UNIVERSITY OF CAMBRIDGE

B|S|G

A new benchmark problem for electromagnetic modelling of superconductors: the high-T_c superconducting dynamo

Mark Ainslie *University of Cambridge*

Francesco Grilli *Karlsruhe Institute of Technology*
Loïc Quéval *University of Paris-Saclay*
Enric Pardo, Asef Ghabeli *Slovak Academy of Sciences*
Fernando Perez-Mendez *University of Cambridge*
Ratu Mataira, Chris Bumby *Robinson Research Institute*
Antonio Morandi *University of Bologna*
Roberto Brambilla *Retired, formerly Ricerca sul Sistema Elettrico*

HTS MOD 2020 [online] 22-23 June 2021

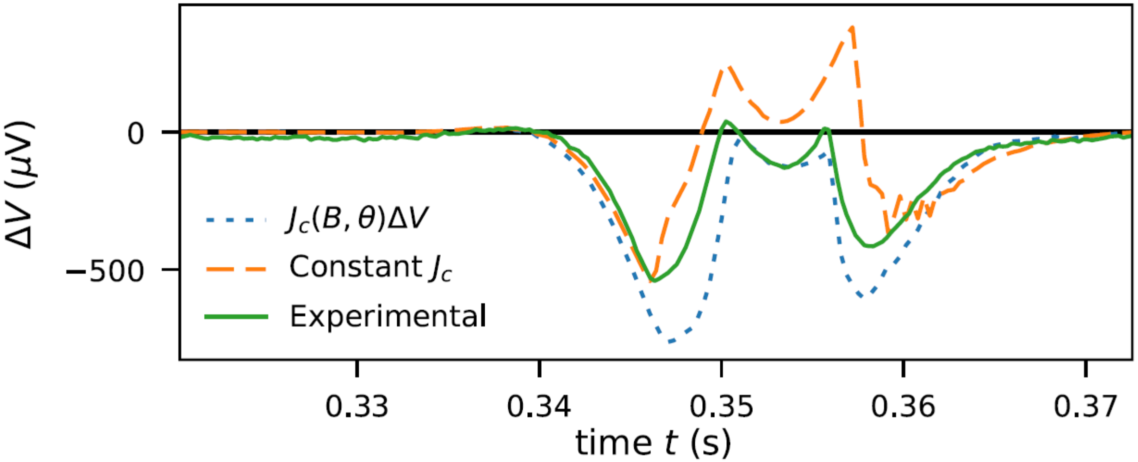
Presentation Outline

- **The high-T_c superconducting (HTS) dynamo**
- **Numerical modelling of the HTS dynamo**
  - Benchmark problem definition
  - Implementation using several different methods
  - Comparison of key results
  - Comparison of modelling frameworks

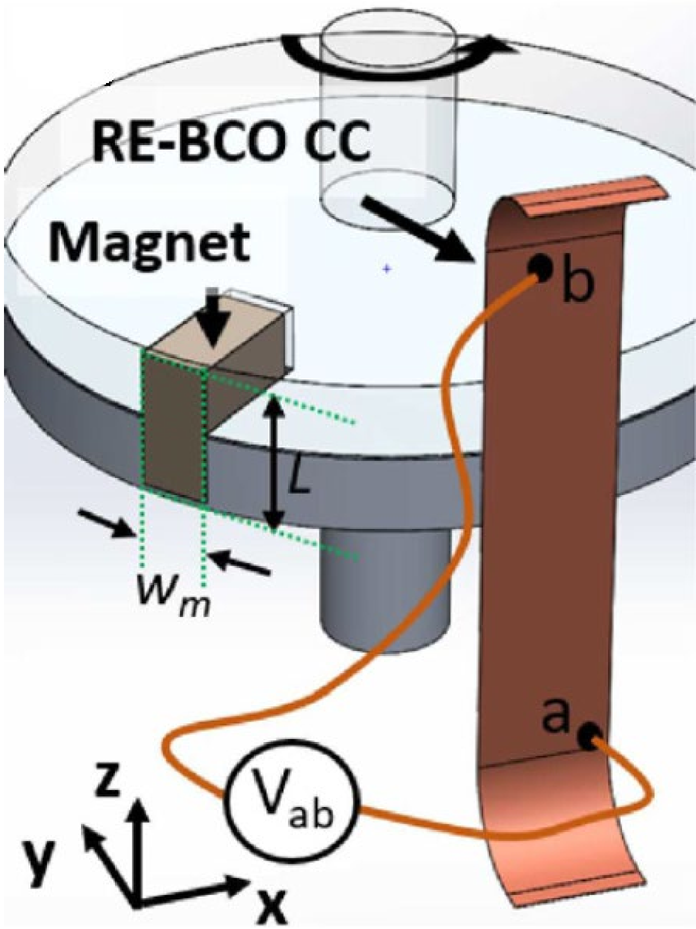
High-T_c Superconducting (HTS) Dynamo

Mataira et al. *APL* 114 (2019) 112601

- **Inject large DC supercurrents into a closed superconducting circuit**
  - Energise HTS coils in NMR/MRI magnets, superconducting rotating machines *without need for current leads*
  - Can be done across a cryostat wall

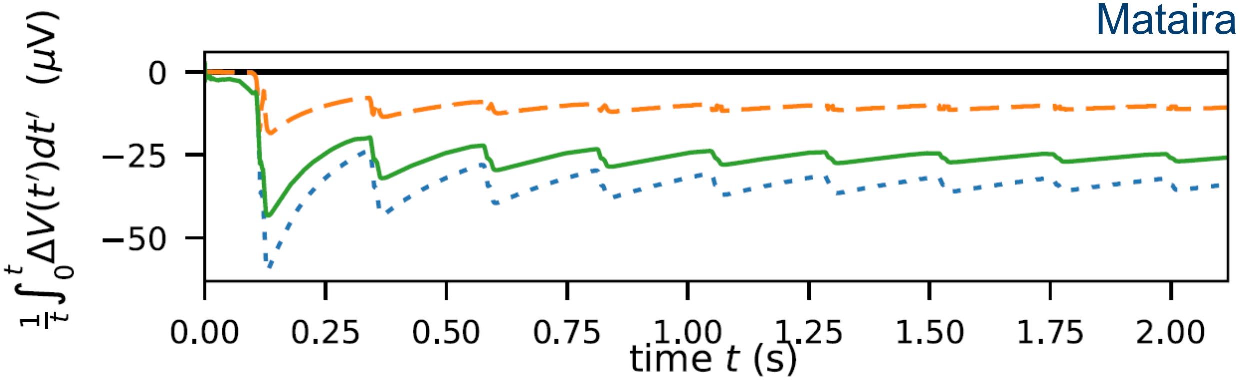


Open-circuit voltage: models and experiments

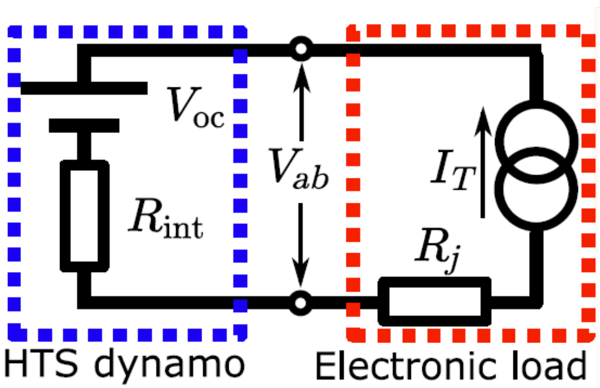


HTS Dynamo – Device Characterisation

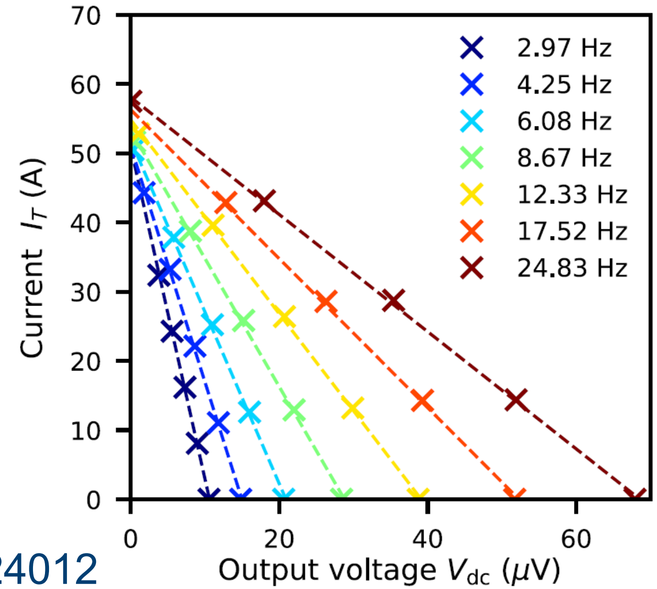
Mataira et al. *APL* 114 (2019) 112601



Cumulative time-averaged voltage



Circuit diagram incl. current supply for *I*-*V* characterisation

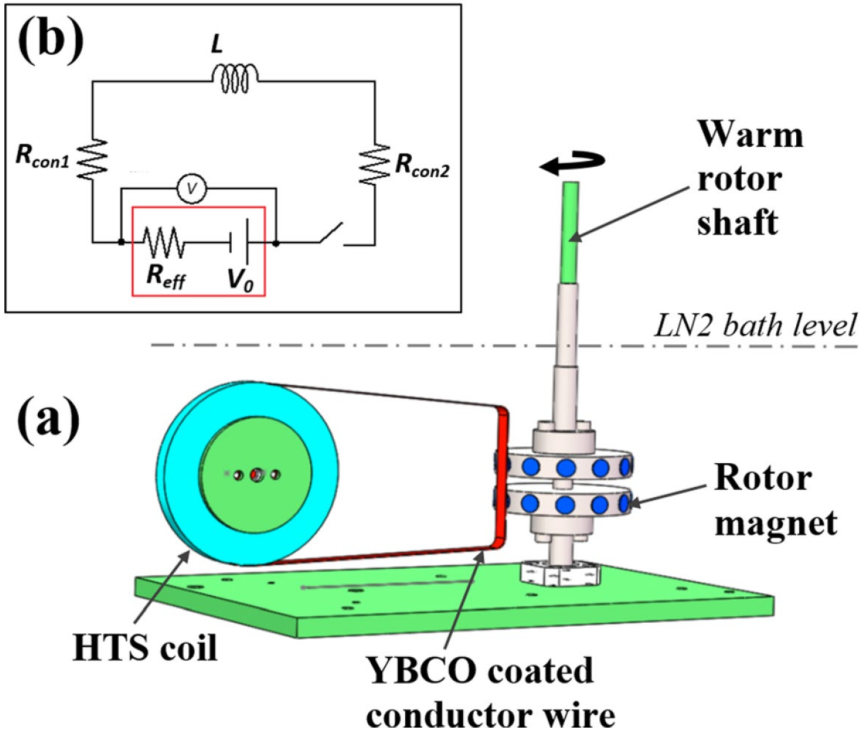


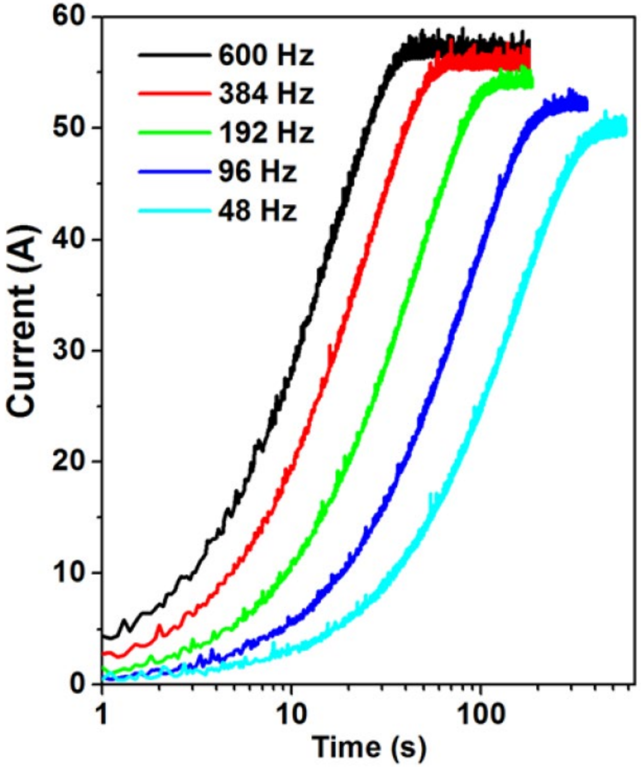
Typical *I*-*V* characterisation of HTS dynamo

Mataira et al. *Phys Rev Appl* 14 (2020) 024012

HTS Dynamo – Current Injection Without Leads

Can be used to drive large DC current into a superconducting coil, without current leads



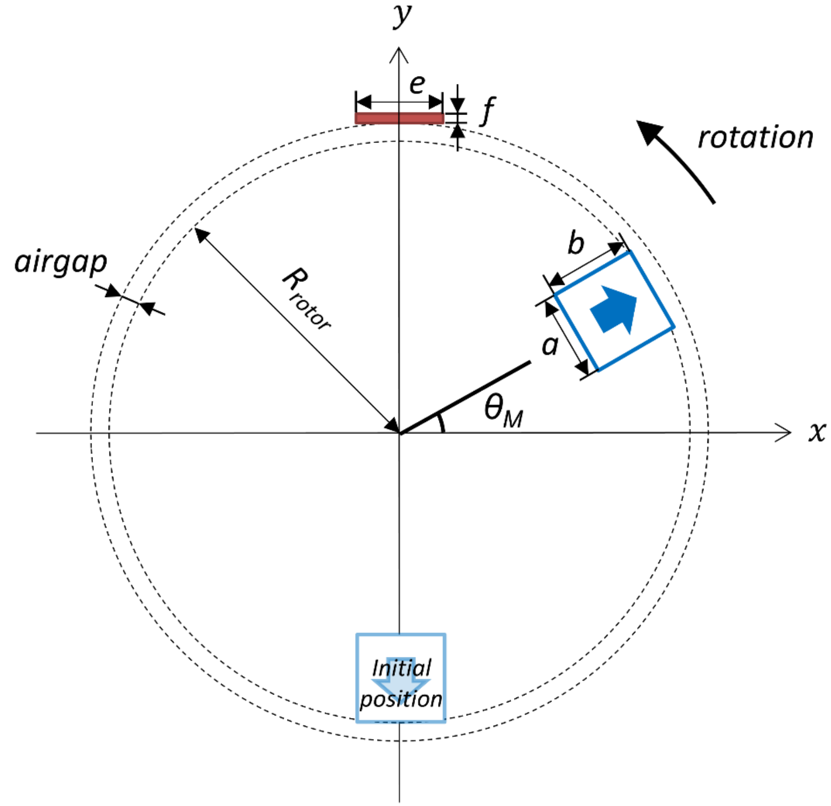


Jiang et al. *APL* 105 (2014) 112601

Benchmark Problem Definition

- **Several numerical models have recently been developed to model the HTS dynamo**
- **Benchmark problem:**
  - A specific simplified geometry with well-defined inputs (i.e. assumptions)
  - An expected set of outputs (i.e. the solution)
  - Allows any modelling technique to be validated & its performance critically compared

2D (infinitely long in *z*-direction)



Geometry of the HTS dynamo 2D benchmark problem

Benchmark Problem Definition

HTS dynamo benchmark parameters

Permanent magnet (PM)	Width, a	6 mm
	Height, b	12 mm
	Active length (depth), L	12.7 mm
	Remanent flux density, B_r	1.25 T
HTS stator wire	Width, e	12 mm
	Thickness, f	1 μm
	Critical current, I_c [self-field, 77 K]	283 A
	n value	20
Rotor external radius, R_{rotor}		35 mm
Distance between PM face and HTS surface, *airgap*		3.7 mm
Frequency of rotation		4.25 Hz
Number of cycles		10

Based on experimental setup in
Badcock et al. *IEEE TAS* 27 (2017) 5200905

General definitions

E-*J* power law: $$\mathbf{E} = \frac{E_0}{J_c}\left|\frac{J}{J_c}\right|^{n-1}\mathbf{J}$$

DC component:

$$V_{DC} = -\frac{L}{T}\int_t^{t+T} E_{ave}(t')\,dt'$$

Equivalent instantaneous voltage:

$$V_{eq}(t) = -LE_{ave}(t)$$

Cumulative time-averaged voltage:

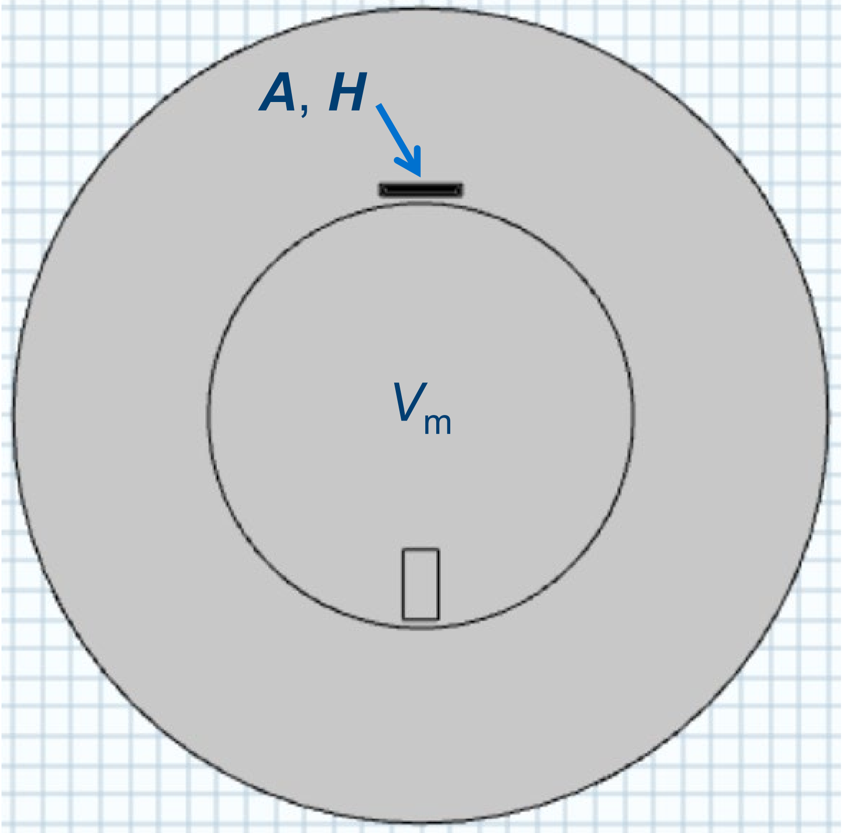
$$V_{cumul}(t) = \frac{1}{t}\int_0^t V_{eq}(t)\,dt$$

$$I(t) = \iint_S J_z(x, y, t)\,dS = 0$$

Numerical Modelling Frameworks – H-A

- **Coupled H-A formulation**
  - Models the *entire* rotating model, with rotating mesh
  - Most of model uses magnetic *scalar* potential, V_m

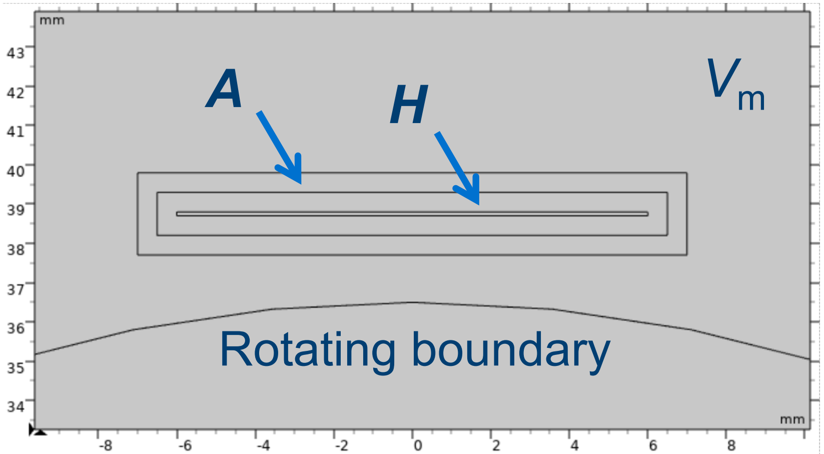
$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$



Numerical Modelling Frameworks – H-A

- **Coupled H-A formulation**
  - Models the *entire* rotating model, with rotating mesh
  - Most of model uses magnetic *scalar* potential, V_m
  - Vector potential, **A**, solved for small region around conductive (current-carrying) subdomain, i.e., the **H**-formulation subdomain including the HTS wire
  - V_m, **A** regions implemented in COMSOL's Rotating Machinery, Magnetic (RMM) interface
  - **H**-formulation: COMSOL's Magnetic Field Formulation (MFH) interface
  - Appropriate coupling between the mixed V_m-**A** and **H-A** formulations at boundaries

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

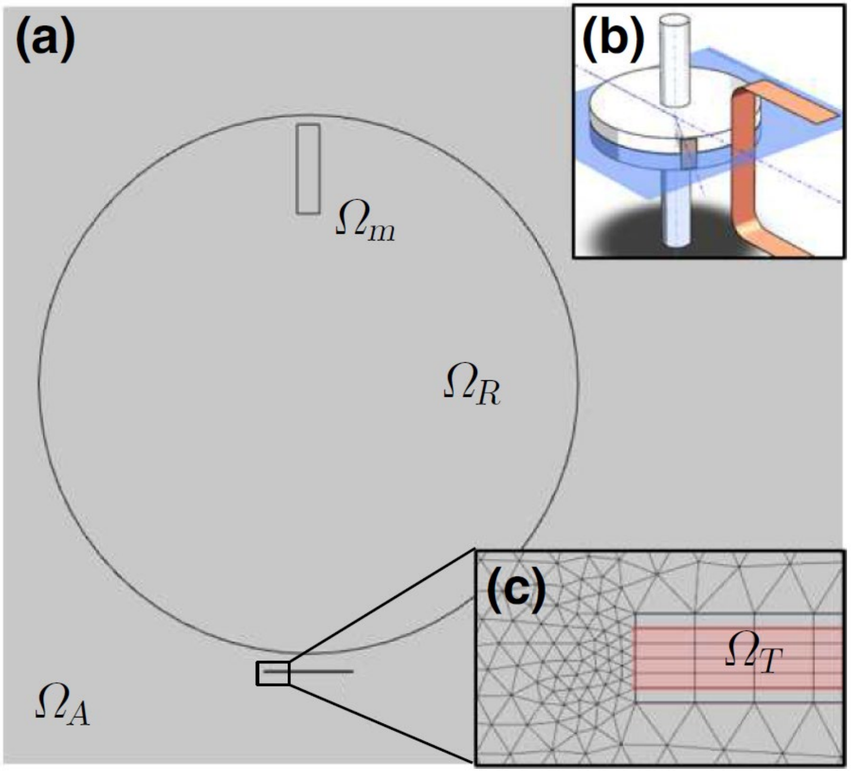


Numerical Modelling Frameworks – H+SC

- ***H*-formulation + shell current**
  - Represents PM as time-dependent sheet current, $\boldsymbol{K}_{\text{sheet}}$
  - Rotor is thus omitted, replaced with boundary condition:

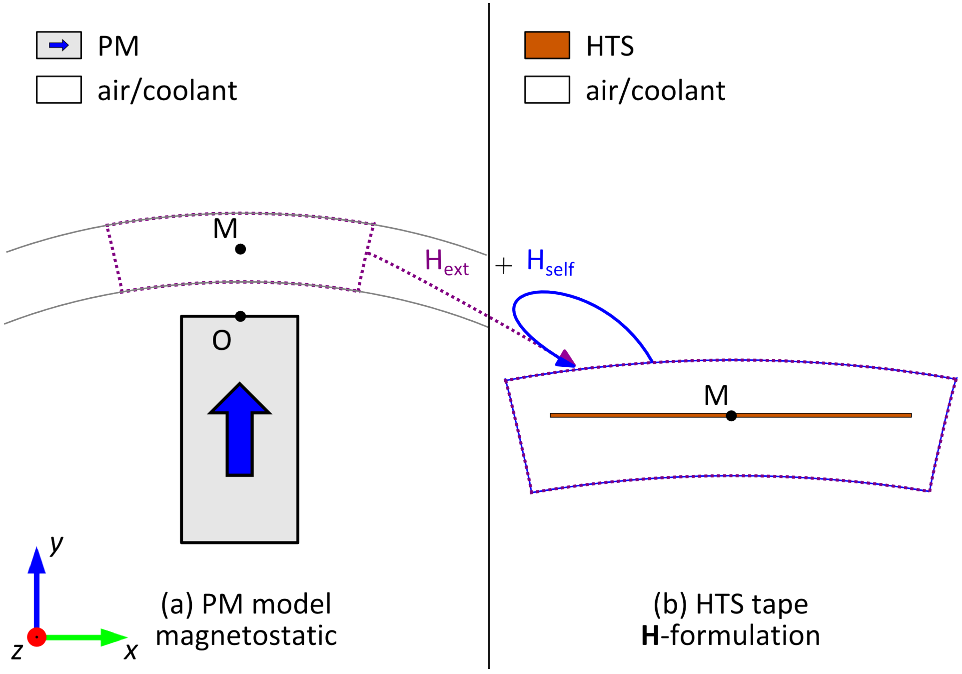
$$\nabla \times \mathbf{H}\big|_{\partial\Omega_R} = -\mathbf{K}_{shell}(\theta - \theta_M(t))$$

  - Avoids rotating mesh or inter-model couplings to capture rotation
  - Large number of mesh elements committed to simulating boundary of rotor domain



Numerical Modelling Frameworks – SEG-H

- **Segregated H-formulation**
  - *Magnetostatic* PM model + *time-dependent* **H**-formulation HTS model
  - Unidirectional coupling between PM and HTS models using boundary conditions + translation (rotation) operator



Numerical Modelling Frameworks – SEG-H

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  - *Magnetostatic* PM model + *time-dependent* **H**-formulation HTS model
  - Unidirectional coupling between PM and HTS models using boundary conditions + translation (rotation) operator
  - HTS model boundary: $\mathbf{H}_{ext}$ + $\mathbf{H}_{self}$
  - $\mathbf{H}_{ext}$ = rotated PM field
  - $\mathbf{H}_{self}$ = self-field from current in HTS wire, from Biot-Savart law

Rotated PM field

$$\begin{bmatrix} H_{ext,x}(x,y,t) \\ H_{ext,y}(x,y,t) \end{bmatrix} = \begin{bmatrix} \cos\theta_M(t) & \sin\theta_M(t) \\ -\sin\theta_M(t) & \cos\theta_M(t) \end{bmatrix} \times \begin{bmatrix} H_{PM,x}(x_{rot},y_{rot}) \\ H_{PM,y}(x_{rot},y_{rot}) \end{bmatrix}$$

$$\begin{bmatrix} x_{rot} \\ y_{rot} \end{bmatrix} = \begin{bmatrix} \cos\theta_M(t) & -\sin\theta_M(t) \\ \sin\theta_M(t) & \cos\theta_M(t) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Self-field

$$H_{\mathrm{self,x}}(x,y,t) = \frac{1}{2\pi} \iint_S \frac{-J_z(x',y',t)\cdot(y-y')}{(x-x')^2+(y-y')^2}\, dx'dy'$$

$$H_{\mathrm{self,y}}(x,y,t) = \frac{1}{2\pi} \iint_S \frac{J_z(x',y',t)\cdot(x-x')}{(x-x')^2+(y-y')^2}\, dx'dy'$$

Numerical Modelling Frameworks – MEMEP

- **Minimum Electromagnetic Entropy Production (MEMEP)**
- Variational method, solves **J** by minimising a functional containing all variables of problem: **A**, **J**, *φ*
- Fast: **J** only exists in HTS wire → mesh only required here

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi$$

$$\nabla \cdot \mathbf{J} = 0$$

- In Coulomb's gauge ($\nabla \cdot \mathbf{A} = 0$) **A** separated into contributions from current density $\mathbf{A}_J$ + external source $\mathbf{A}_a$

$$A_J(r) = -\frac{\mu_0}{2\pi}\int_S dS' J(r') \ln|r - r'|$$

Minimise following function:

$$L = \int_S ds \left[\frac{1}{2}\frac{\Delta A_J}{\Delta t} \cdot \Delta J + \frac{\Delta A_a}{\Delta t} \cdot \Delta J + U(J_0 + \Delta J)\right]$$

where *U* is dissipation factor

$$U(J) = \int_0^J E(J') \cdot dJ'$$

that incorporates *E*-*J* power law

Vector potential from PM, $\mathbf{A}_M$, appears in A_a:

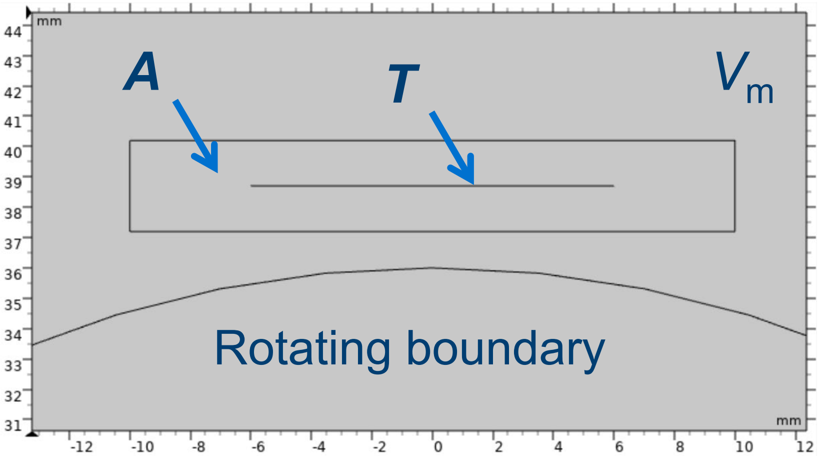
$$\mathbf{A}_M(r) = -\frac{\mu_0}{2\pi} M \int_{\partial S} dl' \mathbf{e}_m \times \mathbf{e}_n(r') \ln|r - r'|$$

Ghabeli & Pardo *SUST* 33 (2020) 035008 112601; DOI:10.1088/1361-6668/ab6958

Numerical Modelling Frameworks – T-A

- **Coupled T-A formulation**
  - Uses **A** to calculate magnetic field in whole domain, current vector potential **T** to calculate current density
  - Four models implementing the **T**-**A** formulation were built
  - 1D (line) or 2D (finite thickness) object
  - SP = Mixed scalar-vector potential (see **H**-**A**; COMSOL's RMM interface)
  - VP = Vector potential only, implemented in COMSOL's Magnetic Fields (MF) interface

$$\mathbf{J} = \nabla \times \mathbf{T}$$



Coupled ***T***-***A*** formulation:
HTS wire is a 1D (line) object;
Mixed scalar-vector potential (SP)

Numerical Modelling Frameworks – IE

- **Current distribution along 1D superconducting layer can be given by an integral equation (IE)**
  - The IE here is written in COMSOL's Partial Differential Equation (PDE) module in 1D:

$$\rho J_s = \mu f(Q + K) + C$$

$$K(x,t) = \int_{-a}^{x} \partial_t H_n(\xi,t)\, d\xi$$

$$Q(x,t) = \frac{1}{2}\int_{-a}^{a} \partial_t J_s(\xi,t) \ln|\xi - x|\, d\xi$$

  - ρ = E-J power law resistivity, J_s = sheet current density, f = thickness, a = half-width
  - Constant C set to zero (transport current = 0 constraint)
  - Coupled to 2D MF interface to obtain external PM field

Numerical Modelling Frameworks – VIE

- **Volume integral equation-based equivalent circuit separates total electromotive force at any point in HTS wire into two contributions:**

  1) due to time-varying field from current induced in HTS wire

  2) due to movement of PM

- **The following equation is satisfied, in weak form, over each element:**

$$\mathbf{E} = -\frac{\partial \mathbf{A}^{int}}{\partial t} - \mathbf{v} \times \mathbf{B}^{PM} - \nabla \varphi$$

  where $\mathbf{A}^{\mathrm{int}}$ = vector potential of current in HTS wire, $\mathbf{B}^{\mathrm{PM}}$ = PM field, $\mathbf{v}$ = velocity of $\mathbf{B}^{\mathrm{PM}}$ at considered point

- State variable **J** obtained by relating **E** and $\mathbf{A}^{\mathrm{int}}$ to **J** via *E*-*J* power law and $A_J(r) = -\frac{\mu_0}{2\pi} \int_S dS' J(r') \ln |r - r'|$. Shell current used for PM field.
- Discretised equations (weighted residual approach) correspond to voltage balance between: non-linear resistor (from HTS wire electric field), coupled inductor (magnetic interaction of induced current), voltage generator (Lorentz-like electromotive force)

Comparison of Key Results – $V_{eq}(t)$

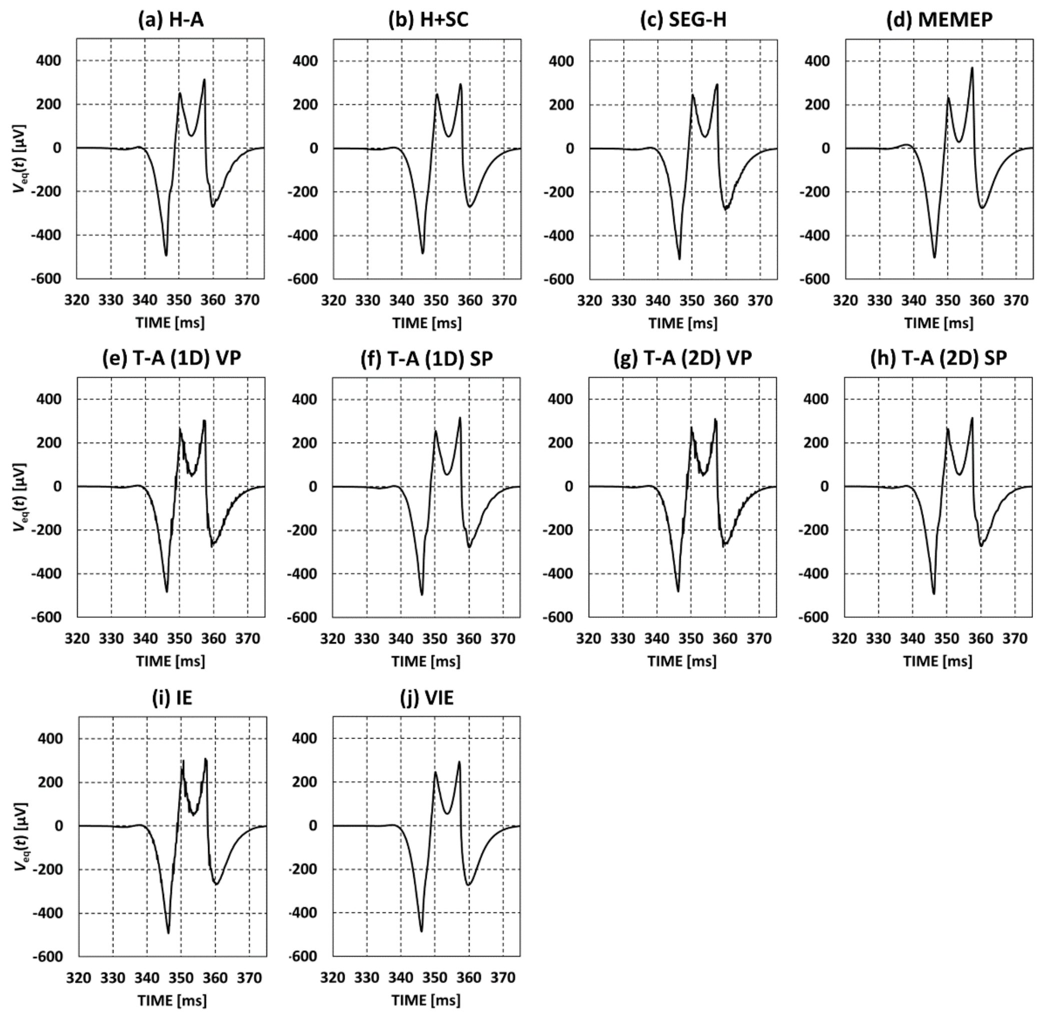
Open-circuit equivalent instantaneous voltage

$$V_{eq}(t) = -LE_{ave}(t)$$

2nd transit of PM past HTS wire, ignoring any initial transient effects in 1st cycle

Qualitatively, four distinct peaks with left-to-right asymmetry, as observed in experiments

Excellent quantitative agreement (see cumulative voltage next)



Comparison of Key Results – $V_{cumul}(t)$

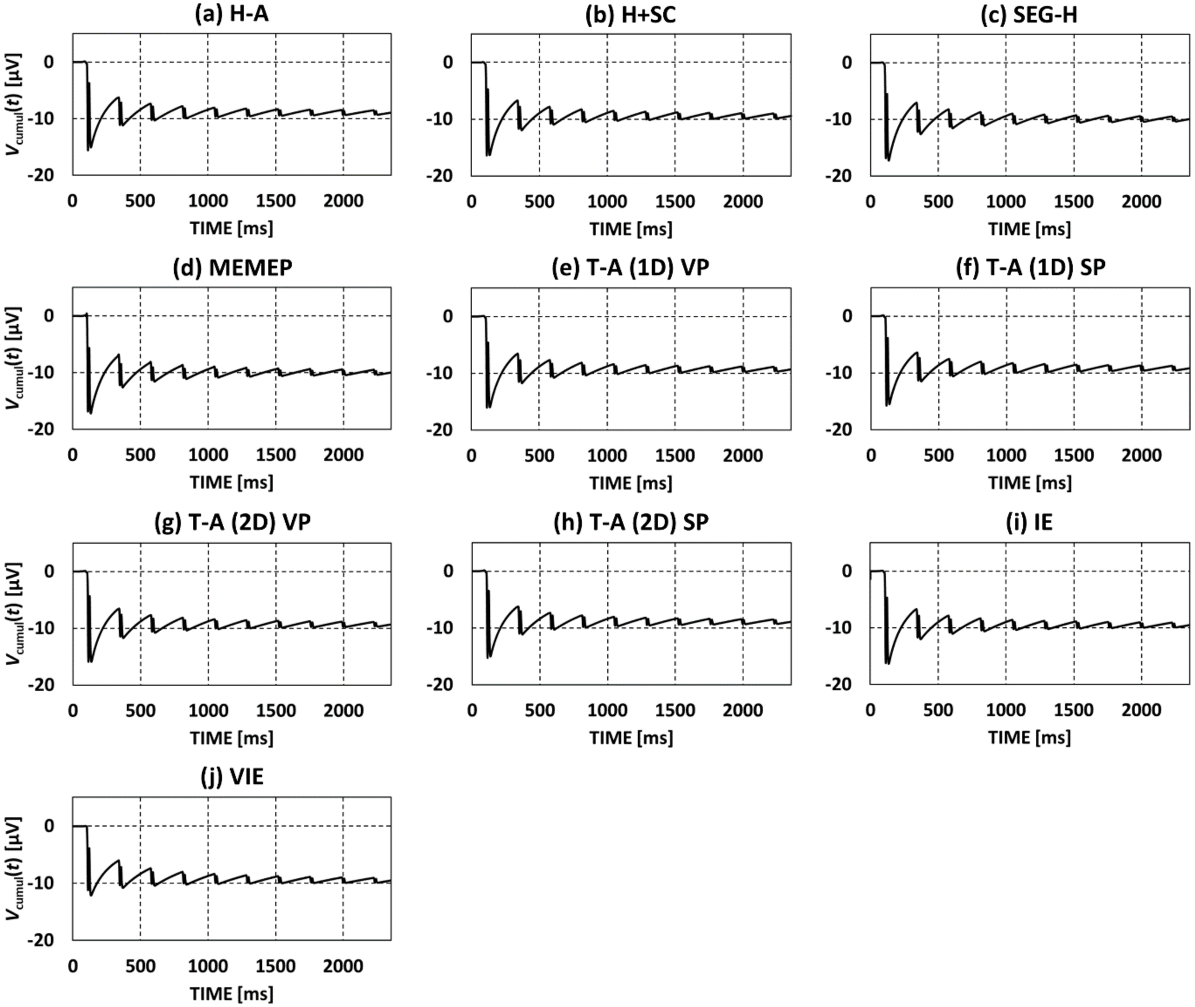
Cumulative time-averaged equivalent voltage

$$V_{cumul}(t) = \frac{1}{t}\int_0^t V_{eq}(t)\,dt$$

V_{cumul} over 10 cycles

Converges to non-zero asymptotic value → V_{DC}

Excellent quantitative agreement:
–9.41 µV average with 0.34 µV standard deviation



Comparison of Key Results – J/J_{c0}, E

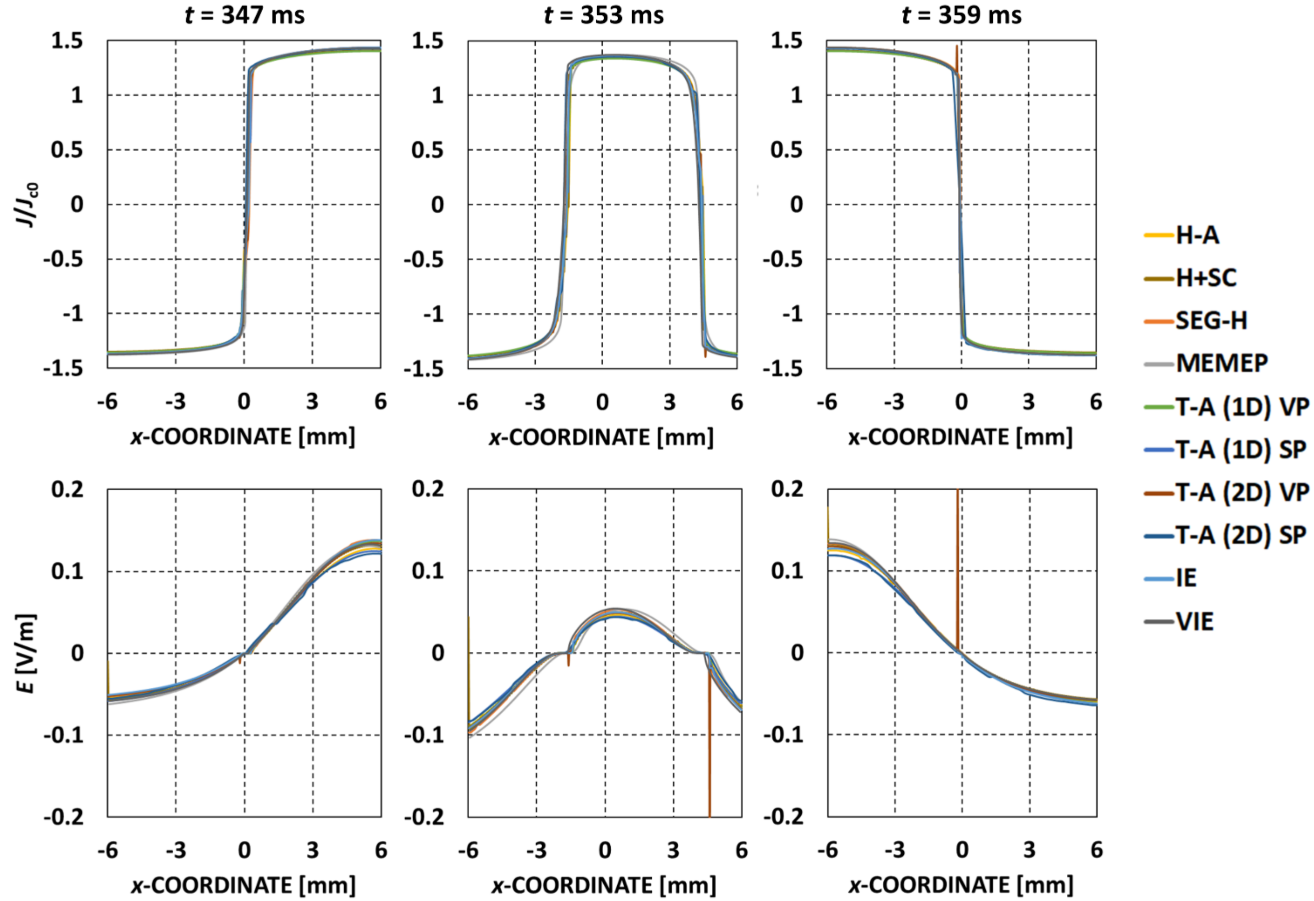
Magnet approaching $\theta_M \approx 81°$

Magnet directly below $\theta_M = 90°$

Magnet leaving $\theta_M \approx 99°$

Normalised critical current density*

Electric field*



*averaged along thickness

Comparison of Modelling Frameworks

Key metrics assessed for each benchmark model

Model	Mesh (SC)	Mesh (total)	DOFs	Rel./abs. tolerance	Approx. time/cycle (min/cycle)	Software implementation
MEMEP	120 (120 × 1)	120	120	1e−4[1]	<0.25[a]	C++
SEG-H	120 (120 × 1)	2653	4071	1e−4/0.1	1.1[b] 2.6[b]	COMSOL 5.4 COMSOL 5.5
VIE	120 (120 × 1)	120	120	1e−3[2]/1e−6[2]	1.6[b]	MATLAB
H-A	120 (120 × 1)	4176	3661	1e−4/0.1	2.1[b]	COMSOL 5.5
T-A (2D) SP	240 (60 × 4)	3800	2863	1e−5/1e−4	3.9[b]	COMSOL 5.5
IE	120 (120 × 1)	5932	12 451	5e−3/0.1	5.1[b]	COMSOL 5.5
T-A (1D) SP	120 (120 × 1)	4876	2779	1e−5/1e−4	6.5[b]	COMSOL 5.5
H+SC	120 (120 × 1)	11 272	16 988	1e−5/1e−3	7.9[b] > 120	COMSOL 5.4 COMSOL 5.5
T-A (1D) VP	120 (120 × 1)	6064	12 715	1e−4/0.1	21.6[b]	COMSOL 5.5
T-A (2D) VP	240 (60 × 4)	5286	13 696	1e−4/0.1	64.6[b]	COMSOL 5.5

PC specifications:[a]Intel® Core™ i7-8700 CPU @ 3.20 GHz, 31.1 GB RAM (10% memory used for MEMEP model), Ubuntu 16.04 LTS, 64-bit[b]Intel® Core™ i9-7900X CPU @ 3.30 GHz, 63.7 GB RAM, Microsoft Windows 10 Pro, 64-bitOther notes:[1]Tolerance for the mutual interaction matrix[2]Default settings for MATLAB/*ode23b* solver

Comparison – Key Findings

- **Clear winner in terms of computational speed = MEMEP**
  - Limited number of DOFs: only HTS wire needs to be meshed
  - Next best performers (SEG-H, VIE) also emphasise reduction of mesh elements
  - The **H**-formulation methods took advantage of artificial expansion technique (1 → 100 μm) to improve computational speed
- **A number of models use a rotating machine-like framework**
  - Stability issues were observed with **T**-**A** formulation when only vector potential used (with COMSOL's MF interface) → noisy voltage waveform, spikes in J/E plots; to compensate 2nd order elements needed = additional computational cost
  - Use of mixed scalar-vector potential (V_m-**A**) resulted in a significant performance improvement & may have useful application to modelling superconducting rotating machines in general

Summary

- **A new benchmark problem for the HTS modelling community was proposed: the HTS dynamo**
  - A permanent magnet rotates past a stationary HTS wire in the open-circuit configuration
  - Specific simplified geometry (2D) with well-defined inputs + expected outputs
  - Allows any modelling technique to be validated + critically compared
- **Several methods were implemented + critically compared**
  - **H**- and **T-A** formulations, MEMEP, integral equation methods
  - All methods showed excellent qualitative/quantitative agreement
  - Methods that emphasised reduced number of mesh elements were fastest
  - Mixed scalar-vector potential rotating machine-like models may have useful application to modelling superconducting rotating machines in general

Additional Resources

- **Paper available (open access) at *Supercond. Sci. Technol.*:**
  - https://doi.org/10.1088/1361-6668/abae04
- **Additional data related to publication:**
  - https://doi.org/10.17863/CAM.54005
- **Several example are available on HTS Modelling Workgroup 'Shared Models' page:**
  - http://www.htsmodelling.com/?page_id=748#dinamo