Coupled multiphysics modelling of the thermal-magnetic-mechanical instability in bulk superconductors during magnetization

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Contents

◆ Background

◆ Research Progress
  ✓ Numerical modelling and formulisms
  ✓ Flux jumps in Bulk MgB2
  ✓ Results and Discussions

◆ Summary
Background: Vortices and Pinning

Type-II Superconductors

Vortex lattice
A. A. Abrikosov
2003
(published 1957)

Normal State

Mixed State
(Vortex matter)

Meissner State

$H_{c1}$

$H_{c2}$

$T_c$

Lorentz force on flux lines ($F_L$)

Transport current ($J$)

Applied magnetic field ($H$)
Background: Challenges in the applications

◆ Mechanical behaviors
  - Electromagnetic force
  - Thermal stress
  - Manufacturing stress
  - …

Delamination of SC films

Degradation of critical parameters

S. Hahn, Nature, 2019

van der Laan and Ekin (2010)
Background: Challenges in the applications

◆ Thermomagnetic instability

Thermal fluctuations
- Electromagnetic force induced motion
- Ac /Magnetization Loss
- Joule heating etc.

Varying magnetic fields

Flux jumps in Nb₃Sn strands

Flux avalanches in SC thin films

CERN-ATS-2013

- 1.9 K
- 4.3 K

Reduced Pinning
Local heating
Positive feedback loop
Vortex motion

Induced Electric potential

MgB₂
Nb₃Sn
a-MoSi
YBCO
Instability phenomenon is widely encountered in nature and engineering, which sharing some common characteristics.

- Brittle fracture
- Electric breakdown
- Thermomagnetic instability

Mechanical breakdown

Lightning

Flux avalanches

<table>
<thead>
<tr>
<th>Loads</th>
<th>Field</th>
<th>Electric potential</th>
<th>Magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stress/strain</td>
<td>Electric-thermal field</td>
<td>Electromagnetic-thermal field</td>
</tr>
<tr>
<td>Equations</td>
<td>Force Equilibrium</td>
<td>Charge Conservation and heat diffusion</td>
<td>Maxwell equations and heat diffusion</td>
</tr>
<tr>
<td>Velocity</td>
<td>supersonic</td>
<td>&lt;μs</td>
<td>Sub-μs or ns</td>
</tr>
</tbody>
</table>
Background: thermomagnetic instability of SCs

Experiments Magneto-optical imaging

- Faraday Effect:

\[ \alpha_F = V B_z d \]

MOI of YBCO thin film at 10 K

Background: thermomagnetic instability of SCs

Jumping in the magnetization curves

![Graph showing magnetization curves with smooth and jumping behaviors.](image)

$\tau = 0.42$  
$\tau = 0.34$


Blocking the channel for transport current in superconductor film

![SEM images showing microstructural degradation.](image)


Noise in the Nb Nano-SQUID

![Graph showing flux noise in a Nb Nano-SQUID.](image)

$T = 4.5 \text{ K}$  
$0.34 \mu \Phi_0/\text{Hz}^{1/2}$


Microstructural degradation in the film during quench –SEM images.

![SEM images showing microstructural changes.](image)
Superconductors, under coupled multi-physical fields, experience severe thermomagnetic and mechanical instability problems.

- AC Loss
- Hysteresis Loss
- Joule Heating

\[ f_L = J \times B \]

\[ J_c(B,T) \]

- Thermal expansion
- Temperature dependent constitutive laws

\[ \text{Displacement (Equilibrium Equation)} \]

\[ \text{Temperature (Heat Equation)} \]

\[ \text{Electromagnetic field (Maxwell Equation)} \]
Background: Instability in bulk SCs

\[ B_{app} = B_{max} \frac{t}{\tau} \exp \left(1 - \frac{2t}{\tau}\right) \]


Ainslie et al. SuST. 29 (2016) 124004
# Modelling framework

## Governing Equations and workflow

### Maxwell Equations
- \( \nabla \times \mathbf{H} = \mathbf{J} \)
- \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \cdot \mathbf{B} = 0 \)

### E-J power law
- \( E = E_0 |J/J_{c\phi}|^n \quad J_{c\phi} = [(1 - \phi)^2 + k]J_c(T, B) \)

### Heat Equations
- \( \rho C \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) + \mathbf{E} \cdot \mathbf{J} \)

### Mechanical equilibrium
- \( \rho \ddot{u}_i = \sigma_{ij,j} + f_i \)

### Hooke’s Law
- \( \sigma = [(1 - \phi)^2 + k][\lambda \langle tr(\varepsilon) \rangle_+ I + 2\mu \varepsilon_+] + [\lambda \langle tr(\varepsilon) \rangle_- I + 2\mu \varepsilon_-] \)

### Geometry relation
- \( \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \)

### Phase field crack
- \( \left( \frac{G_c}{2l_0} + \mathcal{H} \right) \phi - \frac{1}{2} G_c l_0 \Delta \phi = \mathcal{H} \quad \text{in} \Omega \)

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**Thermomagnetic instabilities**

**Mechanical failure**
Research Progress: Instability in bulk SCs

Electromagnetics:
- Bulk – H-formulation
- Thin film – T-A formulation

Thermal field: Comsol Multiphysics
- Heat diffusion

Mechanical stress and failure:
- Solid mechanics+ Phase-field fracture

Axisymmetric

Vacuum Domain

SC Bulk

E-J power law

\[ E = E_0 \left| \frac{J}{J_c} \right|^{n-1} \frac{J}{J_c} \]

\[ J_c(T, B, \varepsilon, \phi) = s_T s_B s_\phi J_{c0} \]

\[ s_T = [1 - (T/T_c)^2]^{1.5} \]

\[ s_B = \frac{B_0}{B_0 + B} \]

\[ s_\phi = (1 - \phi)^2 + k \]

<table>
<thead>
<tr>
<th>para</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_0 )</td>
<td>n value (E-J power law)</td>
<td>103</td>
</tr>
<tr>
<td>( E_0 )</td>
<td>Characteristic voltage</td>
<td>( 1 \times 10^{-4} ) V/m</td>
</tr>
<tr>
<td>( J_{c0} )</td>
<td>Critical current density in zero field</td>
<td>( 3.1 \times 10^9 ) A/m²</td>
</tr>
<tr>
<td>( T_c )</td>
<td>Critical temperature</td>
<td>39 K</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>Normal state resistivity (MgB2)</td>
<td>( 7.0 \times 10^{-8} ) Ω.m</td>
</tr>
<tr>
<td>( E_{bulk} )</td>
<td>Young’s modulus (MgB2)</td>
<td>50 GPa</td>
</tr>
<tr>
<td>( \nu_{bulk} )</td>
<td>Poisson’s ratio (MgB2)</td>
<td>0.33</td>
</tr>
<tr>
<td>( \sigma_{c,bulk} )</td>
<td>Tensile strength (MgB2)</td>
<td>20 MPa</td>
</tr>
<tr>
<td>( \alpha_{bulk} )</td>
<td>Thermal expansion coefficient (MgB2)</td>
<td>( 3.5 \times 10^{-6} ) K⁻¹</td>
</tr>
<tr>
<td>( G_{c,bulk} )</td>
<td>Critical energy release rate (MgB2)</td>
<td>20 N/m</td>
</tr>
</tbody>
</table>
Research Progress: Instability in bulk SCs

Model verification: flux jumps in MgB2

Xia et al, SuST 20(2017) 075004
Threshold magnetic fields Vs Parameters

Field ramp rate

Ambient temperature

\[ B_{f1} [T] \]

\[ \mu_0 H_{at} [T/s] \]

\[ T_{max} [K] \]

\[ T_0 [K] \]

\[ \mu_0 H_{at} [T/s] \]

\[ B_{f1} [T] \]
Model: bulk Superconductor in pulsed field

\[ B_{app} = B_{max} \frac{t}{\tau} \exp\left(1 - \frac{t}{\tau}\right) \]

- Bulk
  - 20 mm (D)
  - 10 mm (H)

- Solenoid Coil
  - 99 mm (ID)
  - 121 mm (OD)
  - 50 mm (H)

\( \mu H_a \) vs. Time \([t]\)
Research Progress: Instability in bulk SCs

Maximum Temperature in the Bulk during a typical pulsed field magnetization process: shows typical multiple time-scale behaviors.
Research Progress: Instability in bulk SCs

Maximum Temperature Distribution within the bulk during flux jumps.
Research Progress: Instability in bulk SCs

Magnetic flux Distribution within the bulk during flux jumps.

$t=1.3468\times 10^{-4}$

$t=2.9196\times 10^{-4}$

$t=0.062251$

$t=0.06251$

$t=0.12$
Research Progress: Instability in bulk SCs

Stress/strain distribution within the bulk during flux jumps.

Temperature [K]

Maximum Stress [MPa]

Time = 0.062868s

Time = 0.12s
Research Progress: Instability in bulk SCs

Influence of the ramp rate on the flux jumps ($\tau=120\text{ms}$)

- $H_0 M$ vs $t/t_{\text{period}}$
- $T_{\text{max}}$ vs $t/t_{\text{period}}$
- Maximum Stress vs $t/t_{\text{period}}$
- Damage vs $t/t_{\text{period}}$

$t=3.7534$

- Damage
- $\sigma_r$
- $\sigma_z$
- $\sigma_{\text{mises}}$
Research Progress: Flux Avalanches in bulk

Infinitely long cylinder bulk under pulsed field
A numerical model is proposed to simulate the Flux jumps, stress/strain and structural degradation in Bulk superconductors during pulsed field magnetization.

Due to the multiple timescales intrinsically involved in the instability behaviors, solving the coupled Multiphysics models numerically to simulate the thermo-magnetic-electro-mechanical instability is still a challenge task ahead.

More efficient simulation methods (such as the FFT based scheme) are needed to simulate the coupled instability behavior.
Thanks for your attention!