Life-HTS

Liège University finite element models for High-Temperature Superconductors

C. Geuzaine

Université de Liège, Institut Montefiore B28, 4000 Liège, Belgium
Joint work with J. Dular and B. Vanderheyden

HTS 2020, June 22 2021
Some background

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- Gmsh (https://gmsh.info) is a 3D finite element mesh generator with a built-in CAD engine and post-processor
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- About 1,300 registered users on the development site
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- About 20,000 downloads per month (70% Windows)
- About 800 citations per year
- Gmsh has become one of the most popular open source finite element mesh generators (> 5000 citations)
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Short demo

2D and 3D $h$-$\phi$ GetDP formulation for twisted HTS wires, with automatic computation of cuts using the Gmsh cohomology solver [Geuzaine, EUCAS 2015]
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To give it a try, download the ONELAB software bundle from https://onelab.info and open models/Superconductors/helix.pro
Life-HTS
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Goals:

- Provide validated templates for modelling systems with high-temperature superconductors, bulk or tapes, with or without ferromagnetic parts
- Provide representative examples that can serve both as benchmarks and as a starting point for users to develop their own models
- Provide great flexibility in the choice of finite element formulations and associated numerical tools
Life-HTS: Why is flexibility needed?

- Life-HTS is about solving Maxwell's equations in the magnetodynamic (magneto-quasistatic) approximation

\[
\begin{align*}
\text{curl } h &= j, \\
\text{curl } e &= -\partial_t b, \\
\text{div } b &= 0,
\end{align*}
\]

with

- \( h \) the magnetic field (A/m),
- \( j \) the current density (A/m\(^2\)),
- \( e \) the electric field (V/m), and
- \( b \) the magnetic flux density (T),

while the displacement current \( \partial_t d \) is neglected.
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- \( \mathbf{b} \) the magnetic flux density (T),

while the displacement current \( \partial_t \mathbf{d} \) is neglected.

- Boundary conditions and constitutive laws relating \( \mathbf{b} \) to \( \mathbf{h} \) and \( \mathbf{e} \) to \( \mathbf{j} \) are needed to obtain a well-posed problem.
Life-HTS: Why is flexibility needed?

- In ferromagnetic materials: classical anhysteretic saturation law, or energy-based hysteresis model [Jacques et al., AIP Advances 2018]
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- In ferromagnetic materials: classical anhysteretic saturation law, or energy-based hysteresis model [Jacques et al., AIP Advances 2018]
- In high-temperature superconductors: $e = \rho(j)j$ with power law

$$\rho(j) = \frac{e_c}{j_c} \left( \frac{||j||}{j_c} \right)^{n-1}$$

with

- $e_c = 10^{-4}$ V/m
- $j_c$ the critical current density
- $n$ the flux creep exponent ($n \in [10, 1000]$)

Life-HTS: Why is flexibility needed?

- Putting it all together on a Tonti diagram, along with scalar and vector magnetic and electric potentials:

\[
\begin{align*}
(\phi, \omega) \xrightarrow{\text{grad}_h} h(t) \xrightarrow{\text{curl}_h} j \xrightarrow{\text{div}_h} 0 \\
\uparrow \quad b = \mu(b)h \quad \uparrow \quad e = \rho(j)j \\
0 \xleftarrow{\text{div}_e} b \xleftarrow{\text{curl}_e} e(a, a^*) \xleftarrow{\text{grad}_e} (v)
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- \( h \)-conform formulations \((h, h-\phi, t-\omega, \ldots)\) satisfy the top exactly
- \( b \)-conform formulations \((a, a-v, a^*, \ldots)\) satisfy the bottom exactly
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- \(h\)-conform formulations \((h, h-\phi, t-\omega, \ldots)\) satisfy the top exactly
- \(b\)-conform formulations \((a, a-v, a^*, \ldots)\) satisfy the bottom exactly
- The choice of the formulation has a significant effect on the numerical performance of the finite element solver
Life-HTS: Why is flexibility needed?

<table>
<thead>
<tr>
<th></th>
<th>$h$-conform</th>
<th>HTS</th>
<th>Ferromagnetic material</th>
</tr>
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<tbody>
<tr>
<td>Constitutive law</td>
<td>$e$</td>
<td>$j$</td>
<td>$b$</td>
</tr>
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[Dular, Geuzaine & Vanderheyden, IEEE TAS 2019]
Life-HTS: Why is flexibility needed?

- In addition to the difficulties inherent to the nonlinear constitutive laws, one should consider:
  - The number of degrees of freedom (depending on the polynomial approximation order) and the structure of the resulting matrices
  - The effect of the (adaptive) time-stepping scheme
  - The “best” formulation choice depends on the application
  - For problems with both HTS and ferromagnetic parts, coupling $h$- and $b$-conform formulation leads to the best results
  - Special care has to be paid to the discretization to ensure stability: tune in for Julien Dular’s talk tomorrow at 10:30
- Additional flexibility is required for handling HTS tapes: thin shell approximation (or not), homogenization of multiple tapes (or not), ...
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Life-HTS: Main features

- $h$- and $b$-conform formulations, uncoupled or coupled ($h$, $h-\phi$, $t-\omega$, $a$, $a-v$, $a^*$, $h-(\phi-)a$, $t-a$, ...):
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- Easy coupling of fields and formulations, staggered or monolithic, for multi-physics coupling (mechanical, thermal)
  - E.g. explicit Jacobian for strongly coupled magneto-thermal problem
Life-HTS: Main features

- Transient analysis with adaptive time stepping (Euler, Crank-Nicholson and BDF schemes) for calculating
  - field maps
  - magnetization
  - eddy currents
  - losses
  - ...

- Linear algebra through PETSc
- Built-in Python and Octave interpreters
- Flexible templating mechanism, allowing one to build a library of generic formulations
- Parameterizable graphical user interface through ONELAB
  - control any simulation parameter
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- Closeness between
  - the input data defining discrete problems (written in plain text .pro files), and
  - the symbolic mathematical expressions of these problems
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  - the symbolic mathematical expressions of these problems
- New models developed through .pro files — no compilation
- Highly portable: exact same .pro files on tablet, laptop or supercomputer
- At the core of template .pro files, weak formulations are written symbolically
Life-HTS: Main features

For example, in a .pro file, the weak formulation: Find \( u(x) \in H^1_0(\Omega) \) such that

\[
- \int_{\Omega} a(x) \nabla u \cdot \nabla u' \, d\Omega = \int_{\Omega} f(x) u' \, d\Omega, \quad \forall u' \in H^1_0(\Omega)
\]

is transcribed as

```plaintext
Formulation{
  { Name MyFirstFormulation; Type FemEquation;
    Quantity {
      { Name u; Type Local; NameOfSpace H1_0; }
    }
  }
  Equation {
    Integral { [ -a[] * Dof{d u}, {d u} ];
      In Omega; Integration I; Jacobian J; }
    Integral { [ -f[], {u} ];
      In Omega; Integration I; Jacobian J; }
  }
}
```
Life-HTS: Main features

Similarly, here is a bare-bones $h$ or $h-$ϕ formulations (they are the same—only the function space HSpace changes!)

```plaintext
Formulation {
  { Name MagDynH; Type FemEquation; 
    Quantity {
      { Name h; Type Local; NameOfSpace HSpace; } 
    } 
  } 
  Equation {
    Integral { Dtdof [ mu[] * Dof{h} , {h} ]; 
      In Omega; Integration Int; Jacobian Vol; } 
    Integral { [ rho[{d h}] * {d h} , {d h} ]; 
      In OmegaC; Integration Int; Jacobian Vol; } 
    Integral { [ dEdj[{d h}] * Dof{d h} , {d h} ]; 
      In OmegaC; Integration Int; Jacobian Vol; } 
    Integral { [ - dEdj[{d h}] * {d h} , {d h} ]; 
      In OmegaC ; Integration Int; Jacobian Vol; } 
  } 
}
```
Examples
### 3D HTS Magnet Motor Pole Modelling

#### Table I: Description of the different formulations in 3D problems

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Function space</th>
<th>Support of DOFs</th>
<th>$\sigma \neq 0$ in $\Omega_c^C$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$-formulation</td>
<td>$\mathcal{H}(\Omega) = { h \in H(\text{curl};\Omega) }$</td>
<td>Edges in $\Omega$, Edges in $\Omega_c^C$, nodes in $\Omega_c^C$</td>
<td>Yes</td>
</tr>
<tr>
<td>$h$-$\phi$-formulation</td>
<td>$\mathcal{H}_\phi(\Omega) = { h \in H(\text{curl};\Omega)</td>
<td>\text{curl} h = 0 } \text{ in } \Omega_c^C }$</td>
<td>Edges in $\Omega_c^C$, Edges in $\Omega_c^C$, nodes in $\Omega_c^C$</td>
</tr>
<tr>
<td>$\bar{a}$-formulation</td>
<td>$\bar{A}(\Omega) = { a \in H(\text{curl};\Omega) }$</td>
<td>Edges in $\Omega_c$, Edges in $\Omega_c$, facets in $\Omega_c^C$</td>
<td>(Yes)</td>
</tr>
<tr>
<td>$a$-formulation</td>
<td>$\mathcal{A}(\Omega) = { a \in H(\text{curl};\Omega) $</td>
<td>Edges in $\Omega_c^C$, Edges in $\Omega_c^C$, facets in $\Omega_c^C$</td>
<td>No</td>
</tr>
<tr>
<td>$h$-$a$-formulation</td>
<td>$h \in \mathcal{H}(\Omega_c^C), a \in \mathcal{A}(\Omega_c^C)$</td>
<td>Edges in $\Omega_c^C$, Edges in $\Omega_c^C$, facets in $\Omega_c^C$</td>
<td>No</td>
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<tr>
<td>$h$-$\phi$-$a$-formulation</td>
<td>$h \in \mathcal{H}_\phi(\Omega_a), a \in \mathcal{A}(\Omega_a)$</td>
<td>Edges in $\Omega_h^C$, Edges in $\Omega_h^C$, facets in $\Omega_a$</td>
<td>No</td>
</tr>
</tbody>
</table>

**Fig. 2:** Current density ($J$) is represented. For (b)-(d), the component along $x$ is depicted.

One eight of the geometry (air domain not shown)

---

**TABLE II: Efficiency of the different formulations**

<table>
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<tr>
<th>Formulation</th>
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<th>CPU time</th>
</tr>
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<tbody>
<tr>
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<td>16,070</td>
<td>1,108</td>
<td>1h16</td>
</tr>
<tr>
<td>$h$-$\phi$-formulation</td>
<td>32,045</td>
<td>1,124</td>
<td>1h25</td>
</tr>
<tr>
<td>$\bar{a}$-formulation</td>
<td>26,964</td>
<td>3,147</td>
<td>3h07</td>
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<tr>
<td>$h$-$a$-formulation</td>
<td>35,532</td>
<td>4,057</td>
<td>5h58</td>
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We run simulations with GetDP. The HTS is always put in a precomputed source field. Systems containing both HTS and FM are advantageously decomposed into two parts: a $\bar{a}$-formulation, choosing $\bar{a}$-formulation for HTS bulk, and a $h$-formulation, only the $h$-formulation is used elsewhere. We will describe the formulations in more details, and compare with results from Comsol.

In the full paper, we will describe the formulations in more details, and compare with results from Comsol.
3D HTS Magnet Motor Pole Modelling

Current density in the bulk during magnetizing pulse and relaxation

[Dular et al., 2021]

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<td>35,532</td>
<td>4,057</td>
<td>5h58</td>
</tr>
<tr>
<td>$h$-$\phi$-formulation</td>
<td>12,172</td>
<td>3,937</td>
<td>3h38</td>
</tr>
<tr>
<td>$\bar{a}$-formulation</td>
<td>29,010</td>
<td>2,955</td>
<td>4h45</td>
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<tr>
<td>$a$-formulation</td>
<td>26,964</td>
<td>3,147</td>
<td>3h07</td>
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<tr>
<td>$h$-$a$-formulation</td>
<td>32,045</td>
<td>1,124</td>
<td>1h25</td>
</tr>
<tr>
<td>$h$-$\phi$-$a$-formulation</td>
<td>16,070</td>
<td>1,108</td>
<td>1h16</td>
</tr>
</tbody>
</table>
Improving HTS magnetic shields with a soft ferromagnetic material

Shielding an axial field with a HTS tube

Shielding with an additional ferromagnetic tube

[Lousberg et al., TAS 2010]
Protecting a bulk HTS against crossed-field demagnetisation with a ferromagnetic layer

Sequence of applied fields

Current distribution in the bulk with a ferromagnetic top layer ($\mu_r = 10, 100$)

[Fagnard et al., SUST 2016]
Magnetic shielding in inhomogeneous fields

<table>
<thead>
<tr>
<th>$J_c$ vs $B$</th>
<th>$J_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>$J_c(B)$</td>
<td>$J_c$</td>
</tr>
<tr>
<td>(c)</td>
<td>(d)</td>
</tr>
</tbody>
</table>

[Hogan et al., SUST 2018]
Magnetic shielding, bulk superconducting cylinders and caps

Induced currents vs. geometries

Tracking stray fields in composite shields

[Fagnard et al., SUST 2019]
Critical states in stacked Niobium films

Peculiar patterns of discontinuity lines in stacks of Nb films

$L = 200 \, \mu m, \, d = t = 300 \, nm$

Needs to include a genuine $J_c(B)$-dependence!

[Raising field stage]

[Decreasing field stage]

[Burger et al., SUST 2019]
Critical states in the presence of a ratchet pinning potential

Experiment: rotation of the central discontinuity line in the decreasing field stage, after magnetization

Model: an anisotropic pinning force reproduces the result

\[ \mu_0 H_a = 3.11 \text{ mT} \]
\[ \mu_0 H_a = 1 \text{ mT} \]
\[ \mu_0 H_a = 0.75 \text{ mT} \]
\[ \mu_0 H_a = 0 \text{ mT} \]

Rotating HTS motor

Pulse magnetization \((h-a\text{-formulation})\)

\[
I_{B\pm}(t) = -I_{C\pm}(t) = \pm I_{\text{max}} \frac{t}{\tau} \exp(1 - t/\tau), \quad I_{A\pm}(t) = 0
\]

3-phase \((A-B-C)\) motor mode \((a\text{-formulation})\)

\[
I_{A\pm}(t) = \pm I_{\text{max}} \sin(\omega t)
\]
\[
I_{B\pm}(t) = \pm I_{\text{max}} \sin(\omega t + 2\pi/3)
\]
\[
I_{C\pm}(t) = \pm I_{\text{max}} \sin(\omega t - 2\pi/3)
\]
2D axisymmetric model of moving bulk superconductors

Comparison between the model predictions and the experimental measurements:

[Houbart & Vanderbemden, 2021]
Coil of HTS Tapes

\( h-a^* \) formulation with thermal coupling; tapes in parallel, series or end-coupled

Current redistribution phenomena for current-driven tapes connected in parallel

Good agreement with reference results from COMSOL

[Schnaubelt, Bortot & Schöps, 2021]
Conclusions and perspectives
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  - Modelling freedom (but some coding is necessary)
  - No licensing cost (from laptop to supercomputer)

- Accessibility, reproducibility and interoperability (free and open source)
- Encapsulated and scriptable
- Easy installation (binary distribution for Windows, Linux and macOS)
- Mature code base (20+ years), successfully used both in academia and in industry
- More "Swiss Army knife" than "bazooka" (but steeper learning curve than e.g. ANSYS or COMSOL; and full customizability makes it harder to document)

The future is exciting!

- More examples (magneto-thermal, quench)
- New formulations (helicoidal coordinates, quasi-2D)
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Thanks for your attention

http://www.life-hts.uliege.be

✉ cgeuzaine@uliege.be