Modelling Pulsed Field Magnetization of Ring-Shaped Bulk Superconductors

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***Abstract*— The motion of flux into ring-shaped bulk high-temperature superconductors during pulsed-field magnetization (PFM) renders finite element modelling of the process more complicated than for disk-shaped bulks. The collision of two opposite-moving flux fronts within the bulk creates a thermal shock which, if poorly modelled, pollutes the numerical solution. In this work, the H-formulation is coupled with a thermal model to simulate the dynamics of ring-bulk PFM. The results are firstly validated against Brandt's analytical equations for a simple zero-field-cooled magnetization case, then extended to analyze PFM under more realistic material assumptions. The limitations, and ways to address these, of finite element modelling to predict the behavior of ring-shaped bulks during PFM are explored.**

Keywords*— HTS modelling, bulk superconductors, ring bulks, pulsed field magnetization, flux jumps

I. INTRODUCTION

Stacks of ring-shaped bulk high-temperature superconductors (HTS) have been shown to be a viable source for the generation of high-strength, uniform magnetic fields for use within benchtop nuclear magnetic resonance (NMR) instruments [1]. Experimental work to date, however, has only explored their suitability when magnetized by field cooling in an NMR-grade background field.

For such ring-bulk stacks to be used more practically, they will need to be magnetized using low-cost, portable magnetization techniques such as pulsed-field magnetization (PFM). PFM has been shown to successfully magnetize disk-shaped HTS [2], but is prone to inducing macroscopic breaks of the critical current in ring-shaped samples through flux jumps [3]. These breaks in current lead to the trapping of negligible, or negative magnetic fields within the bore of the ring.

The behavior of disk-shaped bulk HTS is well understood and has been extensively modelled using finite element (FE) models (FEM) [4], but there is little in the literature regarding the magnetization of ring-shaped devices. Brandt [5] showed that, during zero-field-cooled (ZFC) magnetization, flux penetrates into thin rings from both the inner and outer edges of the sample – introducing two opposite-moving flux fronts into the ring – between which exists a shielded region of zero current and trapped field. In samples of practical height, flux preferentially propagates into the sample at the top and bottom surfaces [6], thus this region of zero field and current is isolated by magnetized regions. As the fronts converge, the region of zero field and current becomes infinitely small, introducing a singularity into FE models, before vanishing. At this point, a large local dB/dt is induced, leading to significant local heating of the bulk at this point. In physical samples, this may locally induce a flux jump within the ring.

If the model is insufficiently finely meshed, this local heating will induce a thermal shock – and subsequent non-physical thermal oscillations – to the result [7]. The maximum element size, e_{max}, is given by

$$e_{max} = \sqrt{\frac{6\kappa\Delta t}{\rho C}}$$

where $\Delta t, \kappa, \rho$ and C are the time-step size, thermal conductivity, mass density and thermal density of the material respectively [7]. As such, bulk HTS materials require a very fine mesh to avoid this issue, introducing multiple degrees of freedom (DoFs) to the model, thus rendering 3D models of PFM intractably large for many desktop PCs.

In this work, the pulsed magnetization of a finite ring is modelled using H- and mixed H-A formulations, with meshes of differing density and element order [8]. They are compared in terms of computational requirements and performance, and the solutions obtained relative to each other. Through this, the limitations of FEM to predict the behavior of ring-shaped bulks during PFM are explored, and methods to address these identified.

II. RING MODELLING

The geometry of the analyzed ring is shown in Fig. 1, assuming for simplicity the 2D axisymmetric case. A bulk HTS ring with inner diameter (ID) 16 mm, outer diameter (OD) 40 mm, height (H) 12 mm and experimentally measured $J_c(T,B)$ is set in an aluminum reinforcement ring of width 5 mm and height 12 mm. It is cooled from the bottom surface by a cold head at constant temperature.

The bulk is magnetized by a coil with dimensions 120/100/50 mm OD/ID/H, applying a pulse with adjustable amplitude and a 13 ms rise time. In all cases, the ring is modelled using the H-formulation with a fully coupled thermal model. The surrounding regions are modelled either using the H-formulation or the A-formulation (in the mixed formulation case).

TABLE I. Impact of element order and model formulation on model solve-time.

Formulation	Element Order	Mesh Density r/z (HTS Region) [mm]	DoFs	Time to solve [s]
H	Quadratic	0.25 / 0.2	53086	11912
H	Linear	0.25 / 0.2	19570	4816
H	Linear	0.125 / 0.1	60446	15257
H	Linear	0.0625 / 0.05	211339	87960
H-A	Quadratic - Linear	0.25 / 0.2	32635	8691
H-A	Quadratic - Quadratic	0.25 / 0.2	39774	10117

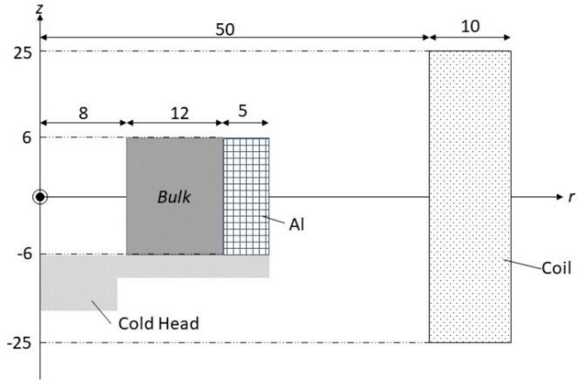

Fig. 1. Geometry of the magnetizing fixture. A pulsed magnetic field is applied to the ring using the solenoid coil.

III. Initial Results

The data in Fig. 2 show the occurrence of a thermal shock during the PFM of a ring with a mesh size 0.25 mm radially and 0.2 mm axially. Fig. 3 shows the subsequent, non-physical, thermal oscillations induced by this shock. By refining the mesh element size in the bulk HTS domain, this shock is damped at the expense of increased model computational time.

By adjusting the element-order of the HTS domain, the thermal shock behavior is damped out whilst the associated time penalty is less than that for a more finely meshed model (Table I).

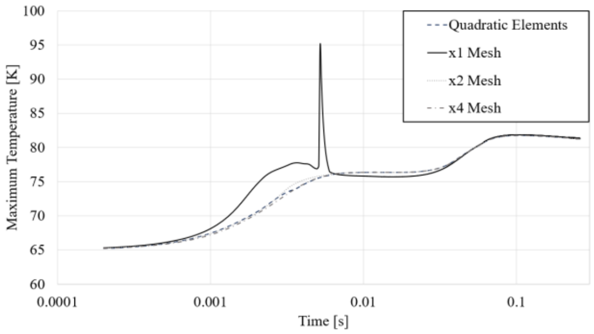

Fig 2. Appearance of thermal shock in PFM of linear element in 0.25/0.2 mm mesh

The detailed findings from this work will be presented at the modelling workshop, including techniques to suppress the numerical errors introduced from the thermal shock and methods to improve the model computational time.

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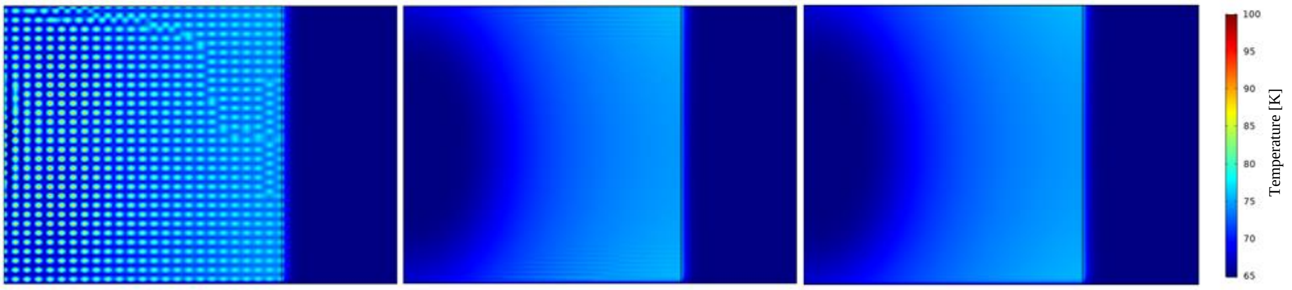

Fig. 3. Thermal profiles for x1, x2 and x4 linear meshes at point of full field penetration.