H-ϕ formulation for the efficient simulation of superconductors

Alexandre Arsenault
Polytechnique Montréal
alexandre.arsenault94@gmail.com

Frédéric Sirois
Polytechnique Montréal
f.sirois@polymtl.ca

***Abstract*—This work shows how to implement the H-ϕ formulation in a commercial finite element framework for efficiently modeling the electromagnetic behavior of high critical temperature superconductors. We show that the results obtained with the H-ϕ and H formulations are in good agreement in both 2-D and 3-D, but the H-ϕ formulation is much faster.**

***Keywords*—H-ϕ formulation, finite element method, HTS tapes**

I. INTRODUCTION

The development and understanding of high temperature superconductor (HTS) applications requires accurate and reasonably quick modeling tools. Currently, one of the most widely used method for simulating the electromagnetic behavior of HTSs is the finite element method (FEM), with Maxwell's equations written in terms of the so-called H-formulation. Although the H-formulation has been shown to accurately model many different applications using HTSs, the need to impose a high resistivity in air domains degrades the matrix conditioning and leads to unnecessarily long computation times [1]. In addition, the non-zero resistivity creates unphysical currents in non-conducting domains [2].

One method proposed to solve this issue is to use the magnetic scalar potential ϕ in non-conducting domains, such that the resistivity and the current density in air domains is automatically set to zero and a scalar dependent variable is used instead of the vector $\mathbf{H}$. This greatly reduces computation times and eliminates unphysical currents in non-conducting domains. Thus, in this contribution, we use the well established H-formulation in conducting domains and ϕ in non-conducting domains to efficiently model HTS applications. Accordingly, this formulation is called H-ϕ.

One issue with the H-ϕ formulation is that the non-conducting domains must be simply connected in order to avoid violating Ampere's law. Several methods have been proposed to solve this problem, such as using a thin conducting layer in the air domains [3] or using "thick cuts" [4].

In this contribution, we show how using "thin cuts" can be used to cut multiply connected domains in a simple way using the commercial FEM program COMSOL Multiphysics. We compare our results with the pure H-formulation and show that both formulations are in agreement.

II. H-ϕ FORMULATION

The H-ϕ formulation is a mixed formulation employing the magnetic field H as the dependent variable in conducting domains and the magnetic scalar potential ϕ in non-conducting domains. The governing equation in conducting domains is therefore the standard H-formulation:

$$\nabla \times (\rho \nabla \times \mathbf{H}) = -\mu_0 \frac{\mathrm{d}\mathbf{H}}{\mathrm{d}t}\,, \tag{1}$$

where ρ is the resistivity.

In non-conducting domains, the current density is zero by definition. According to Ampere's law and vector calculus identities, we can therefore write the magnetic field as $\mathbf{H} = -\nabla\phi$. Using the divergence-free property of the magnetic flux density and assuming $\mathbf{B} = \mu_0 \mathbf{H}$, the governing equation in non-conducting domains is then given by:

$$\nabla \cdot \nabla \phi = 0. \tag{2}$$

In order to avoid violating Ampere's law when imposing transport currents in the conducting domains, the non-conducting domains must be made simply connected. The reason is simple: Ampere's law states that $\int_c \mathbf{H} \cdot d\mathbf{l} = 0$ everywhere in the non-conducting domain by definition of the magnetic scalar potential, where C is an arbitrary closed curve. However, if the non-conducting domain surrounds a conductor with a net current, the right hand side of Ampere's law must be equal to the enclosed current.

To solve this issue, we impose a discontinuity in ϕ equal to the enclosed current along a line (surface) in 2-D (3-D) connecting the air domains to the conductor, therefore "cutting" the air domain to make it simply connected. This can be mathematically stated as:

$$\begin{aligned}\oint_C \mathbf{H} \cdot d\mathbf{l} = -\oint_C \nabla\phi \cdot d\mathbf{l} &= \phi(O) - \phi(\mathrm{d}^+) + \phi(\mathrm{d}^-) - \phi(O)\\ &= \phi(\mathrm{d}^-) - \phi(\mathrm{d}^+) \equiv I_e,\end{aligned} \tag{3}$$

where O represents an arbitrary origin, d^+ and d^- represent each side of the cut and I_e is the enclosed current.

III. APPLICATION EXAMPLE

We implement the H-ϕ formulation in COMSOL Multiphysics 5.5 to model the electromagnetic behavior of a 2-D axisymmetric superconducting pancake coil consisting of 40 turns with an inner radius of 3 cm. The individual tapes have a thickness of 1 μm and a width of 4 mm, and are separated by a distance of 0.15 mm.

The resistivity of the superconductor is represented with the standard power law model [5]:

$$\rho = \frac{E_c}{J_c} \left| \frac{\|J\|}{J_c} \right|^{n-1}, \tag{4}$$

with $J_c = 3 \times 10^{10}$ A/m^2, $n = 40$ and $E_c = 1$ μV/cm. We apply a sinusoidal transport current of 96 A at 50 Hz in the tapes, corresponding to 80% of the critical current.

As shown in Fig. 1, the AC losses computed using the H-ϕ formulation are nearly identical to the losses computed with the H-formulation. Even though 34,418 linear mesh elements are used in both cases, the H-ϕ formulation takes only 12 minutes to compute, while the H-formulation requires 19 hours and 55 minutes. Thus, for this case, the H-ϕ formulation is nearly 100 times faster than the H-formulation. This substantial time difference is mainly due to the elimination of the 40 (one constraint per turn) constraints used to impose the transport current.

For a 3-D example, we model a pair of twisted superconducting filaments carrying a sinusoidal transport current of 80 A at 50 Hz, with a critical current of 100 A per wire. The filaments have a cross-sectional area of 0.1 mm^2 and a twist pitch of 4 mm. The geometry is shown in the inset of Fig. 2, where the cuts are shown in cyan. The cuts are chosen to minimize their surface area and to be easy to handle, which is important because they must be made by the user.

In Fig. 2, we show the AC losses in both filaments calculated with the H and H-ϕ formulations. We find that both formulations are in good agreement. Although we use 283,493 linear mesh elements in both formulations, the computation times are 48 minutes vs. 5 hours and 56 minutes for the H-ϕ and H formulation, respectively (7.4 times faster).

In this contribution, we will show our latest progress in using the H-ϕ formulation to model various HTS applications in 2-D and 3-D, with and without transport current. We demonstrate how to properly use thin cuts in COMSOL to make multiply connected non-conducting domains simply connected. We also compare all the results with the H-formulation.

ACKNOWLEDGMENT

This work was supported by the Fonds de recherche du Qubec Nature et technologies (FRQNT) and TransMedTech Institute and its main funding partner, the Canada First Research Excellence Fund.

REFERENCES

[1] A. Arsenault, F. Sirois, and F. Grilli, "Implementation of the H-ϕ Formulation in COMSOL Multiphysics for Simulating the Magnetization of Bulk Superconductors and Comparison With the H-Formulation," *IEEE Transactions on Applied Superconductivity*, vol. 31, no. 2, pp. 1–11, Mar. 2021.

[2] A. Stenvall, V. Lahtinen, and M. Lyly, "An H-formulation-based three-dimensional hysteresis loss modelling tool in a simulation including time varying applied field and transport current: The fundamental problem and its solution," *Superconductor Science and Technology*, vol. 27, no. 10, 2014.

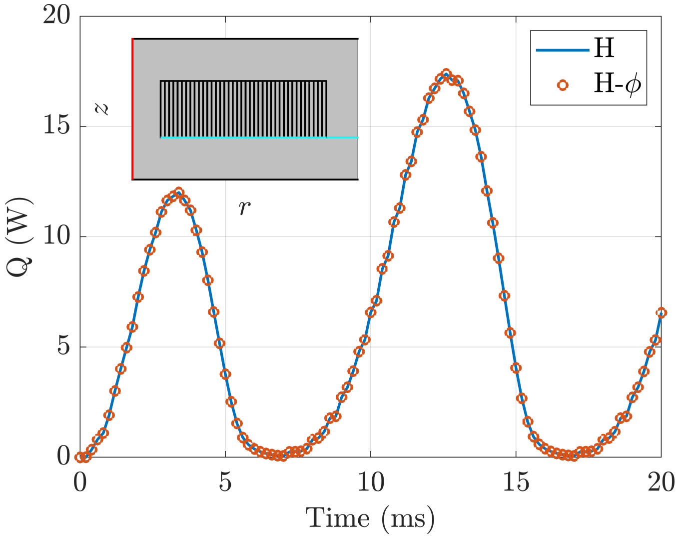

Fig. 1: AC losses in the 40-turn pancake coil calculated for one AC cycle using the H and H-ϕ formulations. The geometry (not drawn to scale) is illustrated in the top left inset of the figure. The cyan lines represent the thin cuts, while the red line shows the symmetry axis.

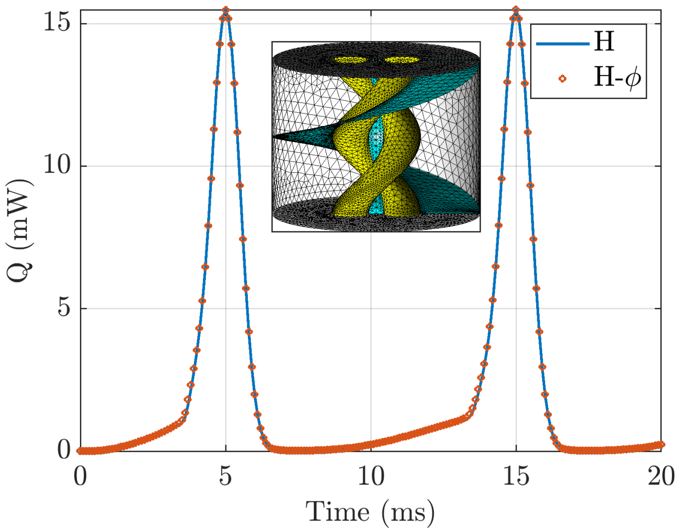

Fig. 2: AC losses in the twisted superconducting filaments calculated for one AC cycle using the H and H-ϕ formulations. The geometry and mesh are showin in the center inset of the figure. The cyan curves represent the thin cuts.

[3] P. J. Leonard and D. Rodger, "A new method for cutting the magnetic scalar potential in multiply connected eddy current problems," *IEEE Transactions on Magnetics*, vol. 25, no. 5, pp. 4132–4134, Sep. 1989.

[4] V. Lahtinen, A. Stenvall, F. Sirois, and M. Pellikka, "A Finite Element Simulation Tool for Predicting Hysteresis Losses in Superconductors Using an H-Oriented Formulation with Cohomology Basis Functions," *J Supercond Nov Magn*, vol. 28, no. 8, pp. 2345–2354, Aug. 2015.

[5] J. Rhyner, "Magnetic properties and AC-losses of superconductors with power law current-voltage characteristics," *Physica C: Superconductivity and its applications*, vol. 212, no. 3-4, pp. 292–300, 1993.