D problem. The 3D model will provide a more sophisticated structure using hexagonal shaped superconductors cooled by a cryocooler.

II. Numerical Modelling

A. *H formulation*

The **H** formulation can be achieved by combining Faraday's and Ampere's law as

$$\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = 0 \qquad (1)$$

where μ_0 is the vacuum permeability and the relative permeability μ_r depends on the material. Considering superconducting material, the value μ_r is usually equal to 1 [6], [8]. For 2D model, we assumed that the electric field **E** and current density **J** are collinear [10], [11]. The relationship between them can be expressed as

$$\mathbf{E} = \rho \mathbf{J} \qquad (2)$$

where ρ is the nonlinear electrical resistivity of the material. There is no need for imposing $\nabla \cdot \mathbf{H} = 0$ in the presented formulation if we choose the curl finite elements as discretization in the numerical model [10].

To model the behavior of the superconductor, we propose to use the percolation law, which is some kind of mix between the Critical State Model and the *E-J* power law model [12], [13], given by

$$E = \begin{cases} 0 & \text{if } J \leq J_{\mathrm{cmin}}, \\ E_c \left(\dfrac{J - J_{\mathrm{cmin}}}{J_c - J_{\mathrm{cmin}}} \right)^n & \text{if } J > J_{\mathrm{cmin}} \end{cases} \qquad (3)$$

with J_c = 100 A/mm^2, $J_{\mathrm{cmin}} = 0.35 J_c$, E_c = 1 μV/cm, and *n*-value = 13.37. Equation (3) can be implemented as a nonlinear resistivity ρ.

B. A formulation

The **A** formulation based on a magnetic vector potential can be expressed as

$$\rho^{-1}\frac{\partial \mathbf{A}}{\partial t}+\nabla\times\left((\mu_0\mu_{\mathrm{r}})^{-1}\nabla\times\mathbf{A}\right)=\mathbf{J}_{\mathrm{app}} \qquad (4)$$

For a 2D model, the magnetic vector potential **A** consists of a single component, *A*, which depends on the scalar current density J_{app}. In the 3D model, we did not perform the Coulomb gauge transformation, i.e. $\nabla\cdot\mathbf{A}\neq 0$. Therefore, we must calculate the double cross-product of the magnetic vector potential in (4). By construction, the **A** formulation satisfies $\nabla\cdot\mathbf{B}=0$ [14].

C. Coupling of A and H formulation

In **H** formulation, the electrical field $\mathbf{E}=-\partial_t\mathbf{A}$ is imposed from **A** formulation in the form of a Neumann boundary condition which can be written as

$$\mathbf{n}\times\mathbf{E}^{\mathbf{H}}\overset{\Gamma}{\leftarrow}\mathbf{n}\times\mathbf{E}^{\mathbf{A}} \qquad (5)$$

On the other hand, we can consider the same approach for **A** formulation and write the Neumann boundary condition as

$$\mathbf{n}\times\mathbf{H}^{\mathbf{A}}\overset{\Gamma}{\leftarrow}\mathbf{n}\times\mathbf{H}^{\mathbf{H}} \qquad (6)$$

where the magnetic field of **H** formulation is imposed in the **A** formulation. Both equations (5) and (6) can be adapted to 2D and 3D models. In the final paper, we will provide more details that will allow researchers to easily write such conditions in a numerical software.

III. MODEL

A PFM process of one single HTS bulk is considered. The problem geometry with a set of boundary conditions is provided in Fig. 1. The electrical circuit of the magnetizer is described in [15]. The equation of the magnetizer can be defined as

$$-u_{\mathrm{c}}+(R_{\lambda}+R_{\mathrm{mc}})\cdot i+L_{\lambda}\frac{di}{dt}+u_{L_{\mathrm{mc}}}=0 \qquad (7)$$

and all parameters in (7) have been summarized in Table 1. The drop voltage across the coil's inductance $u_{L_{\mathrm{mc}}}$ is calculated based on Faraday's law as $d_t\Phi(\mathbf{B})$.

TABLE I. PARAMETERS OF THE ELECTRICAL CIRCUIT

Parameter	**Value**	**Description**
u_{c}	900 V	Initial voltage of the capacitor
R_{mc}	4.6 mΩ	Resistance of the coil
R_{λ}	4 mΩ	System parasitic resistance
L_{λ}	4 μH	System parasitic inductance

IV. RESULTS

The simulations were performed on a computer with processor Intel® Xeon® CPU E5-2630 v4 @ 2.20GHz with 64 GB RAM by using COMSOL software. All formulations defined in PDE form. After the pulse of magnetization, the trapped magnetic field B_{t} in the center of the upper surface of the HTS bulk is about 1.1 T at t = 200 ms. The distribution of the magnetic flux density norm obtained with **H** formulation is shown in Fig. 2 (a). We do not notice any significant differences in the distribution of the magnetic flux density obtained with the hybrid method as shown by the standard deviation in Fig. 2 (b). As expected, the computing time of both models is not the same. The simulation of the hybrid method was 46.55 % faster than the model based on **H** formulation. This difference is related by the number of DoF, which is around 45 % less for a hybrid method (DoF: hybrid method 3 559, **H** formulation 7 905).

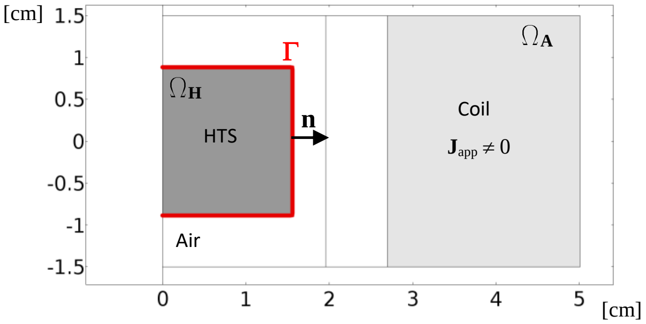

Fig. 1. 2D axisymmetric model of the coil for PFM. The **Γ** boundary separates the two formulations. In the hybrid model, the regions Ω_{H} and Ω_{A} are based on **H** and **A** formulation, respectively. In the second model, all domains are based on the **H** formulation.

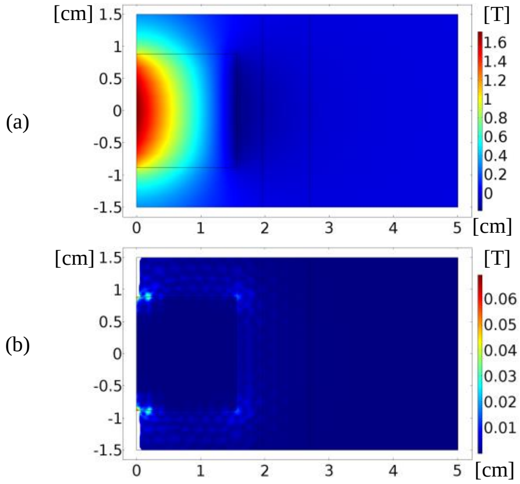

Fig. 2. Distribution of the magnetic flux density norm at t = 200 ms obtained with **H** formulation (a). Standard deviation of the same quantity by using the hybrid method (b).

The 2D hybrid method presented here allows for greater modelling flexibility and a better convergence when non-linear magnetic and conductive materials are included and, in some cases, it may also reduce computation time.

In the final paper, the hybrid method will be presented in more detail. An iron piece will be added in the 2D model in order to give a more general conclusion regarding the computing time. The 3D model, which has not been presented here, will show the magnetization of hexagonal superconductors cooled by a cryocooler. Additionally, the paper will focus on providing numerical tips and tricks associated with the hybrid method that we have developed here using COMSOL.

REFERENCES

[1] H. Fujishiro, T. Tateiwa, A. Fujiwara, T. Oka, and H. Hayashi, "Higher trapped field over 5T on HTSC bulk by modified pulse field magnetizing," *Physica C: Superconductivity and its Applications*, vol. 445–448, pp. 334–338, Oct. 2006, doi: 10.1016/j.physc.2006.04.077.

[2] H. Fujishiro, M. Kaneyama, T. Tateiwa, and T. Oka, “Record-High Trapped Magnetic Field by Pulse Field Magnetization Using GdBaCuO Bulk Superconductor,” *Jpn. J. Appl. Phys.*, vol. 44, no. 9L, p. L1221, Sep. 2005, doi: 10.1143/JJAP.44.L1221.

[3] T. Ida, M. Watasaki, Y. Kimura, M. Miki, and M. Izumi, “Trapped magnetic field measurements on HTS bulk by peak controlled pulsed field magnetization,” *Journal of Physics: Conference Series*, vol. 234, no. 3, p. 032023, Jun. 2010, doi: 10.1088/1742-6596/234/3/032023.

[4] B. Gony, K. Berger, B. Douine, M. R. Koblischka, and J. Leveque, “Improvement of the Magnetization of a Superconducting Bulk using an Iron Core,” *IEEE Transactions on Applied Superconductivity*, vol. 25, no. 3, pp. 1–4, Jun. 2015, doi: 10.1109/TASC.2014.2373494.

[5] M. D. Ainslie *et al.*, “Numerical analysis and finite element modelling of an HTS synchronous motor,” *Physica C: Superconductivity and its Applications*, vol. 470, no. 20, pp. 1752–1755, Nov. 2010, doi: 10.1016/j.physc.2010.05.200.

[6] J. Zou *et al.*, “Numerical modelling and comparison of MgB2bulks fabricated by HIP and infiltration growth,” *Supercond. Sci. Technol.*, vol. 28, no. 7, p. 075009, May 2015, doi: 10.1088/0953-2048/28/7/075009.

[7] M. D. Ainslie *et al.*, “Modelling and comparison of trapped fields in (RE)BCO bulk superconductors for activation using pulsed field magnetization,” *Supercond. Sci. Technol.*, vol. 27, no. 6, p. 065008, 2014, doi: 10.1088/0953-2048/27/6/065008.

[8] M. D. Ainslie and H. Fujishiro, “Modelling of bulk superconductor magnetization,” *Superconductor Science and Technology*, vol. 28, no. 5, p. 053002, May 2015, doi: 10.1088/0953-2048/28/5/053002.

[9] R. Brambilla, F. Grilli, L. Martini, M. Bocchi, and G. Angeli, “A Finite-Element Method Framework for Modeling Rotating Machines With Superconducting Windings,” *IEEE Transactions on Applied Superconductivity*, vol. 28, no. 5, pp. 1–11, Aug. 2018, doi: 10.1109/TASC.2018.2812884.

[10] R. Brambilla, F. Grilli, and L. Martini, “Development of an edge-element model for AC loss computation of high-temperature superconductors,” *Supercond. Sci. Technol.*, vol. 20, no. 1, p. 16, 2007, doi: 10.1088/0953-2048/20/1/004.

[11] Z. Hong, A. M. Campbell, and T. A. Coombs, “Numerical Solution of Critical State in Superconductivity by Finite Element Software,” *Supercond. Sci. Technol.*, vol. 19, no. 12, p. 1246, 2006, doi: 10.1088/0953-2048/19/12/004.

[12] F. Sirois, F. Grilli, and A. Morandi, “Comparison of Constitutive Laws for Modeling High-Temperature Superconductors,” *IEEE Transactions on Applied Superconductivity*, vol. 29, no. 1, pp. 1–10, Jan. 2019, doi: 10.1109/TASC.2018.2848219.

[13] K. Yamafuji and T. Kiss, “Current-voltage characteristics near the glass-liquid transition in high-Tc superconductors,” *Physica C: Superconductivity*, vol. 290, no. 1, pp. 9–22, Oct. 1997, doi: 10.1016/S0921-4534(97)01580-3.

[14] M. S. Kuczmann, “Potential formulations in magnetics applying the finite element method.” 2009.

[15] J. Kapek, K. Berger, and J. Lévêque, “2D Numerical Modeling of a Coil Coupled with an Electrical Circuit used for PFM of Bulk HTS,” in *Symposium de Génie Electrique (SGE 2018)*, Nancy, France, 2018, p. 36, id. 188917, SO-24-PM : 1.