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https://htsmod2020.sciencesconf.org/

Field-Circuit Coupled Simulation of Magnetothermal Dynamics in an HTS Solenoid

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INTRODUCTION

High temperature superconductors (HTS)

- Oxides (CuO2) doping with La, Y-Ga-Ba etc.
- Higher critical temperature and coercive field with respect to low-temperature superconductors (LTS)

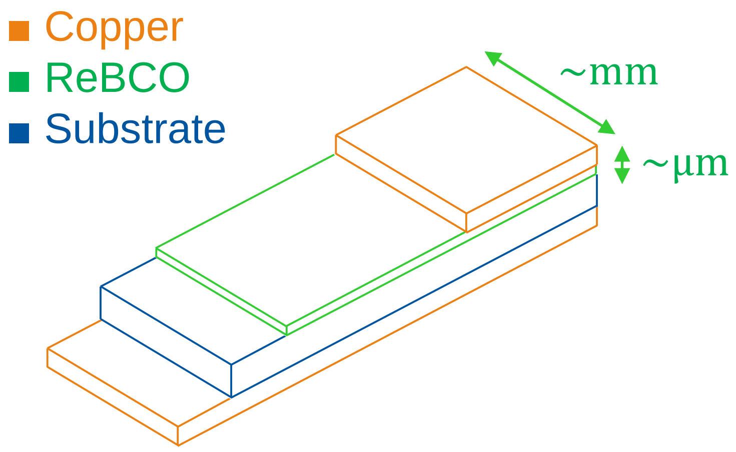
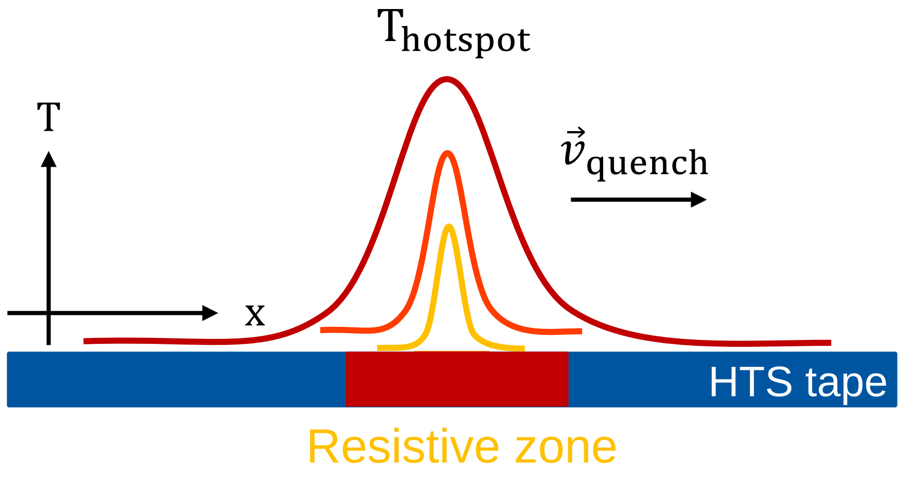


Figure: schematics of a copper-coated ReBCO tape

Quench

Local loss of the superconducting state



High energy-density → potentially irreversible effects!

Mitigation: artificial resistive zone increase → dilution of the Ohmic loss density

Motivation: Simulation of transient effects in circuits of accelerator magnets
Field-Circuit Coupled Systems!

MATHEMATICAL FORMULATION

Magnet domain decomposition

$\Omega_H = \Omega_{H,s} \cup \Omega_{H,c}$ active region → Coils:

- $\Omega_{H,s}$ superconductors ($\sigma \to +\infty$)
- $\Omega_{H,c}$ normal conductors

$\Omega_A = \Omega_{A,c} \cup \Omega_{A,i}$ passive region → Iron yoke, air region:

- $\Omega_{A,c}$ normal conductors
- $\Omega_{A,i}$ insulators ($\rho \to +\infty$)

Field equations [⋆]

$$\nabla \times \rho \nabla \times \mathbf{H} + \mu \partial_t \mathbf{H} + \nabla \times \boldsymbol{\chi}\, u_s = 0$$ **H** formulation in Ω_H

$$\nabla \times \nu \nabla \times \mathbf{A}^\star + \sigma \partial_t \mathbf{A}^\star = 0$$ $\mathbf{A}^\star$ formulation in Ω_A

$$\rho_m C_p \partial_t T - \nabla \cdot k \nabla T - \mathbf{J} \cdot \rho \mathbf{J} = 0$$ Heat balance equation in Ω

$$\int_{\Omega_H} \boldsymbol{\chi} \cdot \nabla \times \mathbf{H}\, d\Omega = i_s$$ Current constraint

$$\boldsymbol{\chi} = -\nabla \xi, \qquad \xi : \ \nabla \cdot \sigma \nabla \xi = 0$$ Voltage distribution function

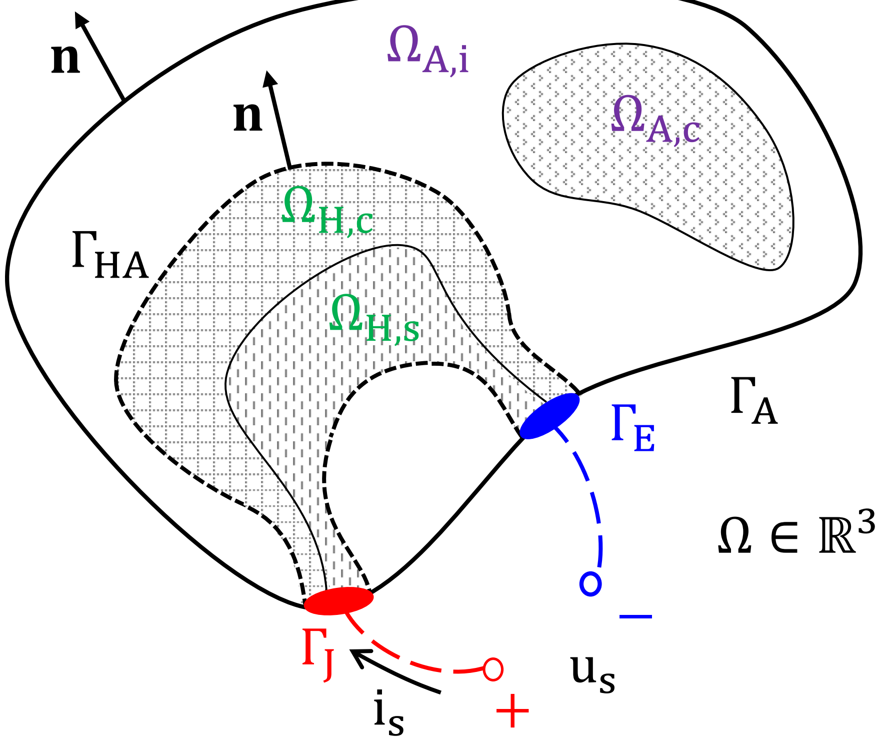


Figure: General representation of the computational domain

[⋆] Bortot, L., et al. "A Coupled A–H Formulation for Magneto-Thermal Transients in High-Temperature Superconducting Magnets." IEEE Transactions on Applied Superconductivity 30.5 (2020): 1-11.

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DISCRETE EQUATIONS

Discretization functions

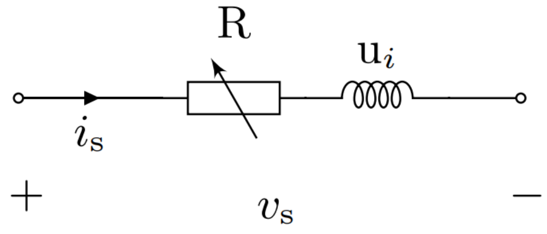
- Edge elements for $\mathbf{H}$, $\mathbf{A}^\star$ (1st and 2nd order)
- Nodal elements for $\boldsymbol{\chi}$, T (1st order)

$$\begin{bmatrix} \mathbf{K}^{\nu}+\mathbf{M}^{\sigma}\frac{\mathrm{d}}{\mathrm{dt}} & -\mathbf{Q} & \mathbf{0} & \mathbf{0} \\ \mathbf{Q}^{\mathrm{T}} & \mathbf{K}^{\rho}+\mathbf{M}^{\mu}\frac{\mathrm{d}}{\mathrm{dt}} & -\mathbf{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}^{\kappa}+\mathbf{M}^{\rho}\frac{\mathrm{d}}{\mathrm{dt}} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{h} \\ \mathbf{u_s} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{i_s} \\ \mathbf{q}(\cdot) \end{bmatrix}$$

$\mathbf{A}^\star$ form	$\int du$	Weak form PDE module
$\mathbf{H}$-form	$\int du$	
Coupling terms	$\int du$	
Heat Balance		Heat transfer in solids
Circuit coupling	$\frac{d}{dt}$	Global ODEs and DAEs

Circuit interface

- External circuit connected via electric ports
- FEM model → one-port component with impedance $\mathrm{Z_{FEM}}$: $\mathrm{u_s} = \mathrm{Z_{FEM}}\ \mathrm{i_s}$



Linearized field-circuit coupling interface for solid conductors

NUMERICAL EXAMPLE (1/2)

HTS solenoid protected by heater strips: 2D axisymmetric model and circuitry

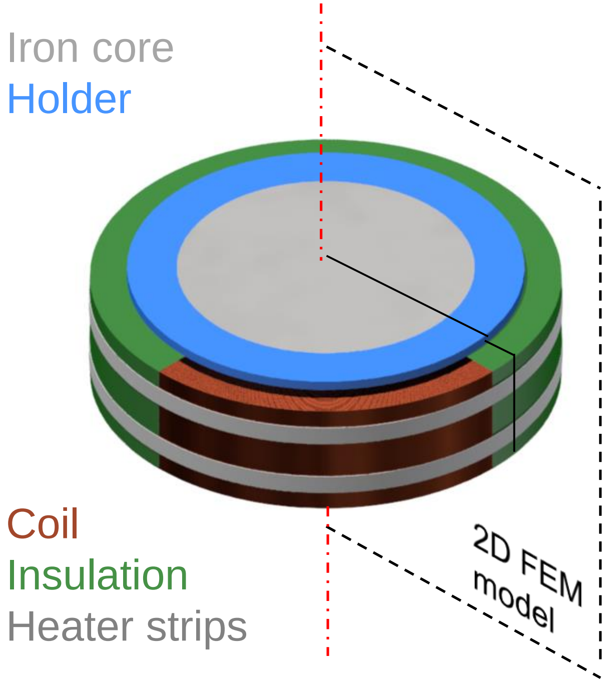


Figure: Rendering of the HTS solenoid. Part of the insulation is removed for illustration purposes.

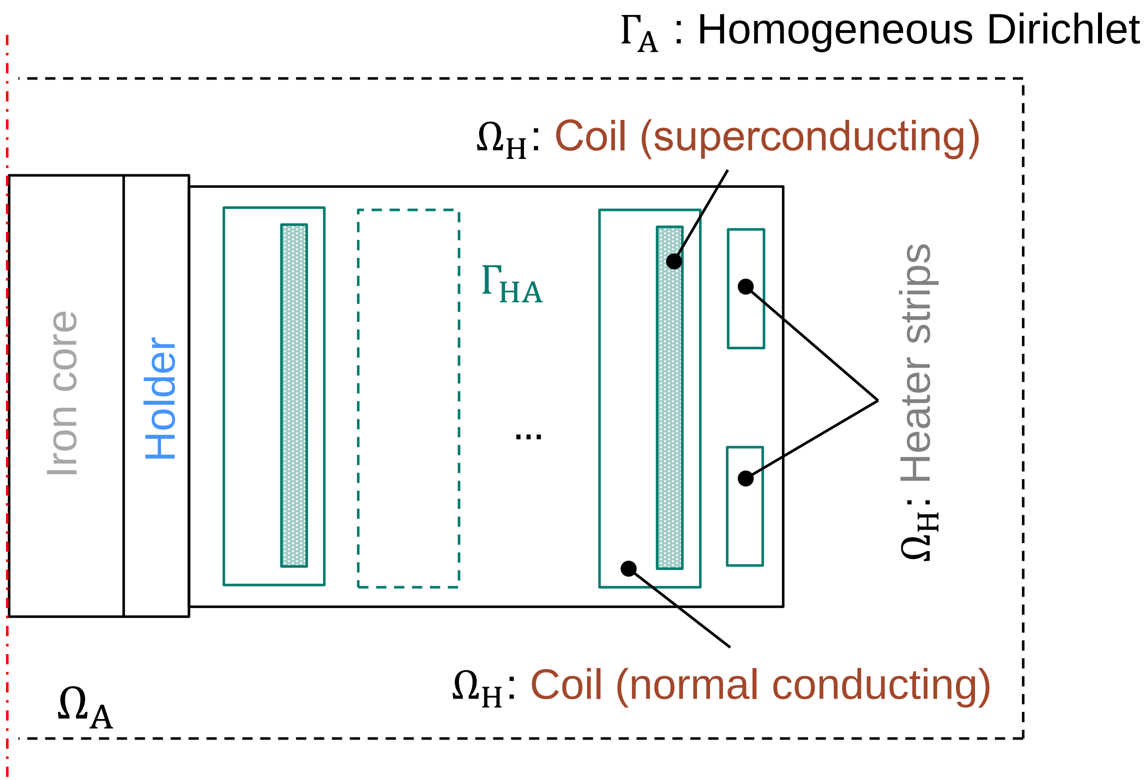
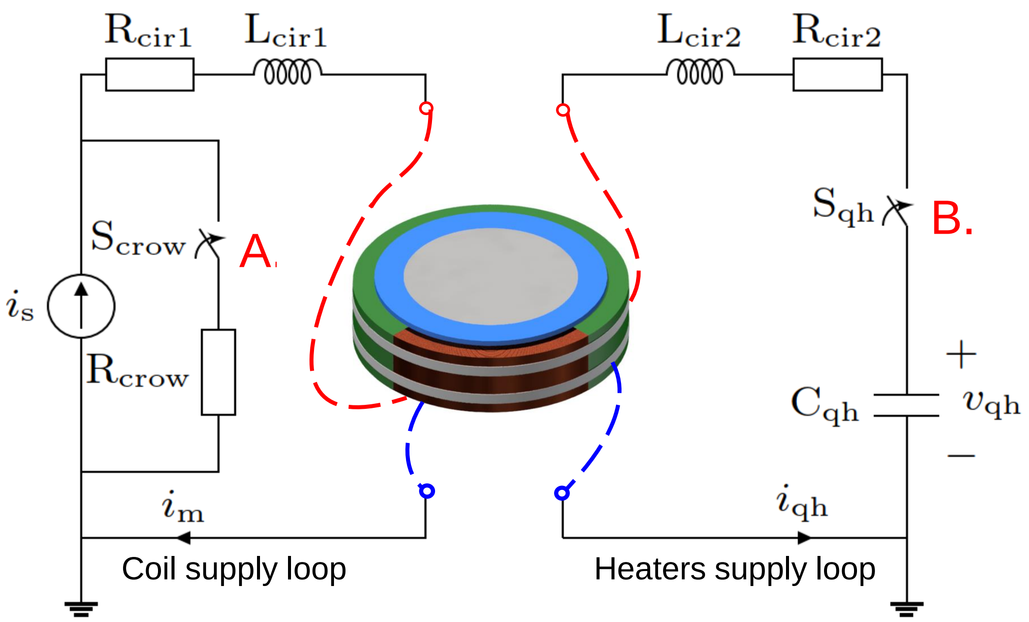
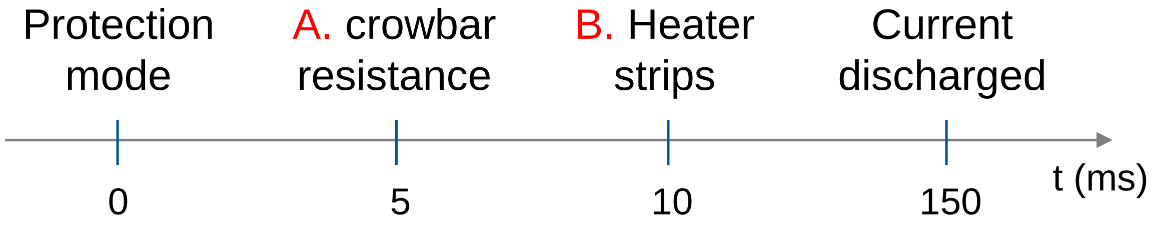


Figure: Domain decomposition

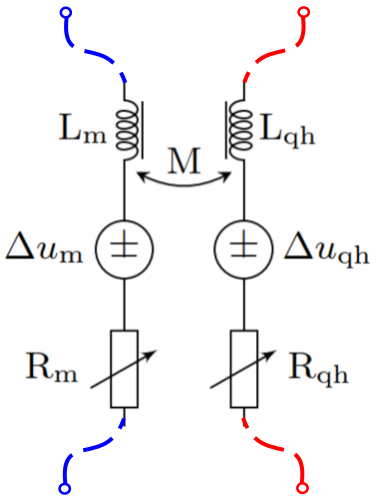
NUMERICAL EXAMPLE (2/2)

Electrical layout





Solenoid lumped-parameter representation



Optimized Schwarz transmission condition for field-circuit coupled simulations with the waveform relaxation algorithm [⋆]

Observations:

$R_{crow} + R_m$ discharge the solenoid current
R_m determined by the quench

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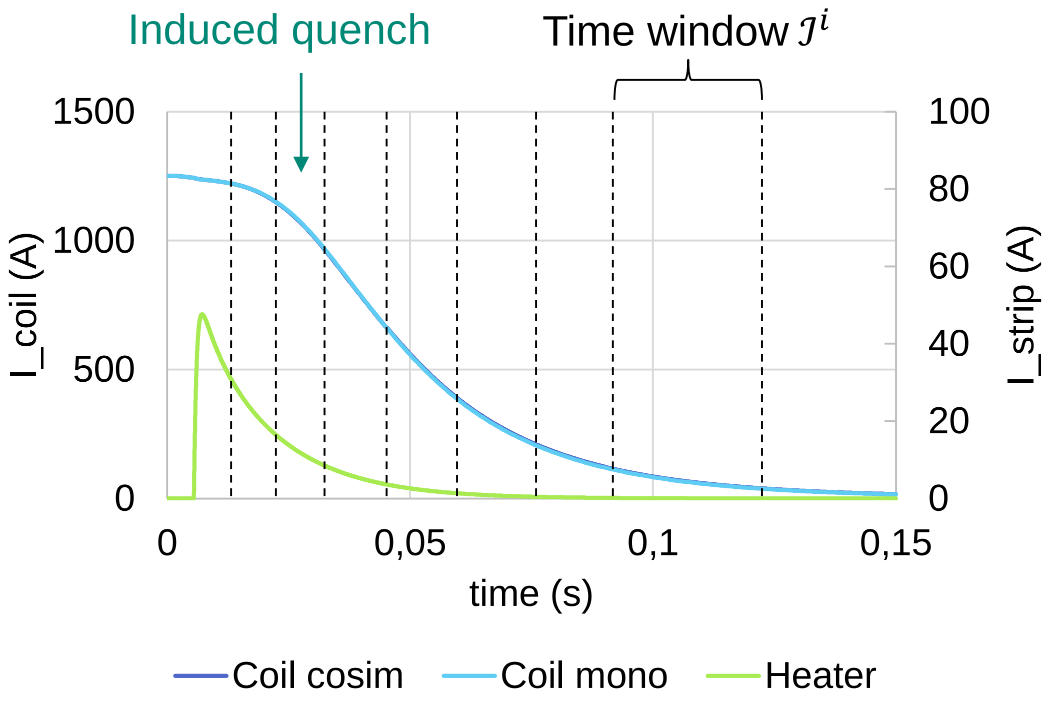
(⋆) Garcia, Idoia Cortes, et al. "Optimized field/circuit coupling for the simulation of quenches in superconducting magnets." IEEE Journal on Multiscale and Multiphysics Computational Techniques 2 (2017): 97-104.

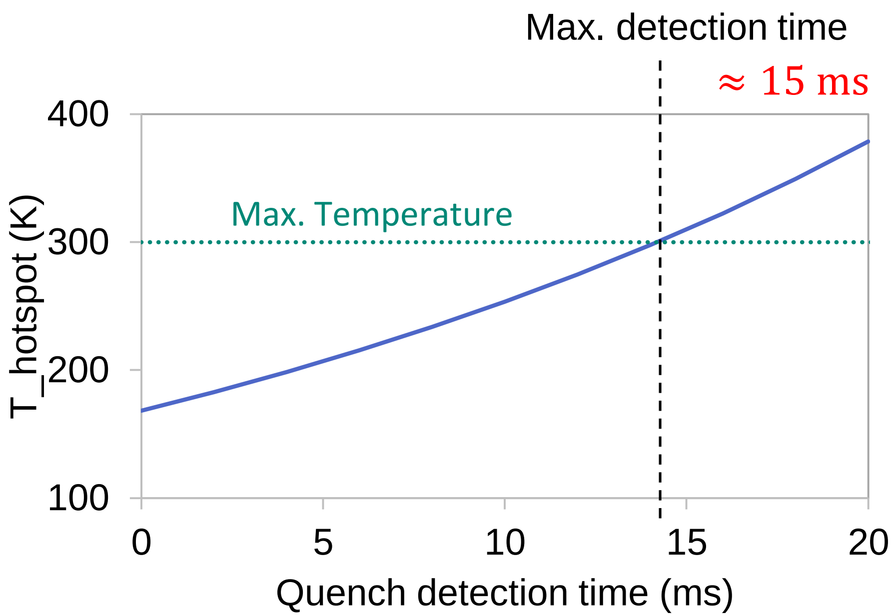
(A) NUMERICAL RESULTS (1/3)

Circuital currents



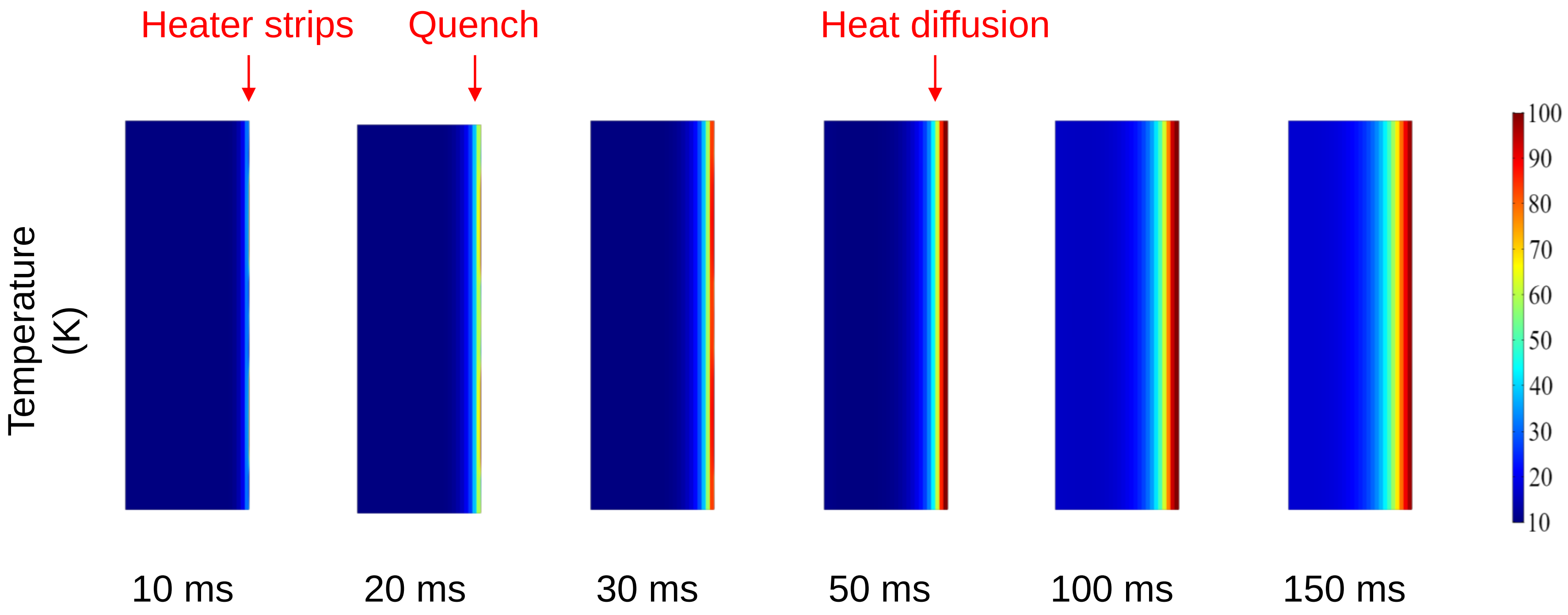
Current decay in the coil (left) and current discharge in the heater strips (right)

Peak temperature



Adiabatic hotspot temperature in the coil, as a function of the quench detection time

NUMERICAL RESULTS (2/3)



Temperature distribution in the superconducting coil, as a function of time

NUMERICAL RESULTS (3/3)

Definitions, at iteration *i*

- x_i signal (current in the magnet)
- ε_{abs} ε_{rel} absolute & relative error
- ε_i convergence error
- $\mathrm{F_{conv}}$ convergence flag

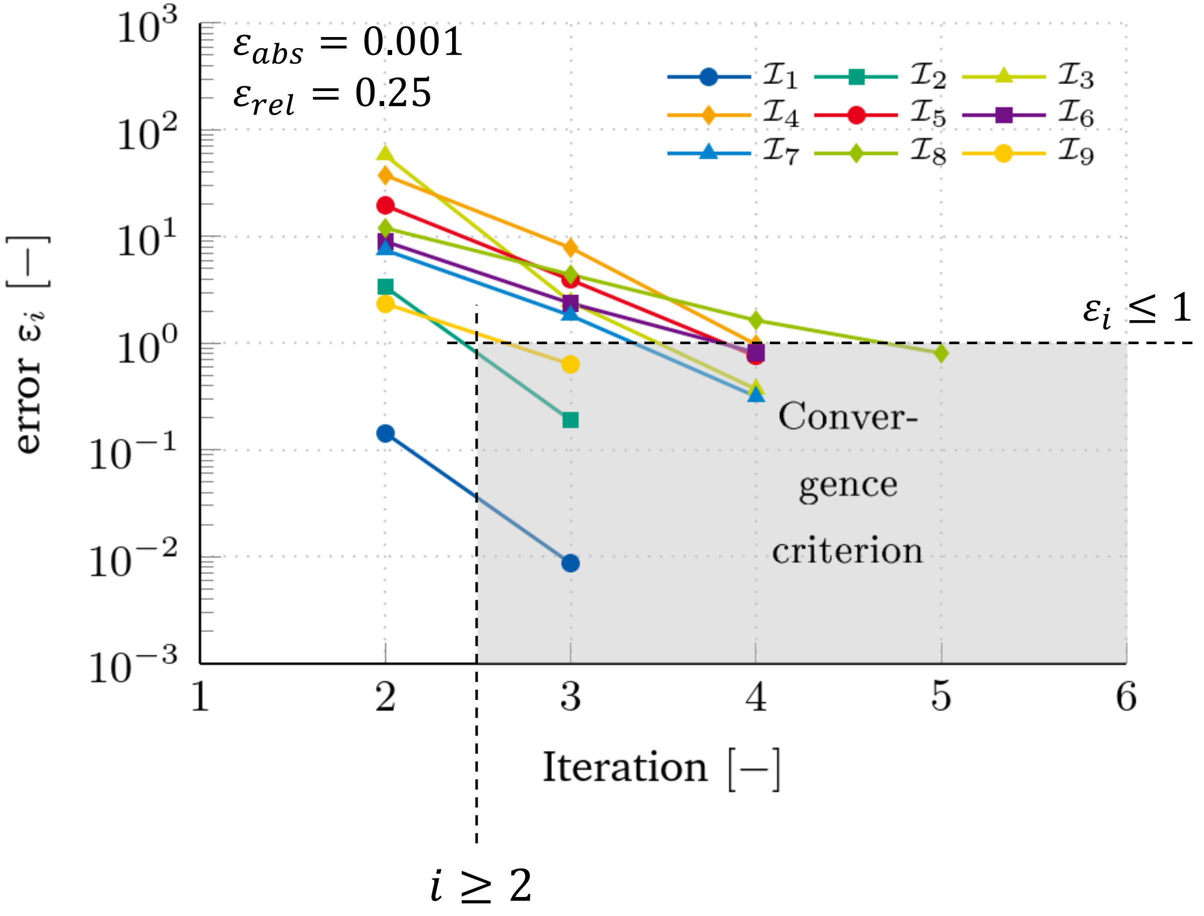
$$\varepsilon_i = \max\left(\frac{|x_i - x_{i-1}|}{\varepsilon_{abs} + |x_i|\varepsilon_{rel}}\right), \quad i \geq 2$$

$$\mathrm{F_{conv}} = \begin{cases} 0, & \text{if } \ i < 2 \\ \varepsilon_i < 1, & \text{if } \ i \geq 2 \end{cases}$$

Enforcement of at least three iterations per time window

Quench as abrupt change in resistivity

- → High influence on the solenoid current
- → More iterations needed!



CONCLUSIONS AND OUTLOOK

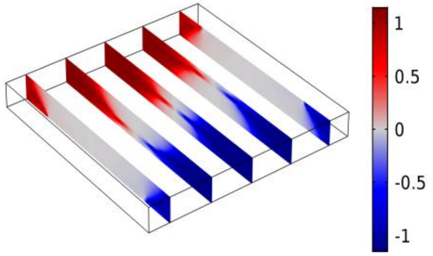


Figure:
normalized induced
current in a HTS bulk

Conclusions

- A-H field formulation for HTS-based accelerator magnets
- Field-circuit coupling interface based on solid conductor model
- Co-simulation of HTS magnets with the waveform relaxation scheme

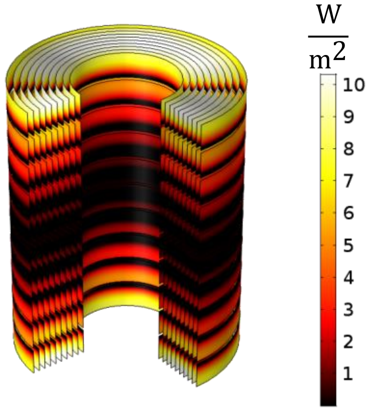


Figure:
Ohmic loss distribution
in a HTS solenoid

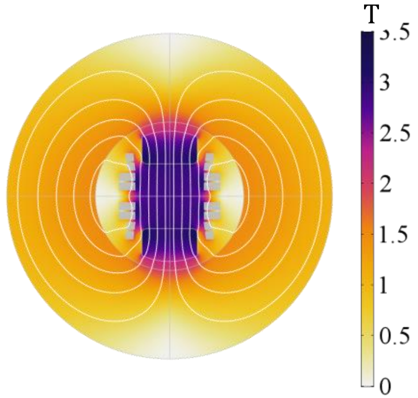


Figure: magnetic flux density
in the Feather-M2
insert dipole magnet

Outlook

- Minimization of the quench detection time
- Protection of HTS magnets in case of a quench
- Dynamics of HTS magnets in accelerator circuits

Thank you for your attention!

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