Karlsruhe Institute of Technology

Homogenization in 3D based on the T-A formulation

Vargas-Llanos, Carlos
Grilli, Francesco

T-A formulation for 3D homogenization

T-A Formulation

$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \boldsymbol{J} = \nabla \times \boldsymbol{T}$

Faraday‘s law

$$\nabla \times \left(\rho_{HTS} \nabla \times \boldsymbol{T}\right) = -\frac{\partial \boldsymbol{B}}{\partial t}$$

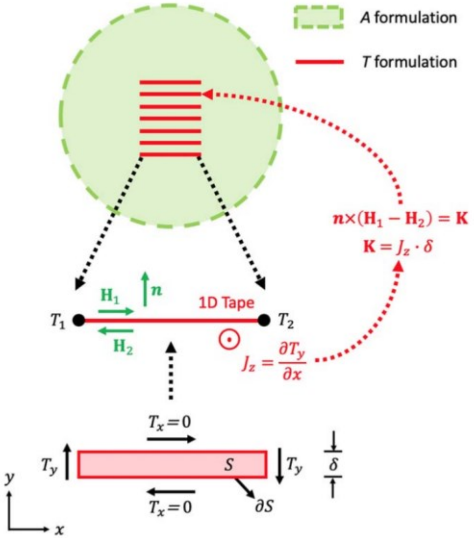
Ampere‘s law

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A}\right) = \boldsymbol{J}$$

(Comsol‘s magnetic field module)

HTS model

$$\rho_{HTS} = \frac{E_c}{J_c(\boldsymbol{B})} \left| \frac{\boldsymbol{J}}{J_c(\boldsymbol{B})} \right|^{n-1}$$



How to enforce the current?

$$I = \iint_S \boldsymbol{J} ds = \iint_s \nabla \times \boldsymbol{T} ds = \oint_{\partial s} \boldsymbol{T} dl$$

$$I = (T_1 - T_2)\delta$$

(Dirichlet Boundary Condition)

δ: *Thickness of the superconducting layer*

*E. Berrospe-Juarez, V. M. R. Zermeno, F. Trillaud and F. Grilli, “*Real-time simulation of large-scale HTS systems: multi-scale and homogeneous models using the T-A formulation*”, Superconductor Science Technology, Vol. 32, p. 065003, 2019.

T-A Homogenization

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \boldsymbol{J} = \nabla \times \boldsymbol{T}$$

Faraday‘s law

$$\nabla \times \left(\rho_{HTS} \nabla \times \boldsymbol{T}\right) = -\frac{\partial \boldsymbol{B}}{\partial t}$$

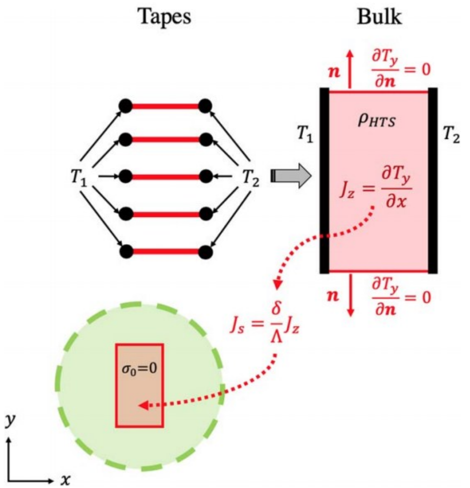
Ampere‘s law

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A}\right) = \boldsymbol{J}$$

(Comsol‘s magnetic field module)

HTS model

$$\rho_{HTS} = \frac{E_c}{J_c(\boldsymbol{B})} \left| \frac{\boldsymbol{J}}{J_c(\boldsymbol{B})} \right|^{n-1}$$



How to enforce the current?

$$\frac{\partial T_y}{\partial \boldsymbol{n}} = 0$$

(Neumann Boundary Condition)

$$I = (T_1 - T_2)\delta$$

(Dirichlet Boundary Condition)

$$J_s = \frac{\delta}{\Lambda} J_z$$

δ: *Thickness of the superconducting layer*
Λ: *Thickness of the unit cell*

*E. Berrospe-Juarez, V. M. R. Zermeno, F. Trillaud and F. Grilli, “*Real-time simulation of large-scale HTS systems: multi-scale and homogeneous models using the T-A formulation*”, Superconductor Science Technology, Vol. 32, p. 065003, 2019.

3D T-A Formulation

Karlsruhe Institute of Technology

Faraday's law

$$\begin{bmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{bmatrix} \vec{n} + \begin{bmatrix} \frac{\partial B_x}{\partial t} \\ \frac{\partial B_y}{\partial t} \\ \frac{\partial B_z}{\partial t} \end{bmatrix} \vec{n} = 0$$

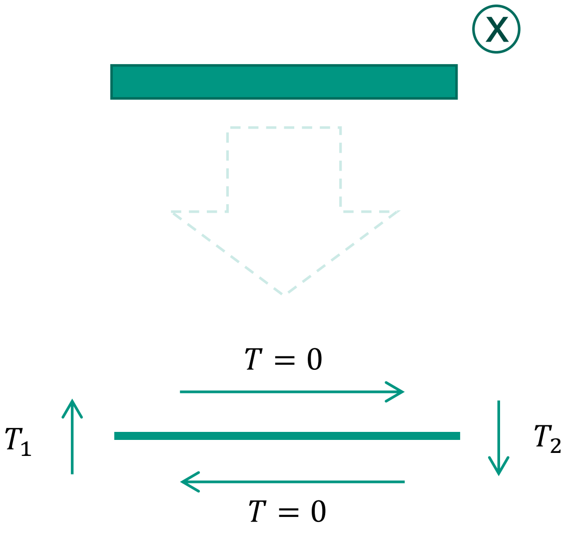
HTS model

$$\rho_{HTS} = \frac{E_c}{J_c(\boldsymbol{B})} \left| \frac{\boldsymbol{J}}{J_c(\boldsymbol{B})} \right|^{n-1}$$

Current vector potential

$$\boldsymbol{J} = \nabla \times \boldsymbol{T}$$

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \frac{\partial(T \cdot n_z)}{\partial y} - \frac{\partial(T \cdot n_y)}{\partial z} \\ \frac{\partial(T \cdot n_x)}{\partial z} - \frac{\partial(T \cdot n_z)}{\partial x} \\ \frac{\partial(T \cdot n_y)}{\partial x} - \frac{\partial(T \cdot n_x)}{\partial y} \end{bmatrix}$$



And the current is enforced as:

$$I = \iint_S \boldsymbol{J} ds = \iint_S \nabla \times \boldsymbol{T} ds = \oint_{\partial s} \boldsymbol{T} dl$$

$$I = (T_1 - T_2)\delta$$

*H. Zhang, M. Zhang, and W. Yuan, "An efficient 3D finite element method model based on the T-A formulation for superconducting coated conductors", Superconductor Science Technology, Vol. 30, p. 024005, 2017.

3D T-A Homogenization

Faraday's law

$$\begin{bmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{bmatrix} \vec{n} + \begin{bmatrix} \frac{\partial B_x}{\partial t} \\ \frac{\partial B_y}{\partial t} \\ \frac{\partial B_z}{\partial t} \end{bmatrix} \vec{n} = 0$$

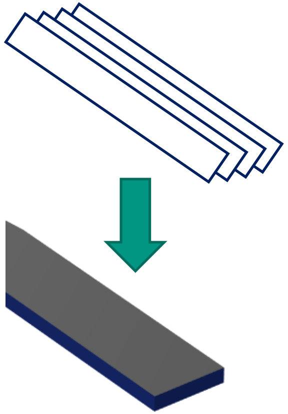
Current vector potential

$$\boldsymbol{J} = \nabla \times \boldsymbol{T}$$

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} \frac{\partial (T \cdot n_z)}{\partial y} - \frac{\partial (T \cdot n_y)}{\partial z} \\ \frac{\partial (T \cdot n_x)}{\partial z} - \frac{\partial (T \cdot n_z)}{\partial x} \\ \frac{\partial (T \cdot n_y)}{\partial x} - \frac{\partial (T \cdot n_x)}{\partial y} \end{bmatrix}$$

HTS model

$$\rho_{HTS} = \frac{E_c}{J_c(\boldsymbol{B})} \left| \frac{\boldsymbol{J}}{J_c(\boldsymbol{B})} \right|^{n-1}$$

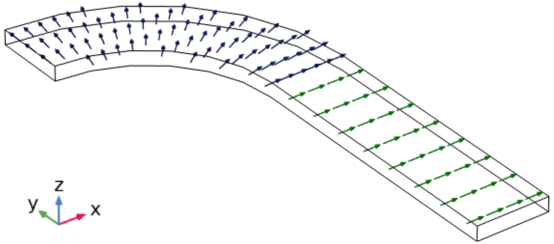


- How can we get $\vec{n}$ inside a volume?
- How can we enforce the current?

3D T-A Homogenization

How can we get $\vec{n}$ inside a volume?

Circular section (centered in (0,0,0))	Straight section (centered in (0,0,0))
$n_x = \frac{X}{\sqrt{X^2+Y^2}}$	$n_x = 1$
$n_y = \frac{Y}{\sqrt{X^2+Y^2}}$	$n_y = 0$
$n_z = 0$	$n_z = 0$



How can we enforce the current?

Neumann Boundary Condition

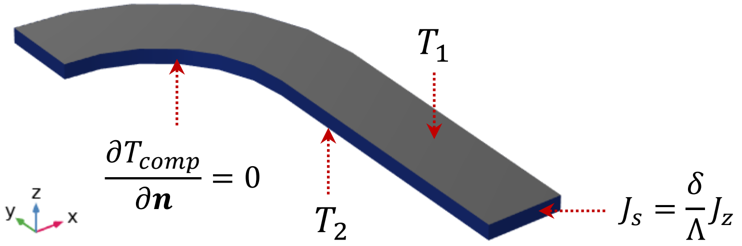
$$\frac{\partial T_{comp}}{\partial \boldsymbol{n}} = \frac{\partial\left(n_x \cdot T_x + n_y \cdot T_y\right)}{\partial \boldsymbol{n}} = 0$$

Dirichlet Boundary Condition

$$I = (T_1 - T_2)\delta$$

$$J_s = \frac{\delta}{\Lambda} J_z$$

δ: Thickness of the superconducting layer
Λ: Thickness of the unit cell



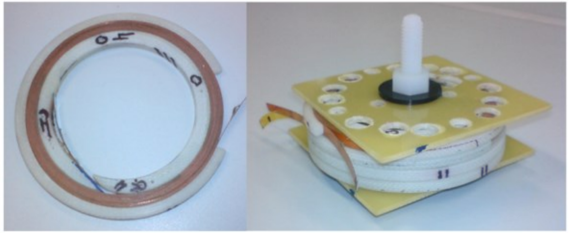
Circular coil

Brief description of the circular coil

Brief description of the circular coils

Internal radious	30 mm
External radious	33.9 mm
Turns	24
Frequency	36 Hz

n	25
Ec	1e-4 V/m
Tape width	4 mm
Ic tape 77 K	128 A



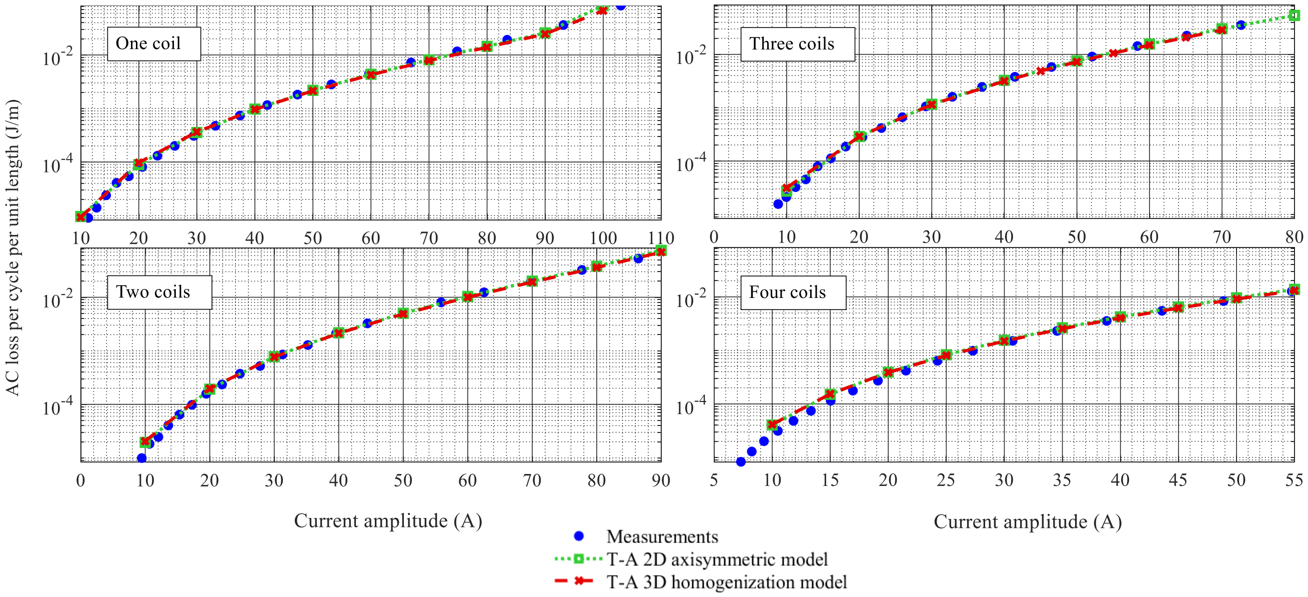
Outcomes

- Validation against measurements.
- Validation against a 2D Solution.
- Comparison of the normalized current penetration.

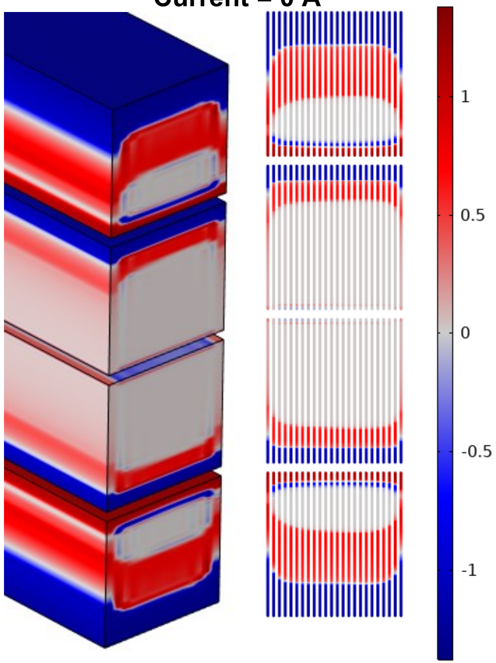
*E. Pardo, J. Šouc and L. Frolek, "*Electromagnetic modelling of superconductors with a smooth current–voltage relation: variational principle and coils from a few turns to large magnets*", Superconductor Science Technology, Vol. 28, p. 044003, 2015.

Circular coil models - Validation

AC transport losses



Normalized current penetration
– 55 A peak transport current simulation –
Current = 0 A



T – A
3D Homogenization

T – A
2D Axisymmetric

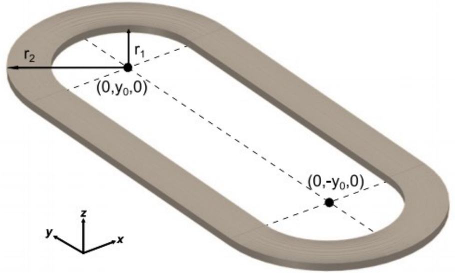
*E. Pardo, J. Šouc and L. Frolek, "*Electromagnetic modelling of superconductors with a smooth current–voltage relation: variational principle and coils from a few turns to large magnets*", Superconductor Science Technology, Vol. 28, p. 044003, 2015.

Racetrack coil verification

Racetrack coil verification

Brief description of the racetrack coil

Internal radious (r1)	35 mm
External radious (r2)	55 mm
Straight part lenght (y_0)	75 mm
Turns	50
Frequency	50 Hz



HTS Tape characteristics

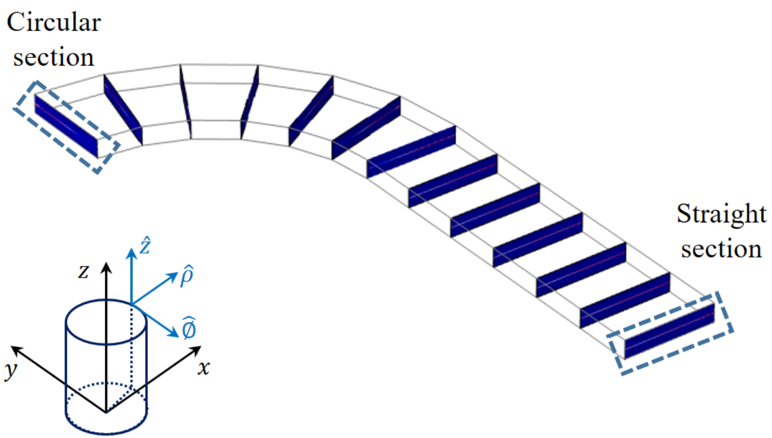
n	21
Ec	1e-4 V/m
Tape width	4 mm
Ic tape 77 K	160 A

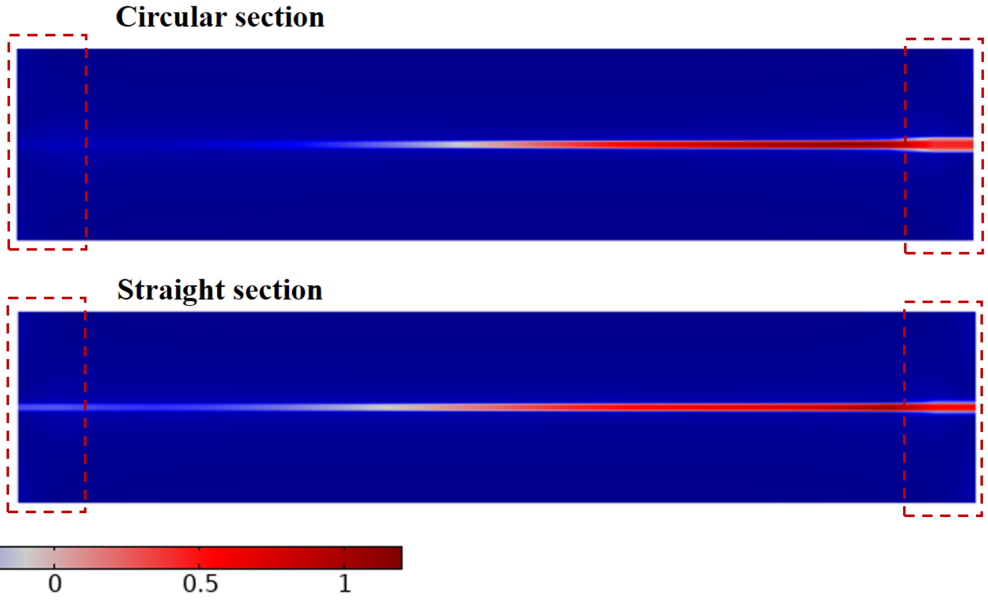
$J_{co} = 49GAm^{-2}$
$k = 0.275$
$B_c = 32.5\ mT$
$b = 0.6$

$$J_c(B_\parallel, B_\perp) = \frac{J_{c0}\ f_{\mathrm{HTS}}}{\left[1 + \sqrt{(B_\parallel k)^2 + B_\perp^2} \middle/ B_c\right]^b}$$

*V. Zermeno and F. Grilli, "*3D modeling and simulation of 2G HTS stacks and coils*", Superconductor Science Technology, Vol. 27, p. 044025, 2014.

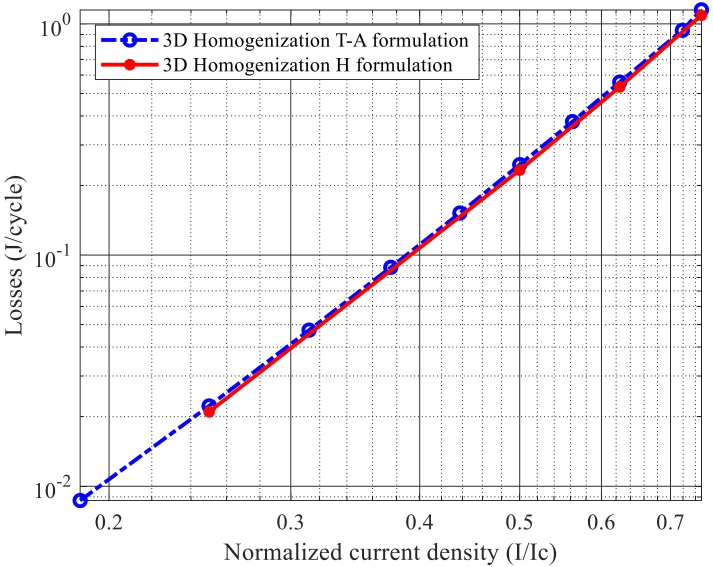
Racetrack coil – Transport current = 120 A – t=0.01425
Normalized current behaviour (J/Jc)





Racetrack coil verification

AC transport losses as a function of normalized current density

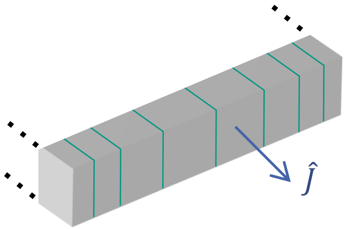


- Degrees of freedom:
  - T-A 3D homogenization (178484).
  - H 3D homogenization (176138).
- Mesh:
  - T-A 3D homogenization: coarse.
  - H 3D homogenization: fine.
- The calculations were made with a 3.60 GHz Intel Core i7-4960X computer with 12 logical processors and 64GB of RAM.

*V. Zermeno and F. Grilli, "*3D modeling and simulation of 2G HTS stacks and coils*", Superconductor Science Technology, Vol. 27, p. 044025, 2014.

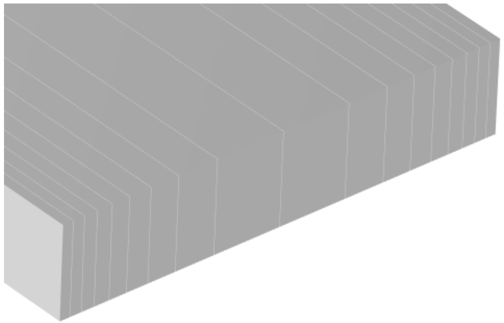
Racetrack coil verification

3D Homogenization H formulation



Possible current sharing between sub-domains

- 2D integral constraint.
- Use of anisotropic resistivity.
- Manual discretization of the homogenized bulk.



3D Homogenization T-A formulation

This restriction is included in the essence of the T-A homogenization by applying the thin strip approximation which constraints the current to flow in the plane parallel to the tape.

*V. Zermeno and F. Grilli, “*3D modeling and simulation of 2G HTS stacks and coils*”, Superconductor Science Technology, Vol. 27, p. 044025, 2014.

Racetrack coil verification

Computation time

Normalized current (I/Ic_tape)	Time H formulation	Time T-A formulation	Percentage reduction
0.25	17.38 h	3.19 h	81.64
0.5	29.74 h	5.66 h	80.98
0.625	35.16 h	6.76 h	80.77
0.75	41.18 h	8.06 h	80.43

Fair comparison between T-A and H homogenization 3D?

- Degrees of freedom:
  - T-A 3D homogenization (178484).
  - H 3D homogenization (176138).
- Mesh:
  - T-A 3D homogenization: coarse.
  - H 3D homogenization: fine.
- The calculations were made with a 3.60 GHz Intel Core i7-4960X computer with 12 logical processors and 64GB of RAM.

*V. Zermeno and F. Grilli, "*3D modeling and simulation of 2G HTS stacks and coils*", Superconductor Science Technology, Vol. 27, p. 044025, 2014.

Thanks!