



7th international workshop on numerical modelling of high temperature superconductors

Coupled multiphysics modelling of the thermal-magnetic-mechanical instability in bulk superconductors during magnetization

Ze Jing¹ and Mark Ainslie²

¹ Xidian University, Xi'an, China

² University of Cambridge, Cambridge, UK

23 June 2021

Contents

◆ Background

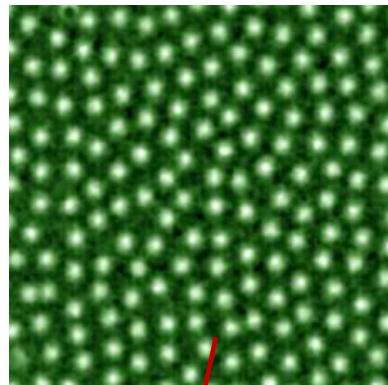
◆ Research Progress

- ✓ Numerical modelling and formulisms
- ✓ Flux jumps in Bulk MgB₂
- ✓ Results and Discussions

◆ Summary

Background: Vortices and Pinning

Type-II Superconductors

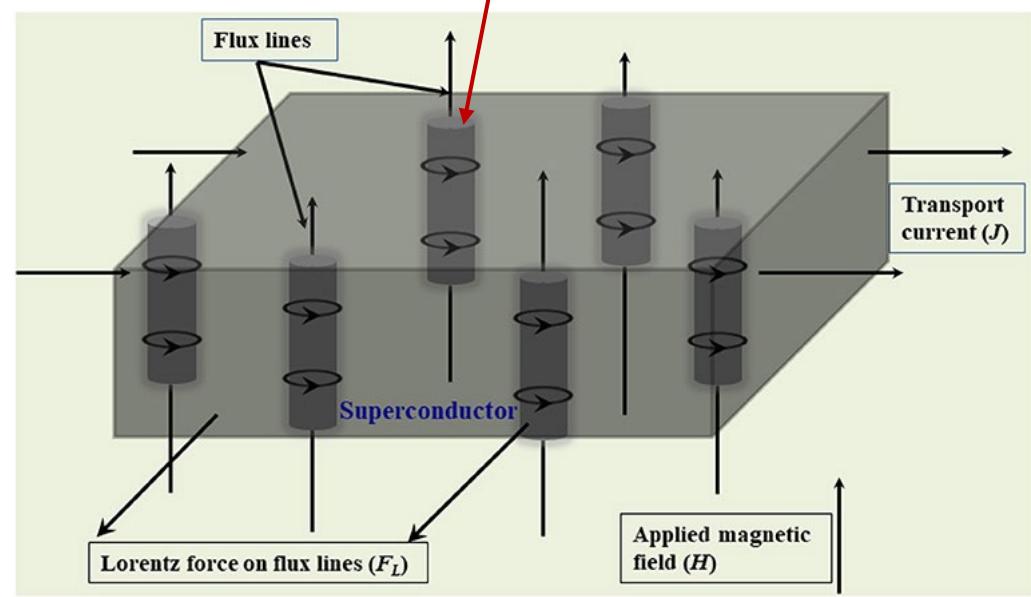
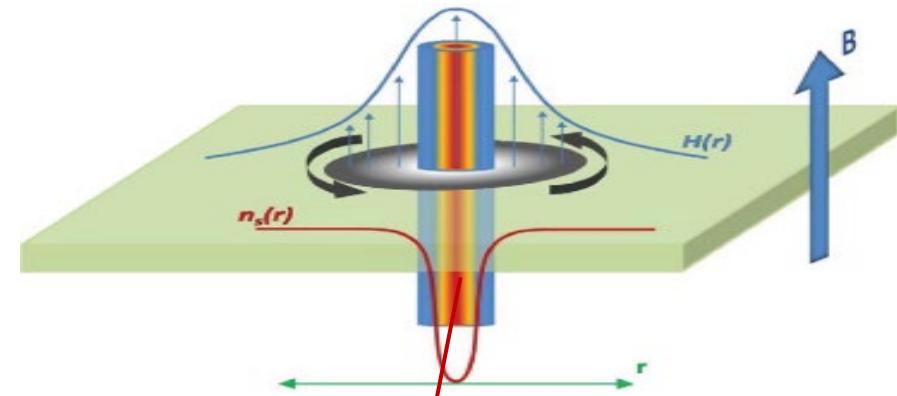
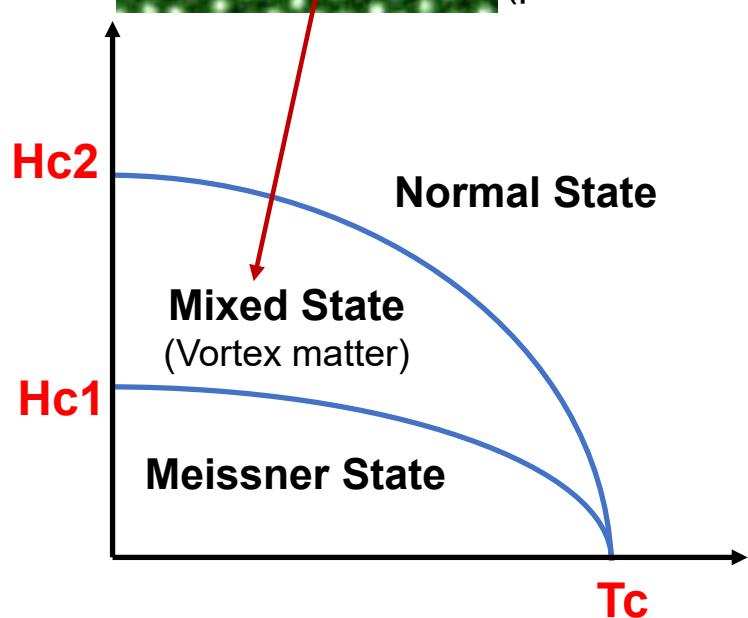


Vortex lattice
A. A. Abrikosov



2003

(published 1957)



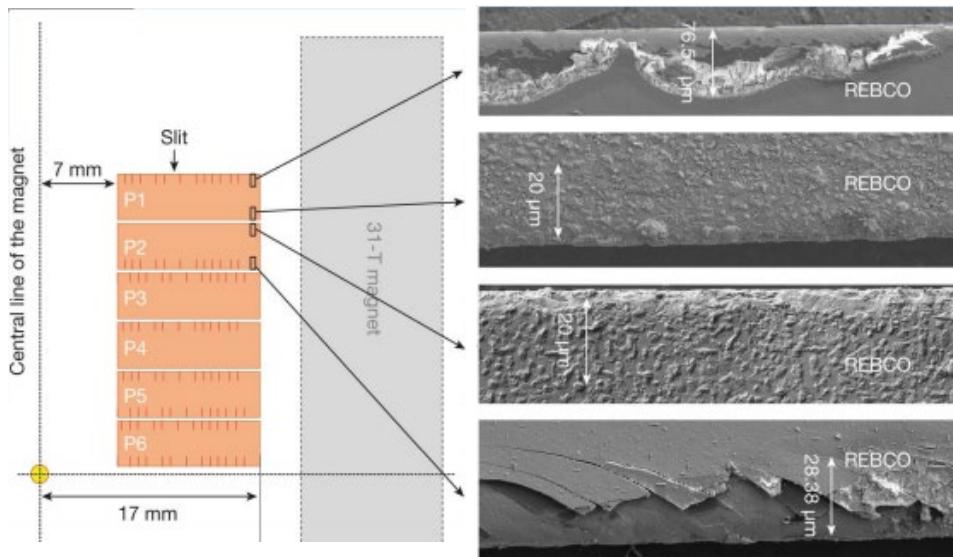
Background: Challenges in the applications

◆ Mechanical behaviors

- Electromagnetic force
- Thermal stress
- Manufacturing stress
- ...

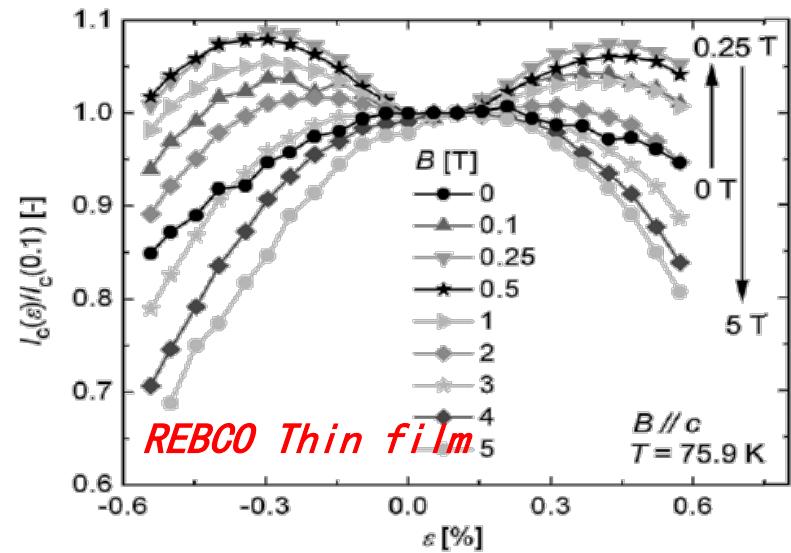


Delamination of SC films



S. Hahn, Nature, 2019

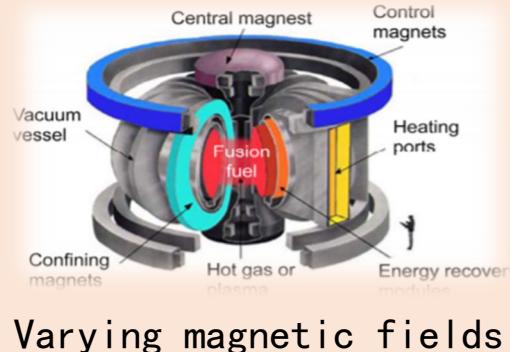
Degradation of critical parameters



van der Laan and Ekin (2010)

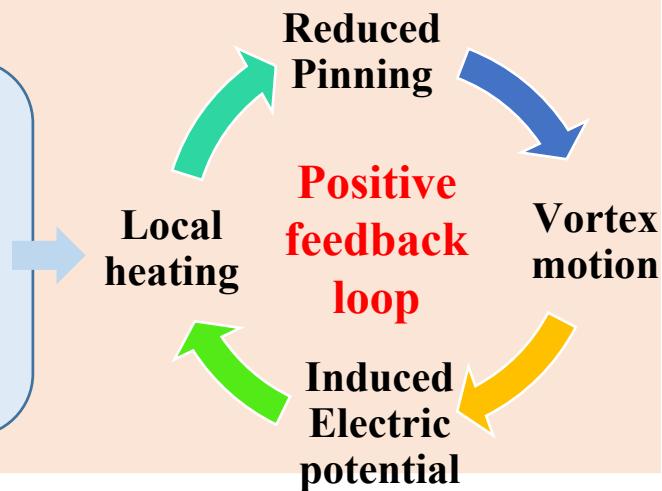
Background: Challenges in the applications

◆ Thermomagnetic instability

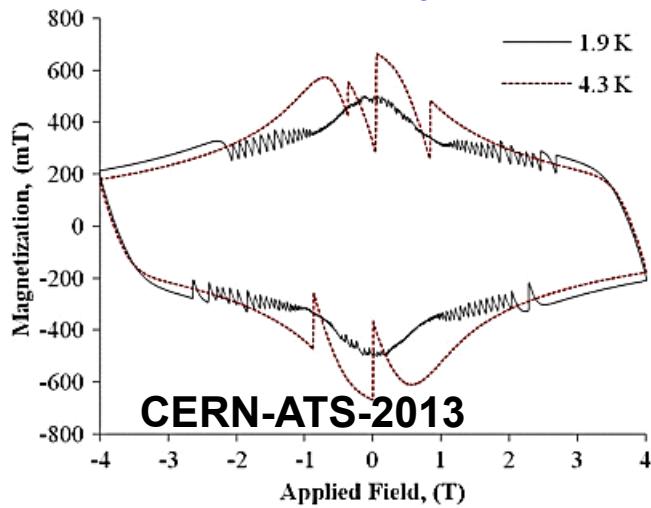


Thermal fluctuations

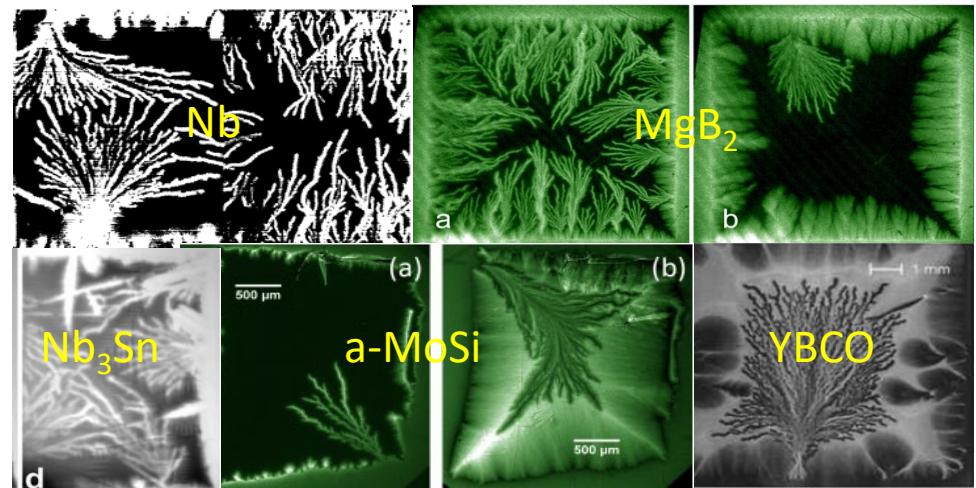
- Electromagnetic force induced motion
- Ac /Magnetization Loss
- Joule heating etc.



Flux jumps in Nb₃Sn strands



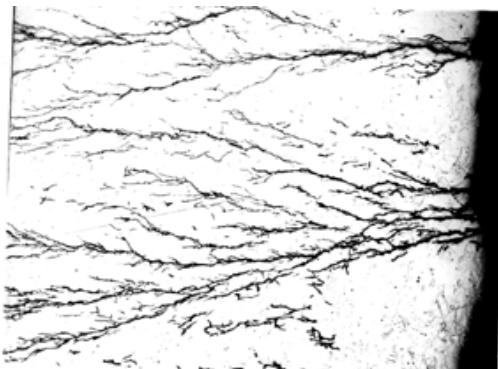
Flux avalanches in SC thin films



Background: Thermomagnetic instability

- ✓ Instability phenomenon is widely encountered in nature and engineering, which sharing some common characteristics

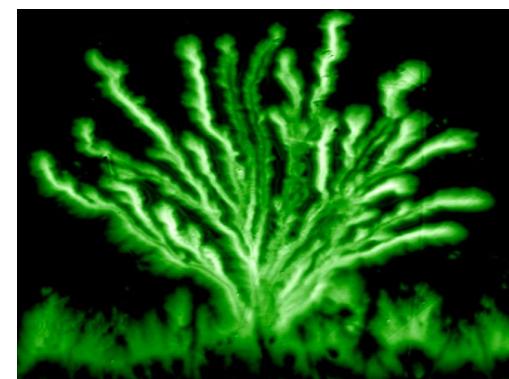
Brittle fracture



Electric breakdown



Thermomagnetic instability



Mechanical breakdown

Lightning

flux avalanches

Loads

force

Electric potential

Magnetic field

Field

Stress/strain

Electric-thermal field

Electromagnetic-thermal field

Equations

Force Equilibrium

Charge Conservation
and heat diffusion

Maxwell equations
and heat diffusion

Velocity

supersonic

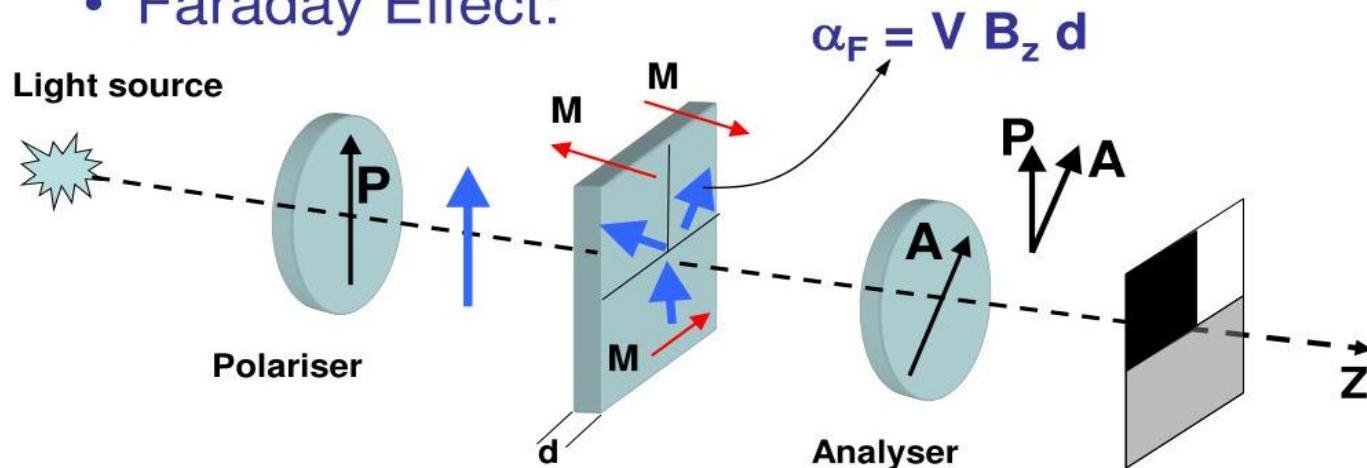
< μ s

Sub- μ s or ns

Background: thermomagnetic instability of SCs

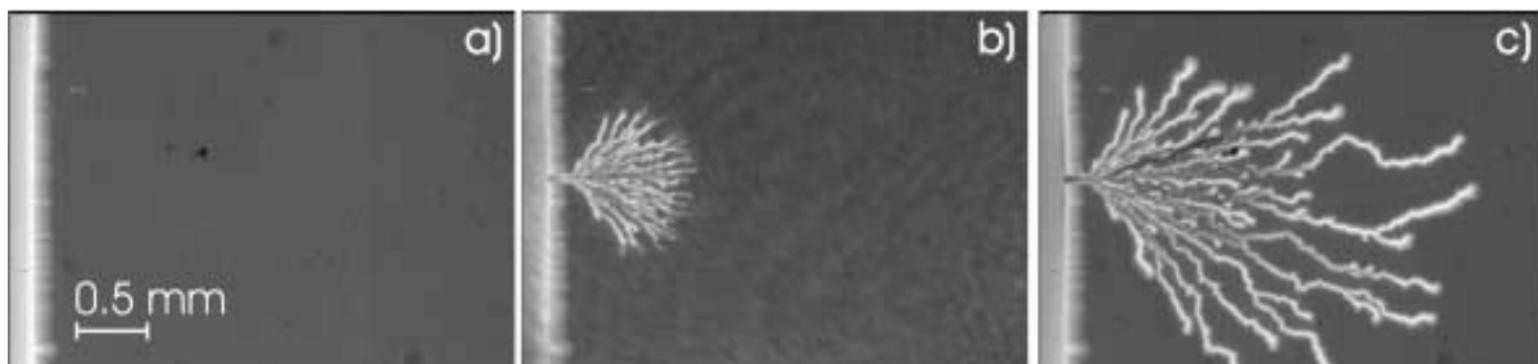
Experiments》 Magneto-optical imaging

- Faraday Effect:



MOI of YBCO thin film at 10 K

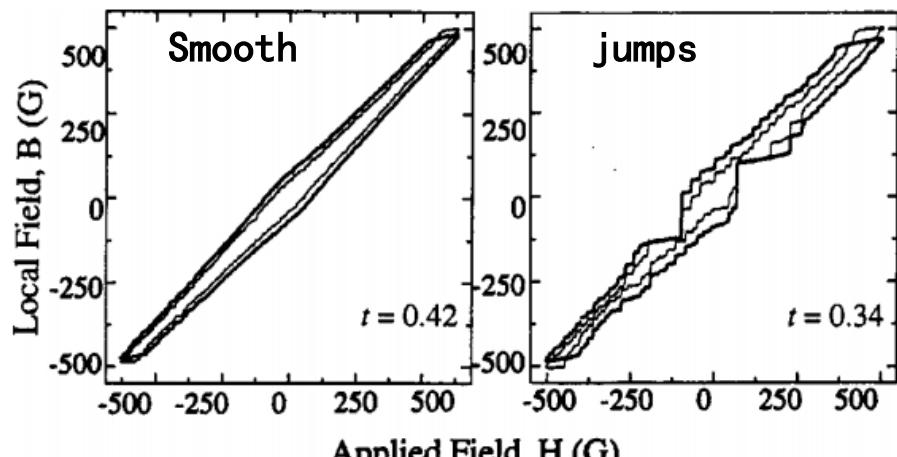
$\sim \text{km/s}$



Europhys. Lett., 59 (2002) 599–605

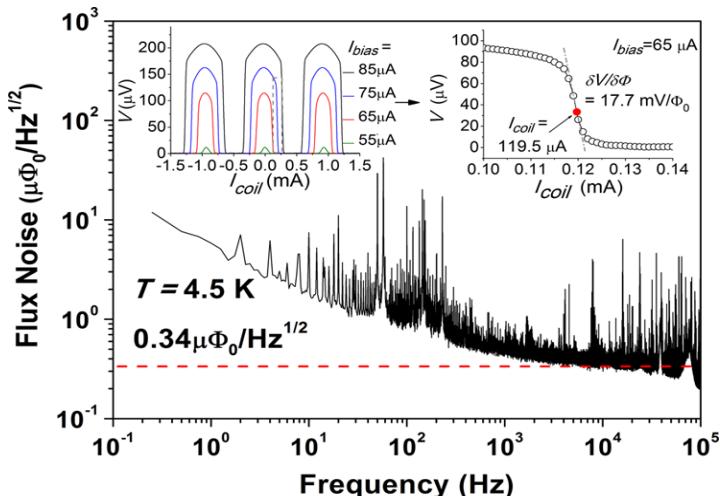
Background: thermomagnetic instability of SCs

Jumping in the magnetization curves

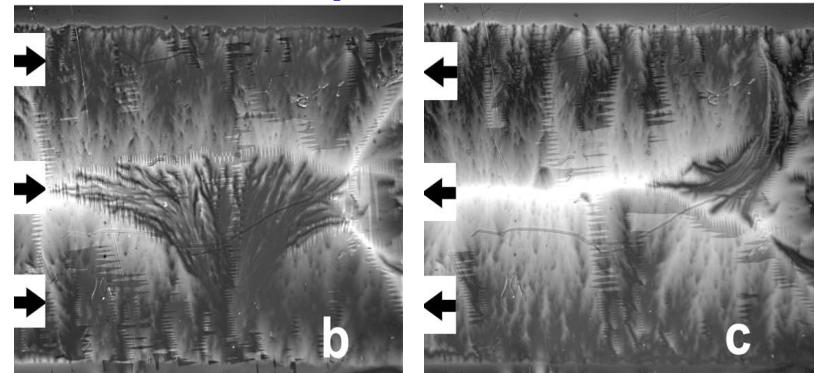


Nowak et al., Phys. Rev. B, 55 (1997) 702

Noise in the Nb Nano-SQUID

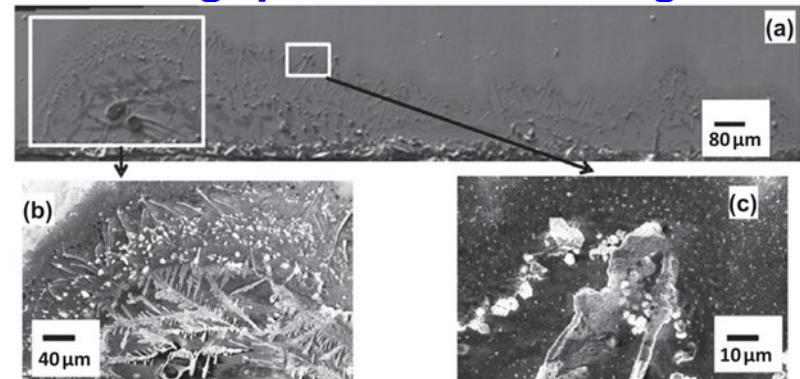


Blocking the channel for transport current in superconductor film



V.B. Alexander et al. Appl. Phys. Lett. 2002

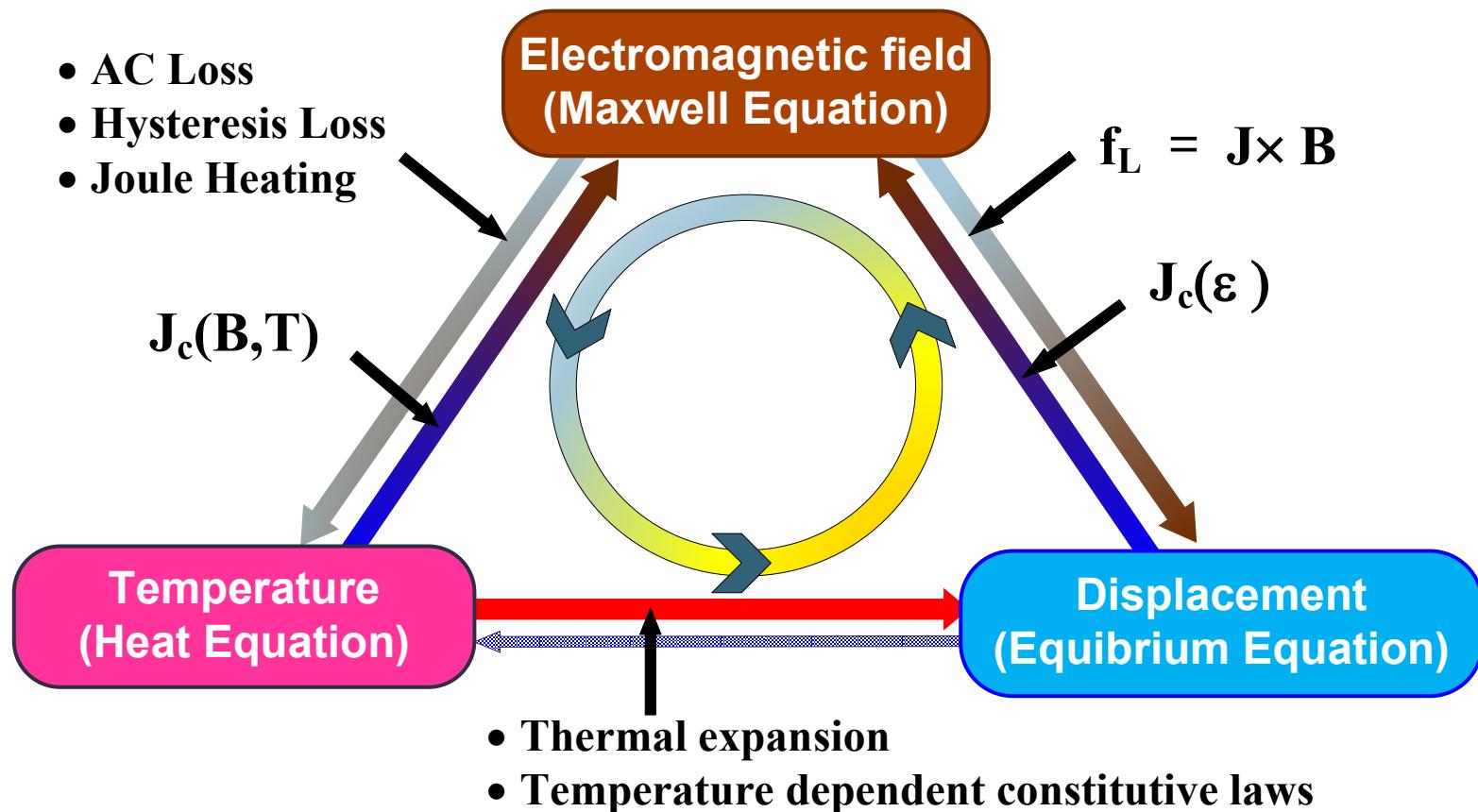
Microstrural degradation in the film during quench –SEM images.



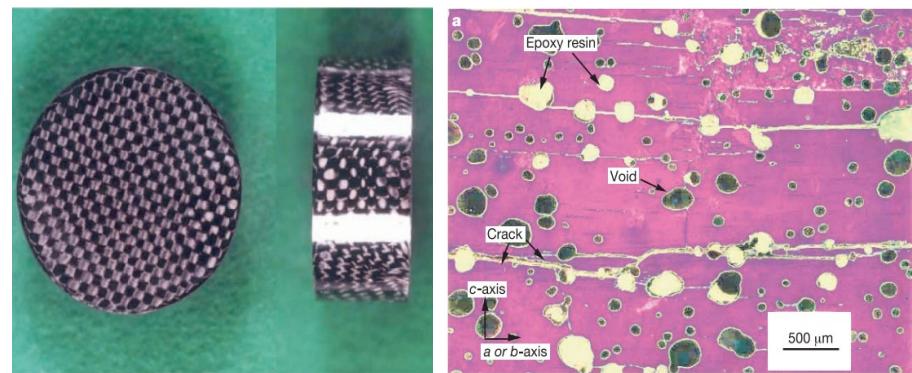
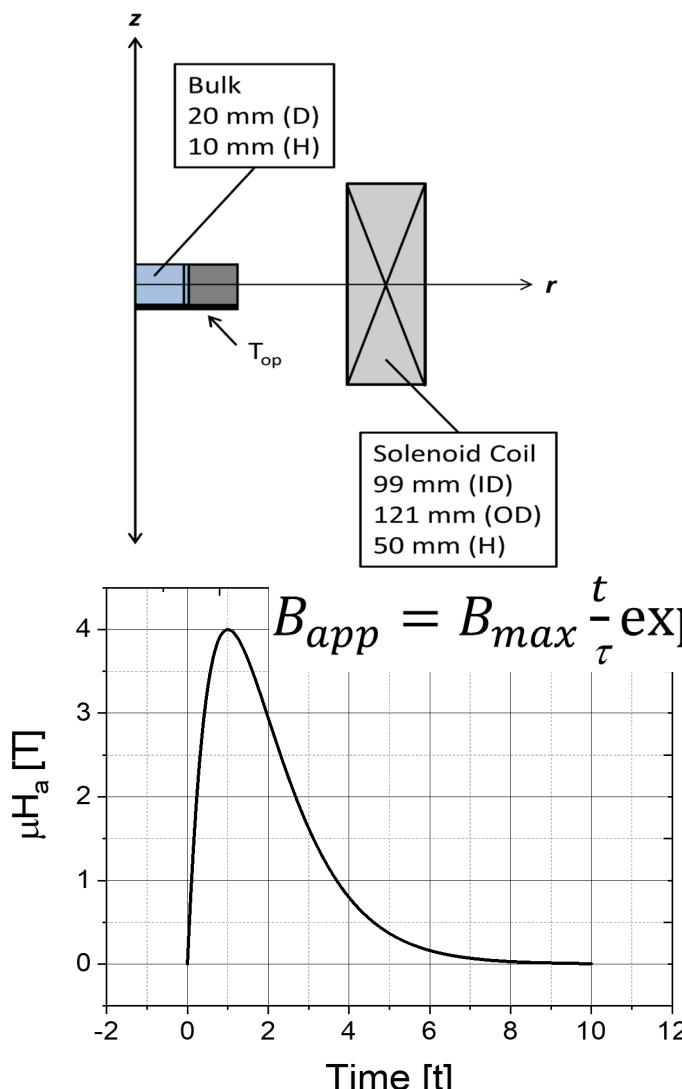
H. Song et al. Acta Mater. 60 (2012) 6991–7000

Background: Multiphysics coupling in application of SCs

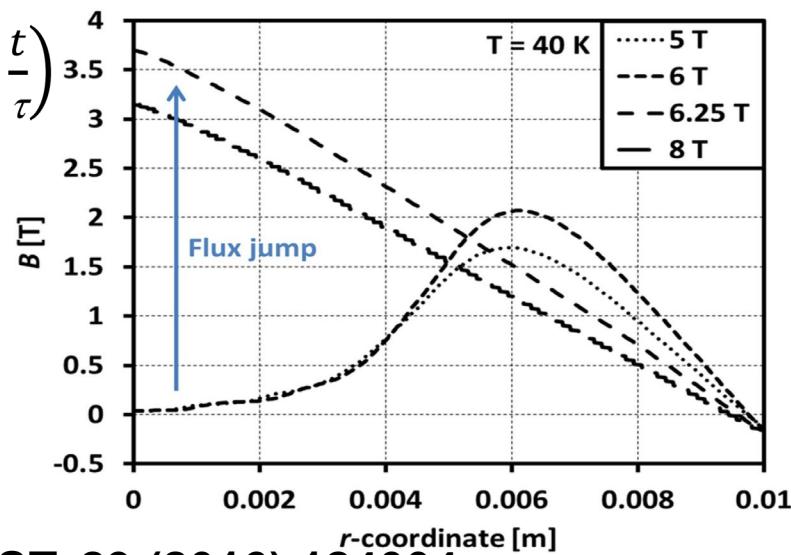
- Superconductors, under coupled multi-physical fields, experience severe thermomagnetic and mechanical instability problems.



Background: Instability in bulk SCs



Nature, 421: 517, 2003



Ainslie et al. SuST. 29 (2016) 124004

Modelling framework

Governing Equations and workflow

Thermomagnetic instabilities

Maxwell Equations

$$\nabla \times \mathbf{H} = \mathbf{J}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \cdot \mathbf{B} = 0$$

E-J power law

$$\mathbf{E} = E_0 \left| \mathbf{J}/J_{c\phi} \right|^n \quad J_{c\phi} = [(1 - \phi)^2 + k] J_c(T, B)$$

Mechanical failure

Mechanical equilibrium $\rho \ddot{\mathbf{u}}_i = \sigma_{ij,j} + \mathbf{f}_i$

Hooke's Law $\sigma = [(1 - \phi)^2 + k] [\lambda \langle \text{tr}(\boldsymbol{\varepsilon}) \rangle_+ \mathbf{I} + 2\mu \boldsymbol{\varepsilon}_+]$

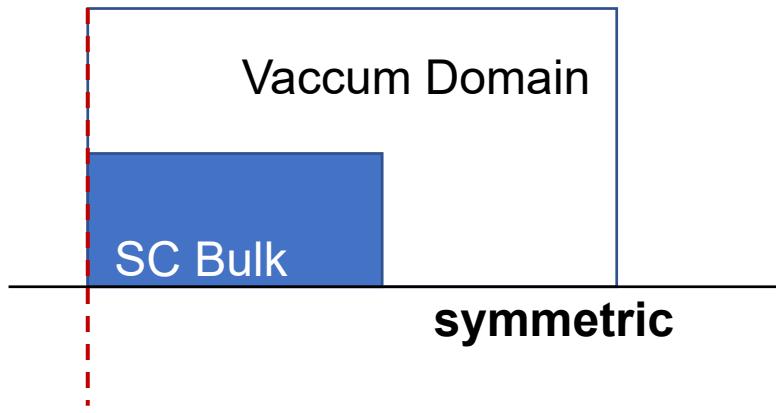
Geometry relation

$$\boldsymbol{\varepsilon}_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) + [\lambda \langle \text{tr}(\boldsymbol{\varepsilon}) \rangle_- \mathbf{I} + 2\mu \boldsymbol{\varepsilon}_-]$$

Phase field crack $\left(\frac{G_c}{2l_0} + \mathcal{H} \right) \phi - \frac{1}{2} G_c l_0 \Delta \phi = \mathcal{H} \quad \text{in } \Omega$

Research Progress: Instability in bulk SCs

Axisymmetric



E-J power law
$$E = E_0 \left| \frac{J}{J_c} \right|^{n-1} \frac{J}{J_c}$$

$$J_c(T, B, \varepsilon, \phi) = s_T s_B s_\phi J_{c0}$$

$$s_T = [1 - (T/T_c)^2]^{1.5}$$

$$s_\phi = (1 - \phi)^2 + k$$

$$s_B = \frac{B_0}{B_0 + B}$$

Electromagnetics:

Bulk – H-formulation

Thin film – T-A formulation

Thermal field: **Comsol Multiphysics**

Heat diffusion

Mechanical stress and failure:

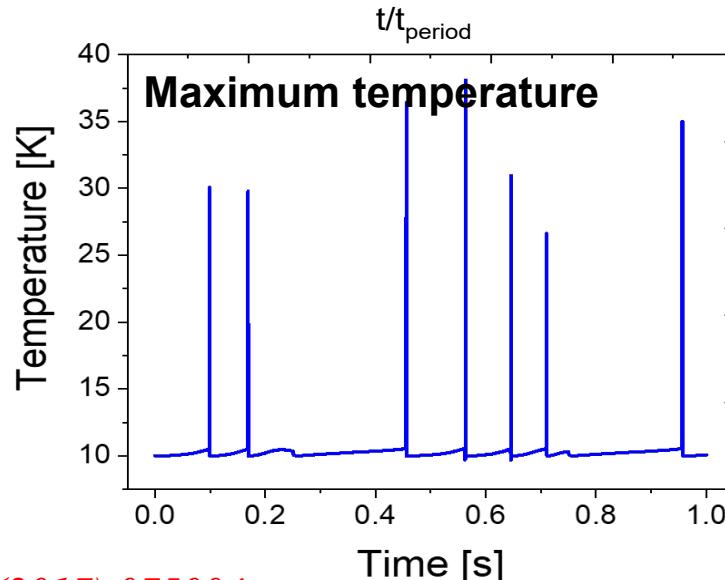
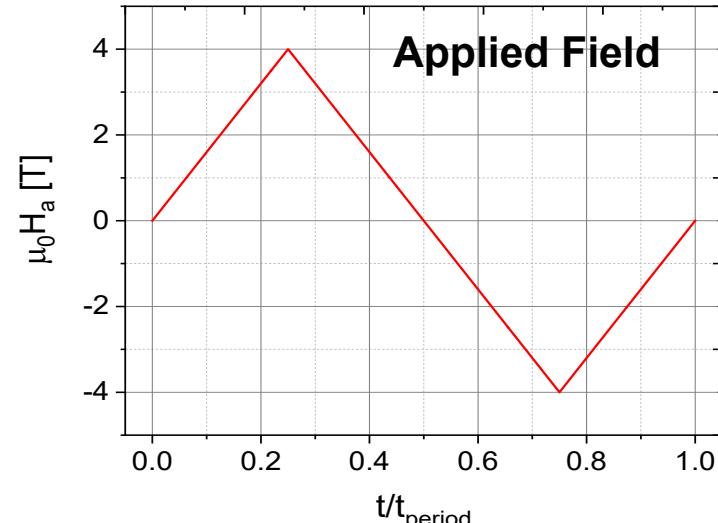
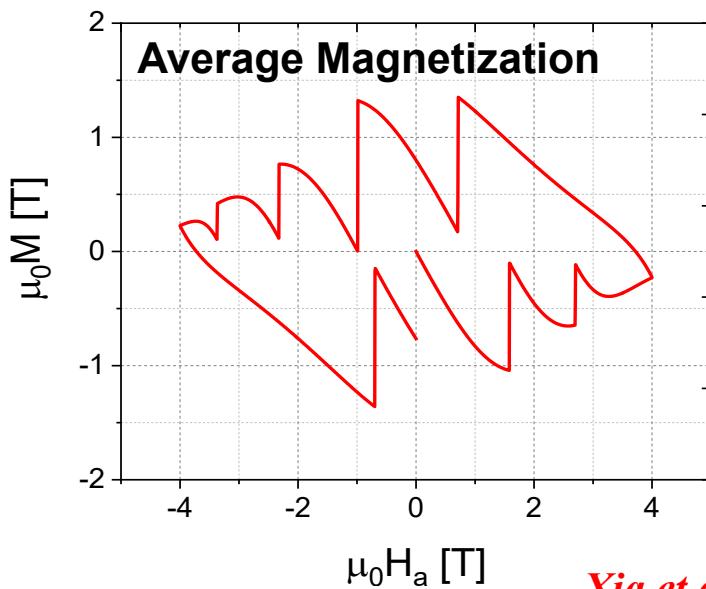
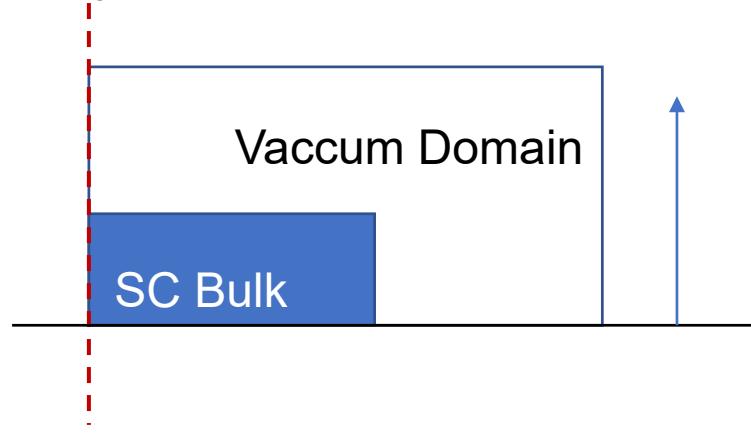
Solid mechanics+ Phase-field fracture

| para | Description | Value |
|-------------------|--|--------------------------------------|
| n_0 | n value (E-J power law) | 103 |
| E_0 | Characteristic voltage | 1×10^{-4} V/m |
| J_{c0} | Critical current density in zero field | 3.1×10^9 A/m ² |
| T_c | Critical temperature | 39 K |
| ρ_0 | Normal state resistivity (MgB2) | 7.0×10^{-8} Ω.m |
| E_{bulk} | Young's modulus (MgB2) | 50 GPa |
| ν_{bulk} | Poisson's ratio (MgB2) | 0.33 |
| $\sigma_{c,bulk}$ | Tensile strength (MgB2) | 20 MPa |
| α_{bulk} | Thermal expansion coefficient (MgB2) | 3.5×10^{-6} K ⁻¹ |
| $G_{c,bulk}$ | Critical energy release rate (MgB2) | 20 N/m |

Research Progress: Instability in bulk SCs

Model verification: flux jumps in MgB₂

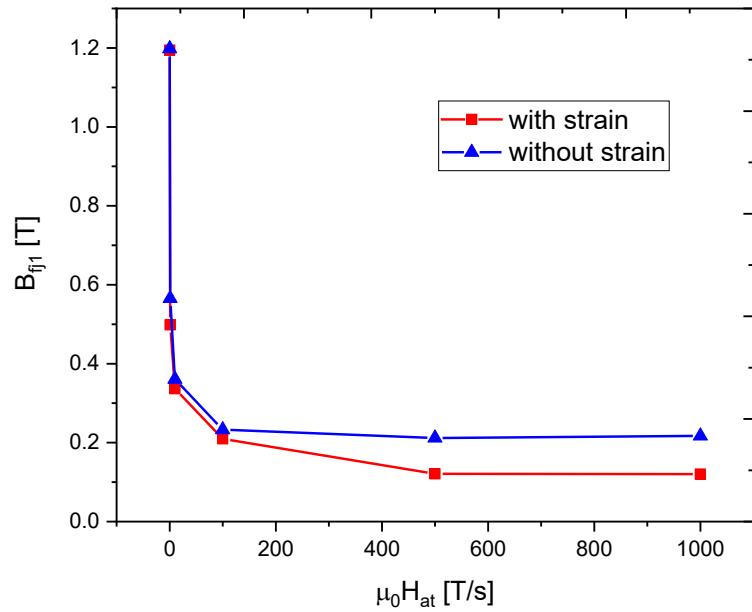
Axisymmetric



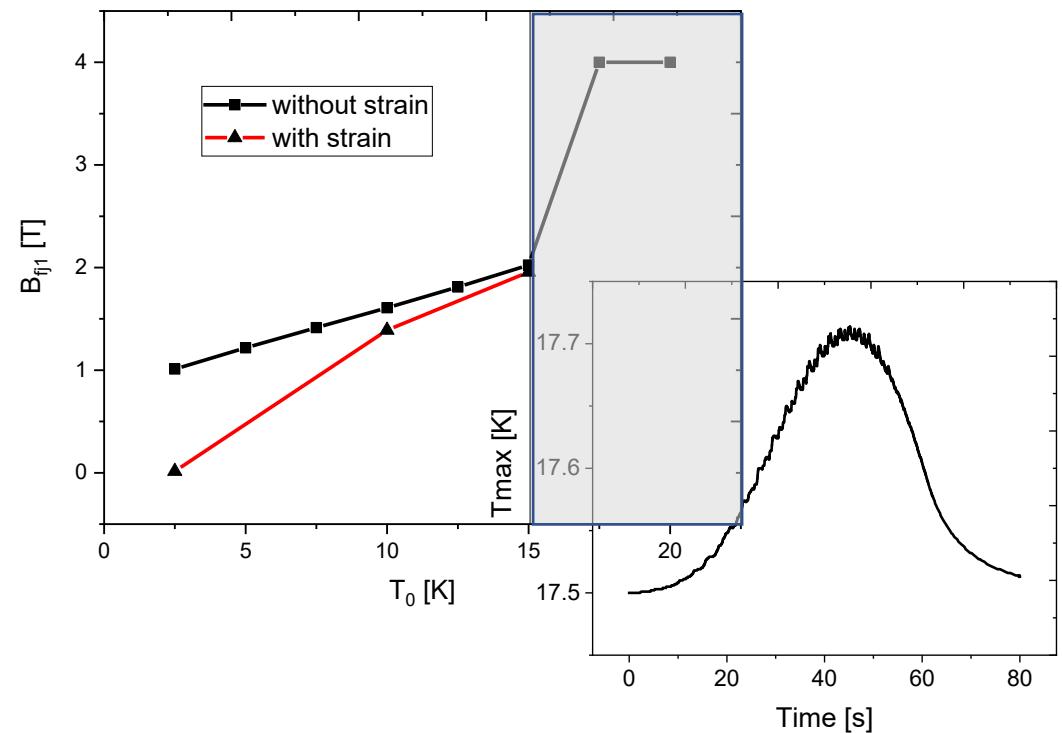
Research Progress: Instability in bulk SCs

Threshold magnetic fields Vs Parameters

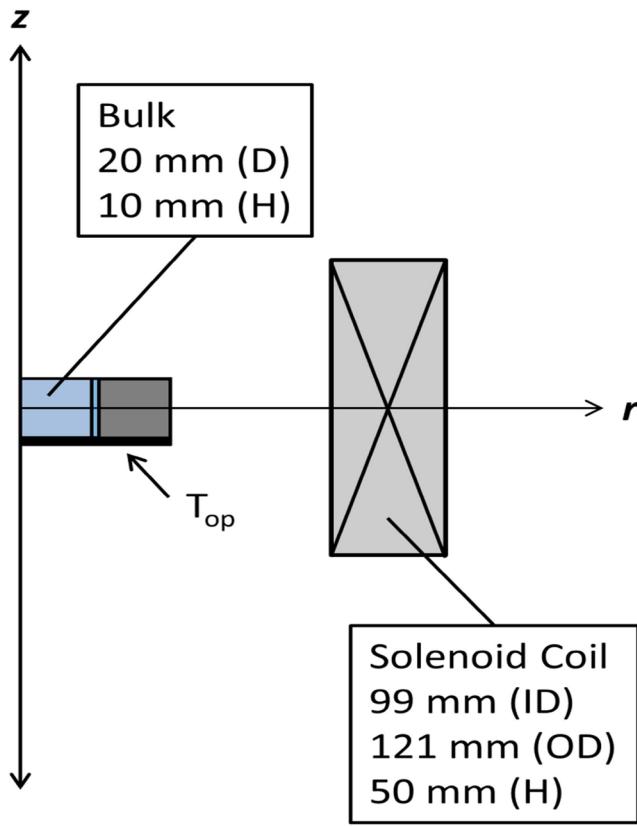
Field ramp rate



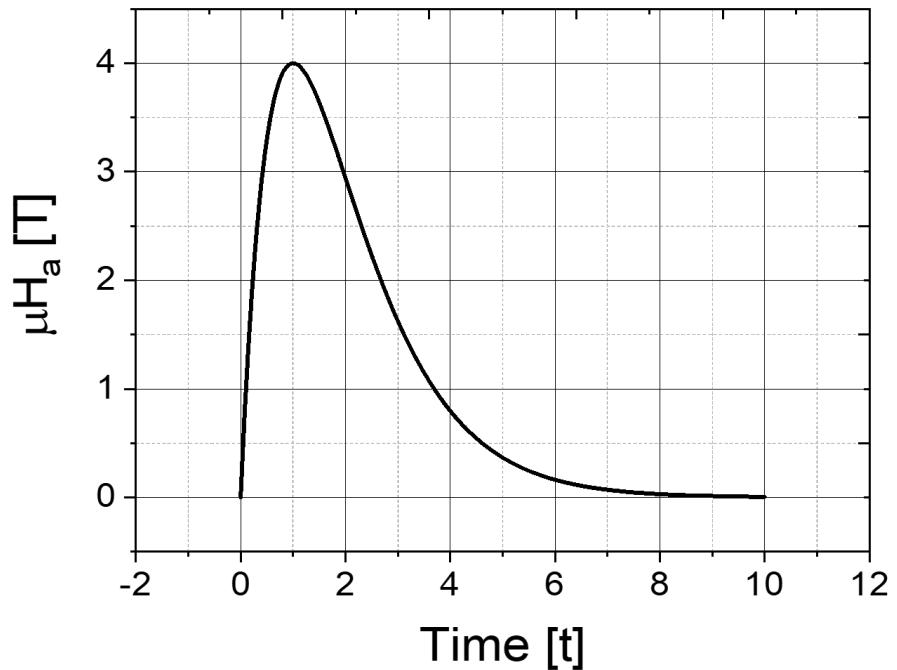
Ambient temperature



Model: bulk Superconductor in pulsed field

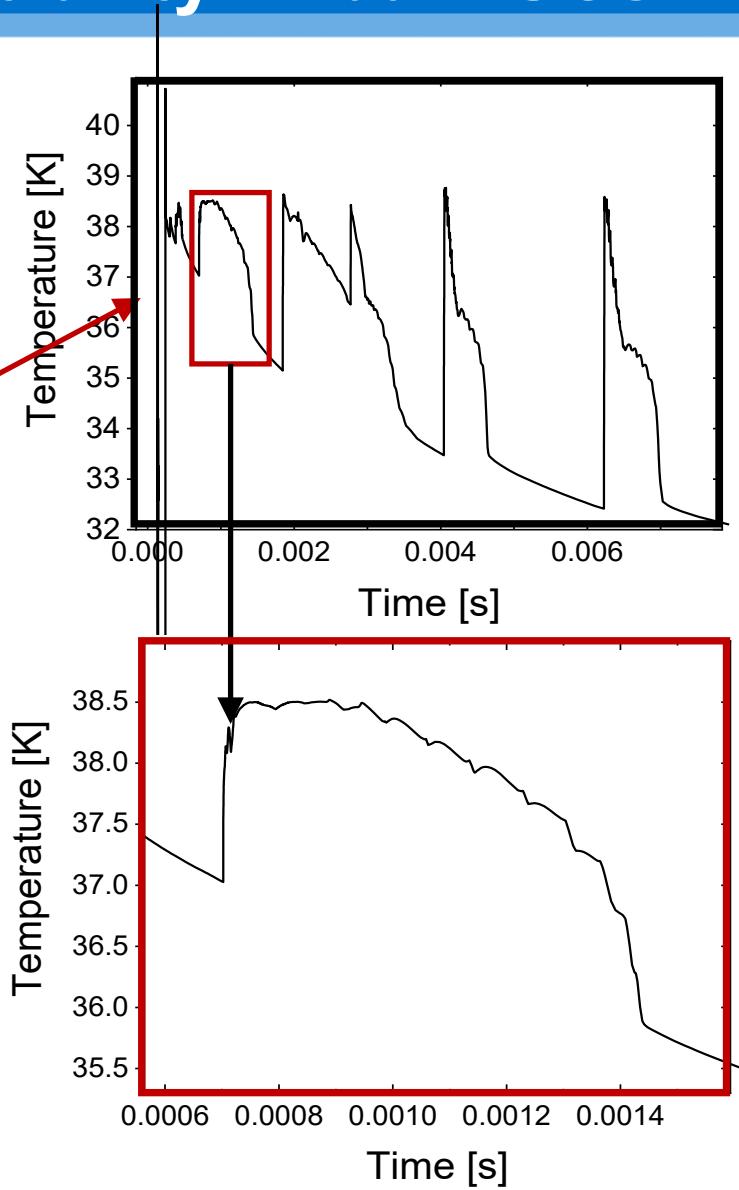
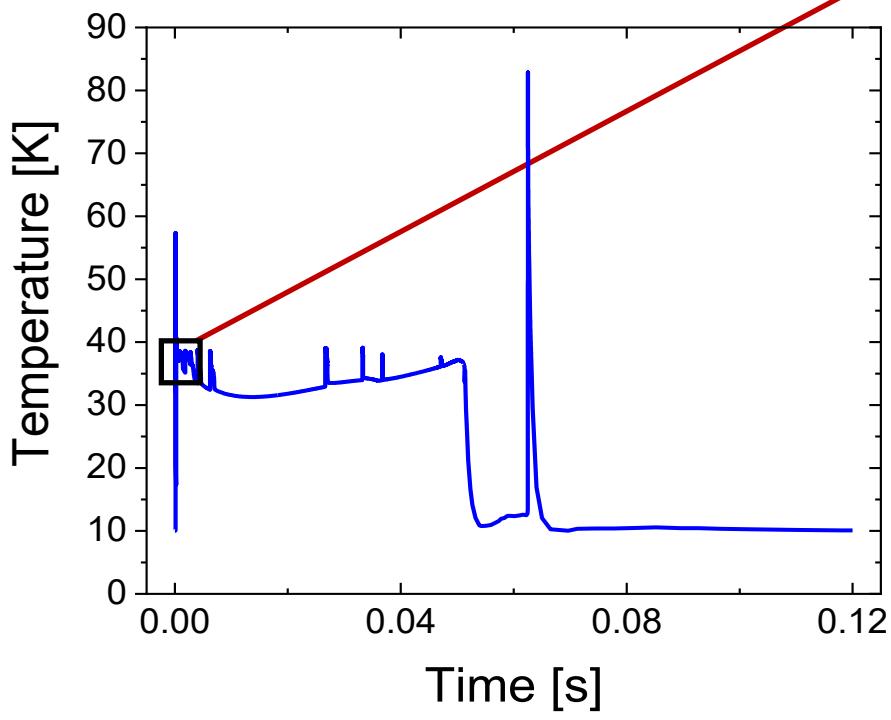


$$B_{app} = B_{max} \frac{t}{\tau} \exp\left(1 - \frac{t}{\tau}\right)$$



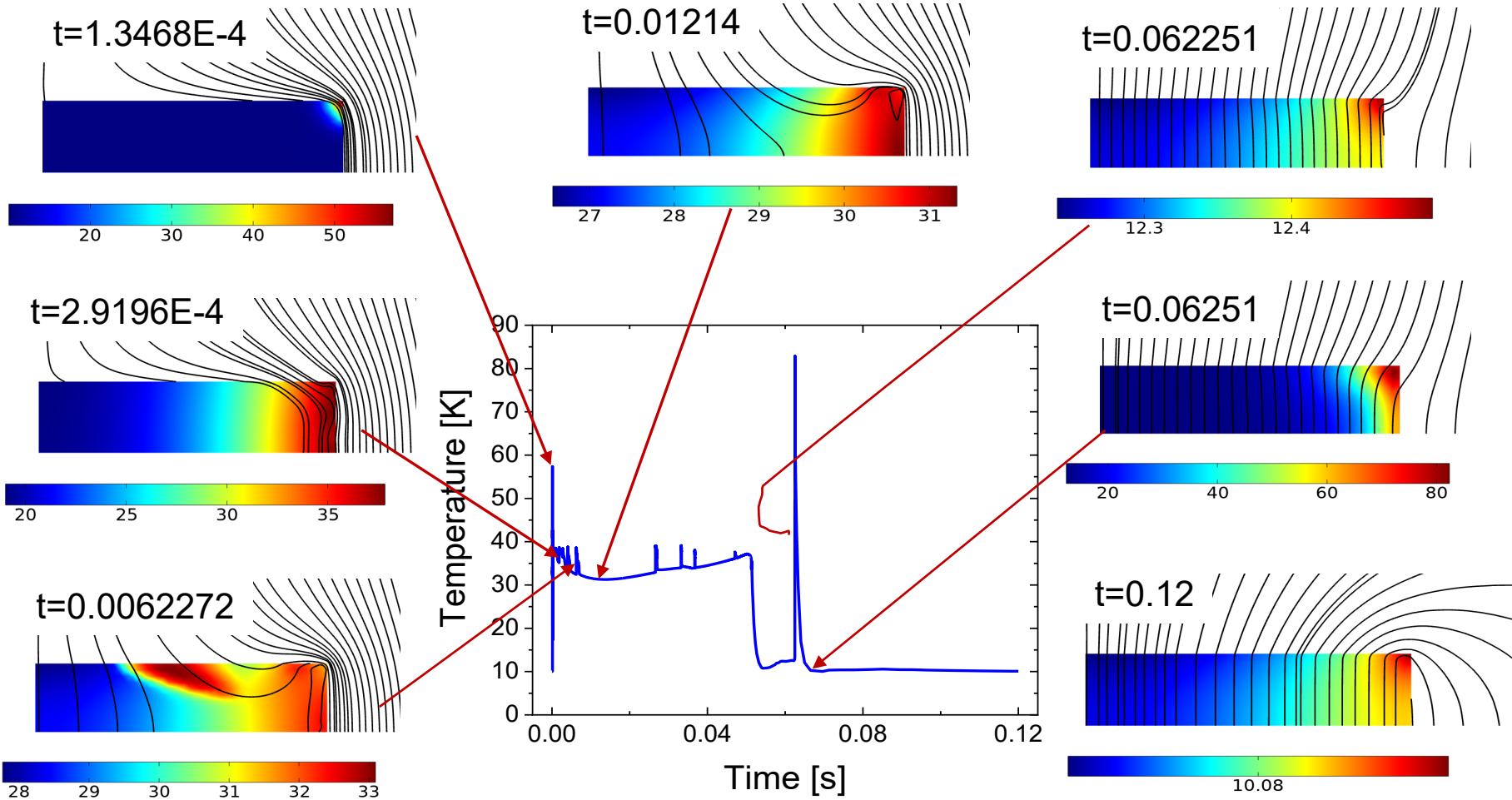
Research Progress: Instability in bulk SCs

Maximum Temperature in the Bulk during a typical pulsed field magnetization process: shows typical multiple time-scale behaviors.



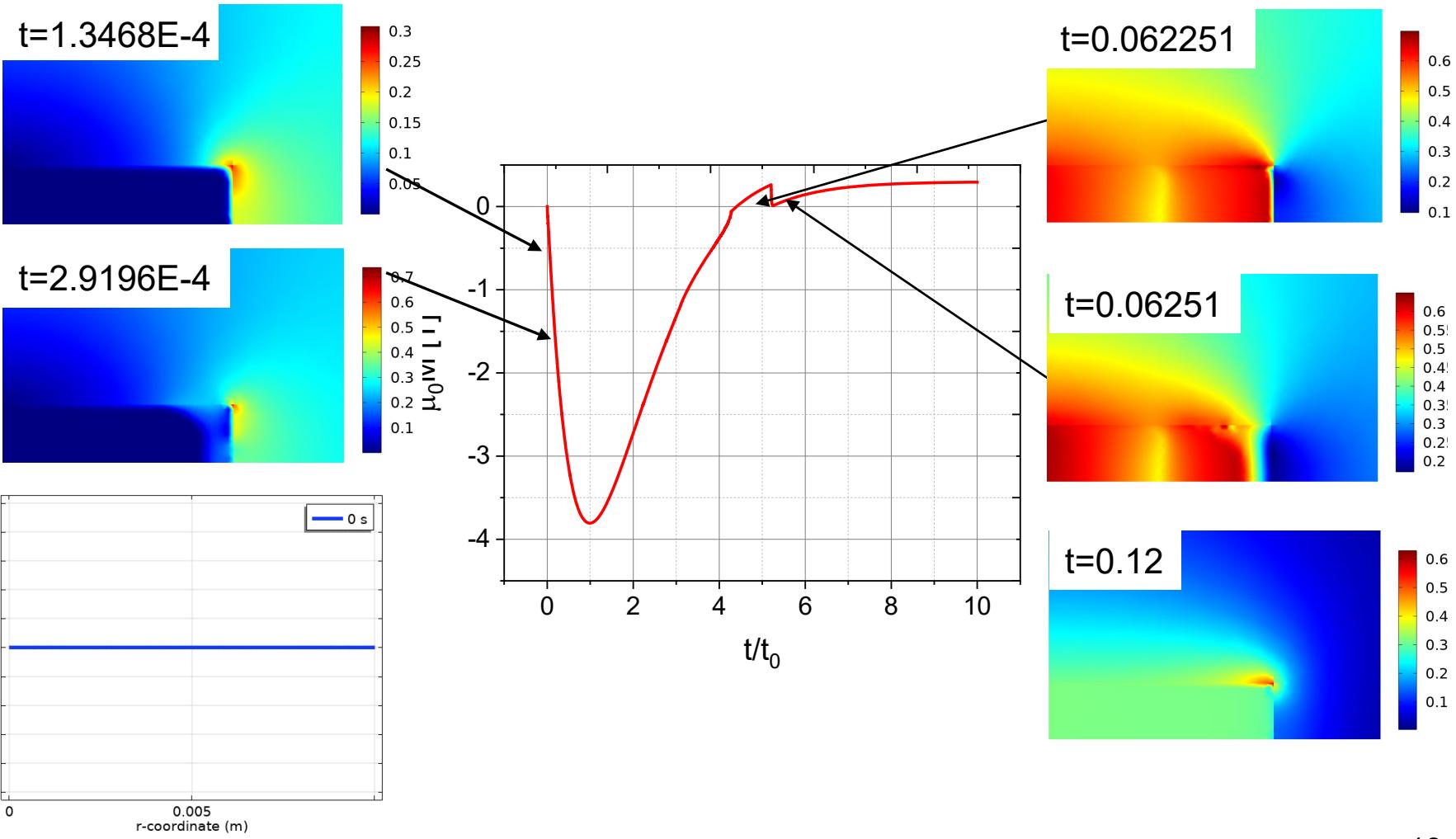
Research Progress: Instability in bulk SCs

Maximum Temperature Distribution within the bulk during flux jumps.



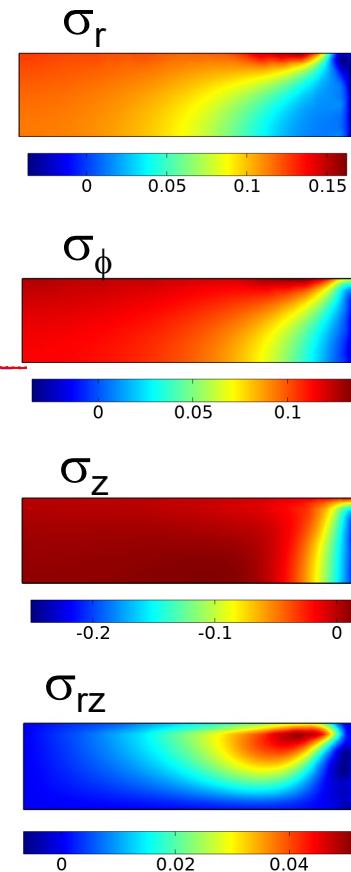
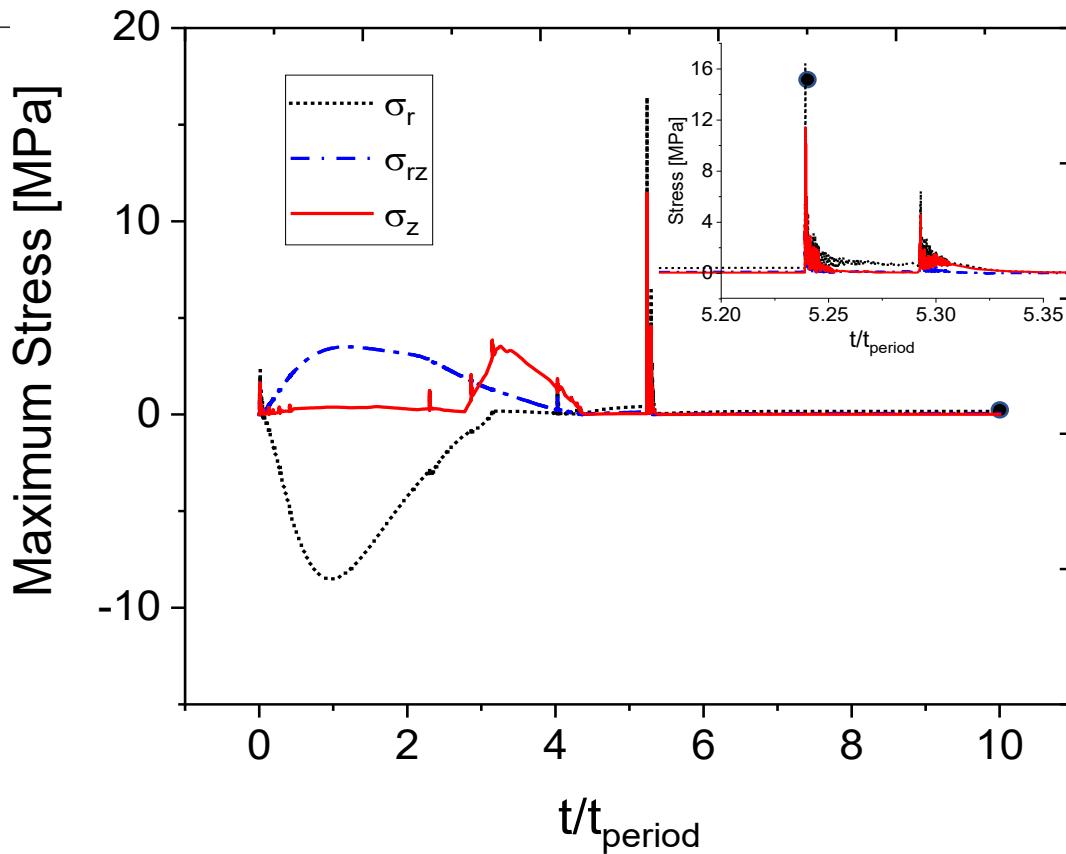
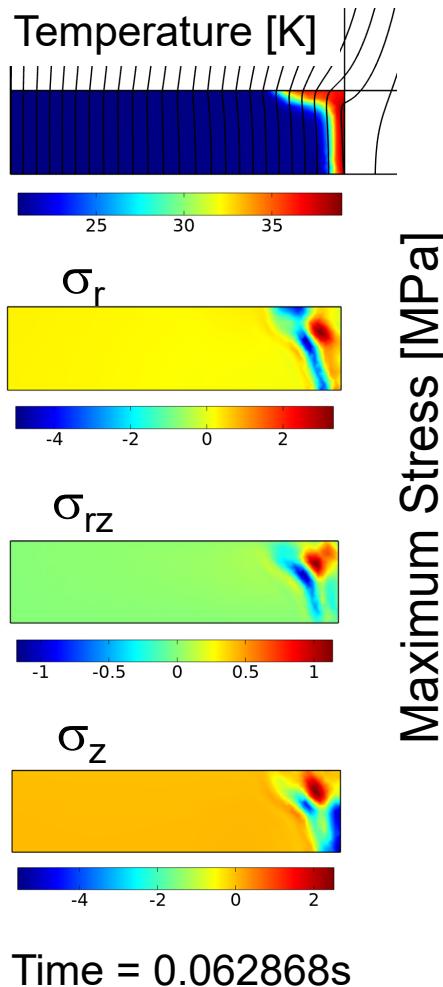
Research Progress: Instability in bulk SCs

Magnetic flux Distribution within the bulk during flux jumps.



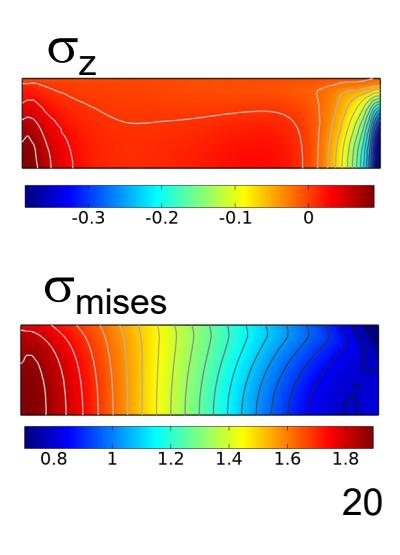
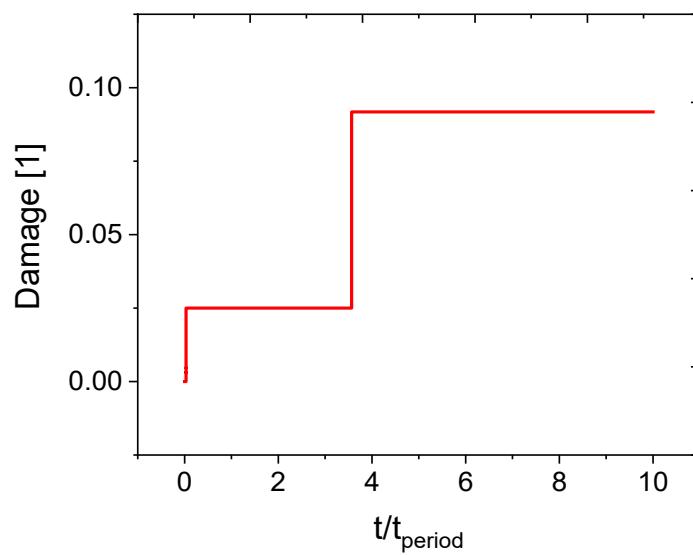
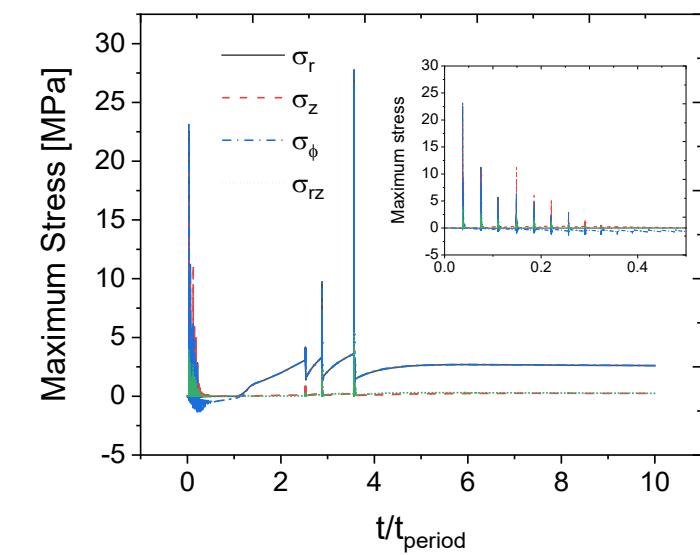
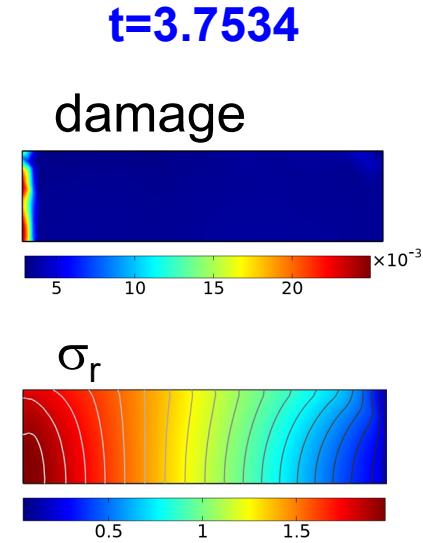
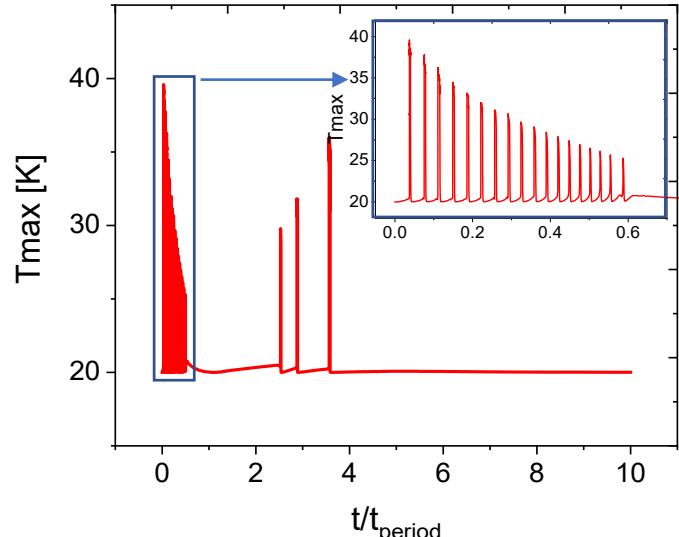
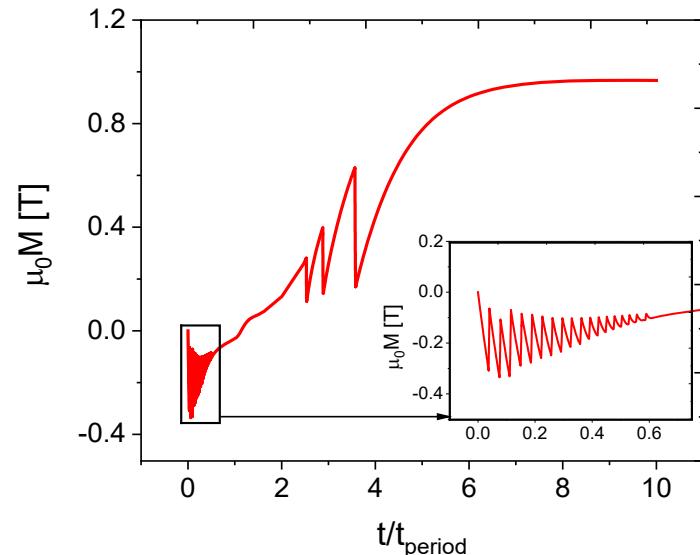
Research Progress: Instability in bulk SCs

Stress/strain distribution within the bulk during flux jumps.



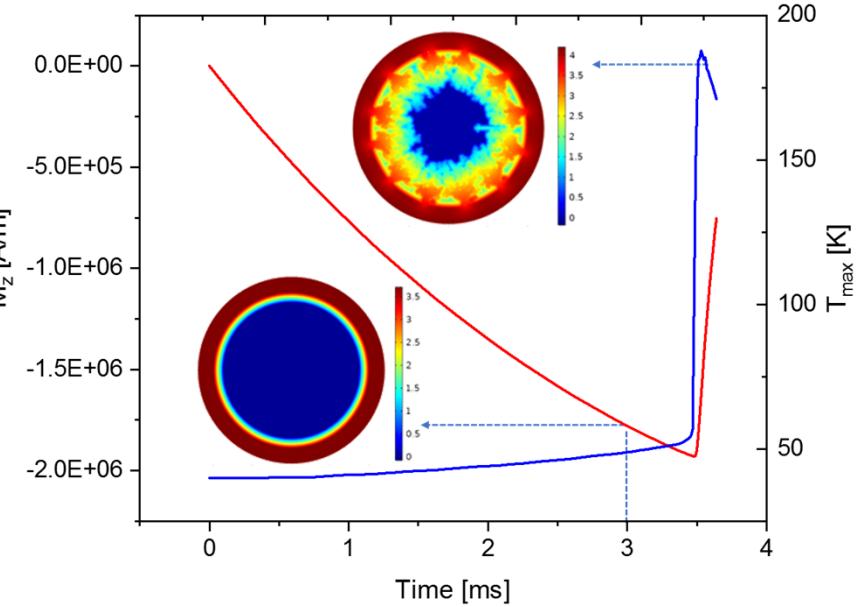
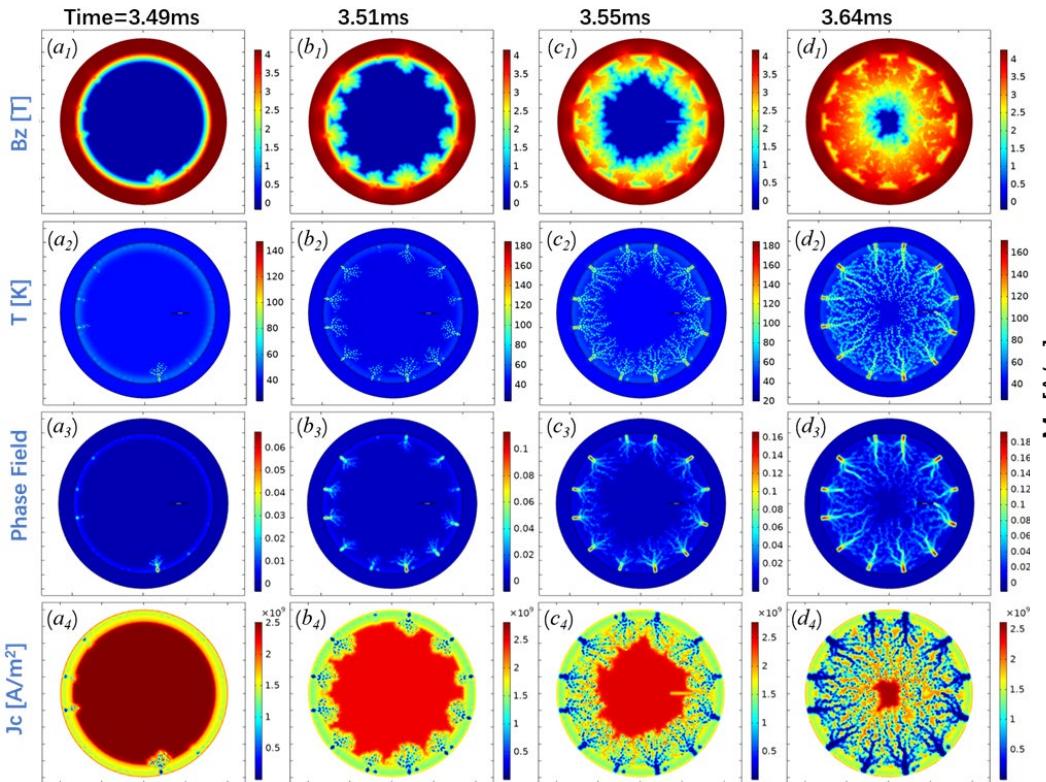
Research Progress: Instability in bulk SCs

Influence of the ramp rate on the flux jumps ($\tau=120\text{ms}$)



Research Progress: Flux Avalanches in bulk

Ininitely long cylinder bulk under pulsed field



Summary

- A numerical model is proposed to simulate the Flux jumps, stress/strain and structural degradation in Bulk superconductors during pulsed field magnetization.
- Due to the multiple timescales intrinsically involved in the instability behaviors, solving the coupled Multiphysics models numerically to simulate the thermo-magnetic-electro-mechanical instability is still a challenge task ahead.
- More efficient simulation methods (such as the FFT based scheme) are needed to simulate the coupled instability behavior.

Thanks for your attention!