Fast and efficient HTS modelling using ANSYS *A*-*V* formulation

Kai Zhang :: Insertion Device Group :: Paul Scherrer Institute

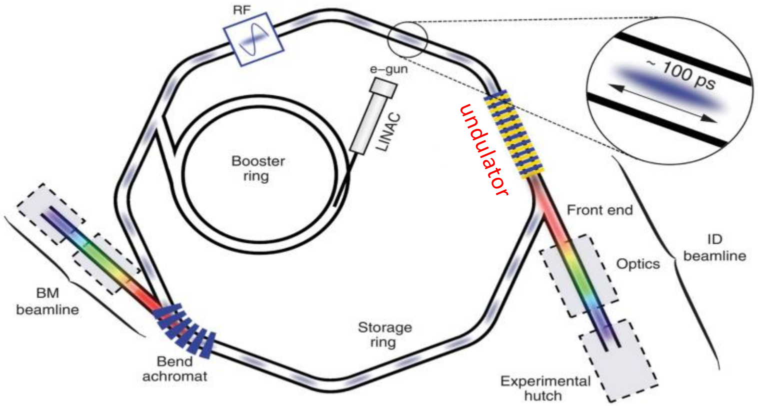
Acknowledgement: Mark Ainslie (Cambridge), Lucas Brouwer (LBNL), Ryota Kinjo (RIKEN Spring-8), Marco Calvi, Thomas Schmidt and Sebastian Hellmann (PSI)

7th International Workshop on Numerical Modelling of High Temperature Superconductors
23.06.2021, Virtual (Nancy, France)

Outline

- HTS for undulator applications
- HTS modelling using ANSYS ***A***-*V* formulation
  - A. Resistivity-adaptive algorithm (RAA)
    - ✓Critical state model: 2D
  - B. Direct iteration method
    - ✓Flux creep model (***E-J*** power law): 2D and 3D
    - ✓Critical state model: 2D
  - C. Backward computation method
    - ✓Critical state model: 2D and 3D
- Conclusion

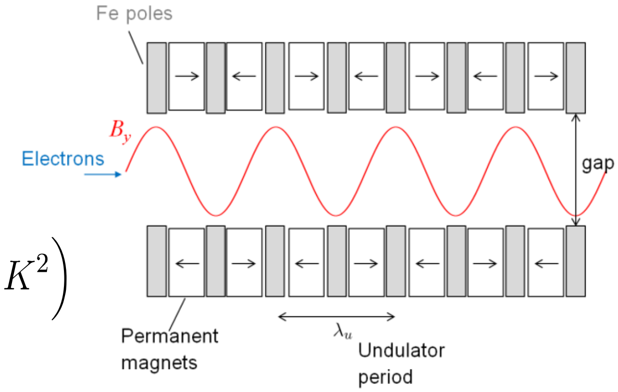
PSI Light sources: Storage Rings and FELs

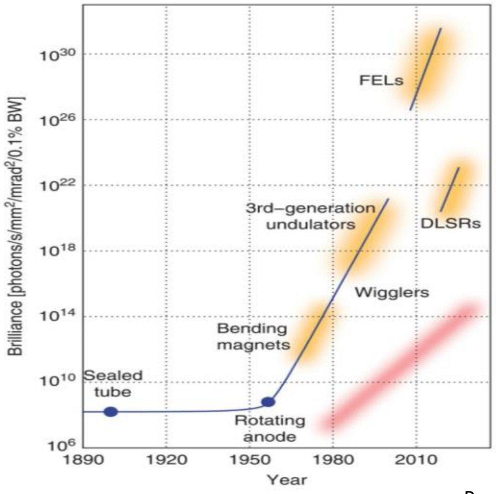


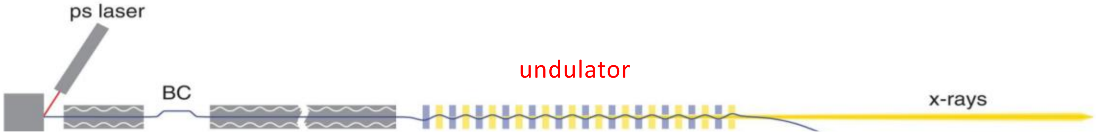
Storage Rings

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1+\frac{1}{2}K^2\right)$$

$$K = \frac{B_0 e}{mc}\frac{\lambda_u}{2\pi}$$







Free-Electron Lasers

SwissFEL, Aramis beamline (PM undulator)

15mm period

3mm gap

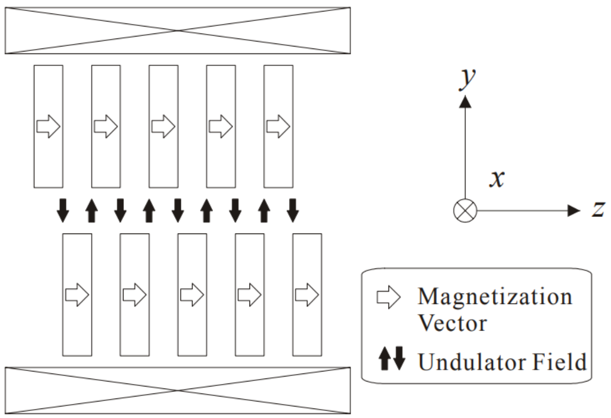
B_0 = 1.28T

3500 periods

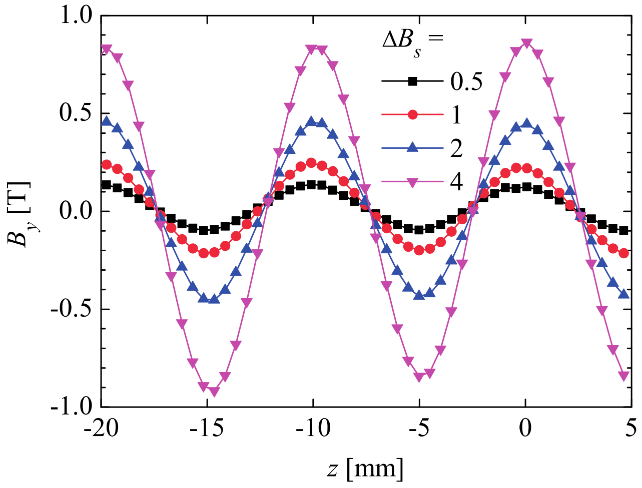
40m long

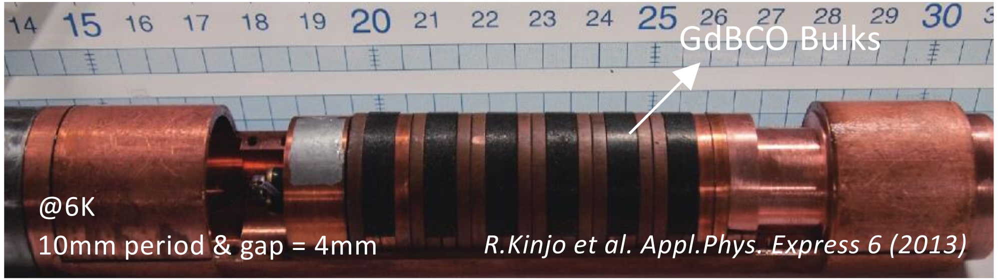
In future, we want to go to 10mm period …

Staggered-array bulk Re-Ba-Cu-O undulator



Kinjo R et al 2008 Proc. 30th FEL Conf. 473-76





Ideally, B_y can reach 2T when ΔB_s = 10T.

This HTS undulator concept is attractive to the new hard x-ray beamlines planned for both SLS2.0 and SwissFEL at PSI.

Our big Challenge is large-scale HTS magnetization simulation and optimal design of the magnetic field (fast simulation desired)

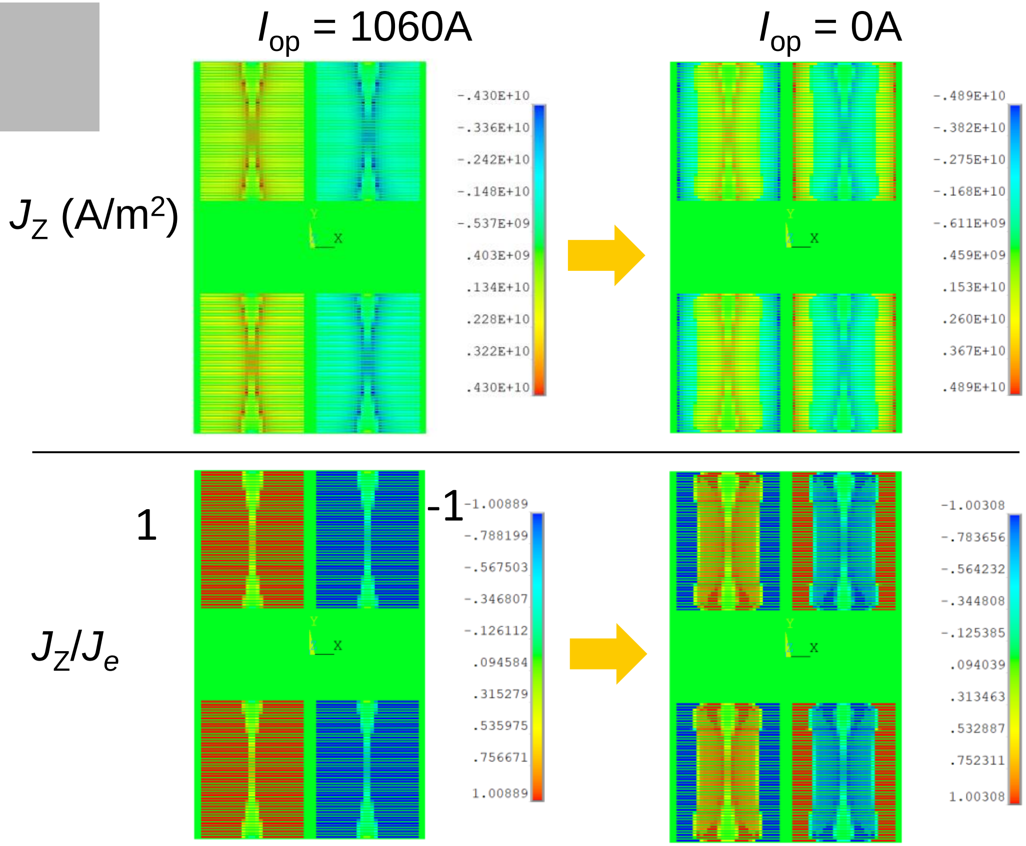
ANSYS ✓

A: Resistivity-Adaptive Algorithm (RAA)

- First proposed by Hidetoshi Hashizume et al in 1992
  - ✓ Initial electric-conductivity σ of all HTS elements is assumed sufficiently large
  - ✓ If $|\boldsymbol{J}| > |J_{\mathrm{c}}|$, then $\sigma^{i+1} = \frac{|J_{\mathrm{c}}|}{|\boldsymbol{J}|}\sigma^{i}$
- Further developed by Chen Gu et al in 2005 and 2013, and by Stefania Farinon et al in 2010 and 2014 through using ANSYS Parametric Design Language (APDL)
  - ✓ Initial resistivity ρ_0 of all HTS elements is set to a low value
  - ✓ Update $\rho^{i+1} = \max\left\{\frac{|\boldsymbol{J}|}{J_{\mathrm{c}}}\rho^{i}, \rho_0\right\}$, until $\left|\frac{\rho^{i+1}-\rho^{i}}{\rho^{i}}\right| \leq \varepsilon$ for all HTS elements

Hashizume H et al 1992 IEEE Trans. Magn. 28 1332-35
Gu C and Han Z 2005 IEEE Trans. Appl. Supercond. 15 2859-62
Farinon S et al 2010 Supercond. Sci. Technol. 23 115004
Farinon S et al 2014 Supercond. Sci. Technol. 27 104005
Gu C et al 2013 IEEE Trans. Appl. Supercond. 23 8201708

A: RAA – Screen currents



$$J_e(B_{//},B_\perp) = J_{e0}\cdot\left(1+\sqrt{(k|B_{//}|)^2+|B_\perp|^2}/B_c\right)^{-b}$$

On the left is the simulated screen currents inside a periodical FEA model (transport current I_{op} ramps to 1060A and drops to 0A).

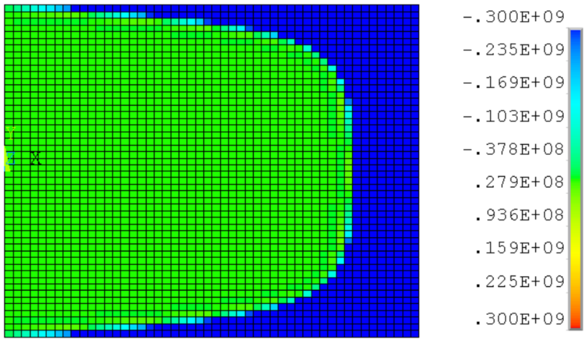
The rigorous critical state in the outer layer can still be reached during I_{op} drops, but there is a **slight decay in the inner layer** (this issue was also emphasized in [Gu C. et al 2013] and remained to be solved).

Computation speed: **~5 min for 60000 DOFs**

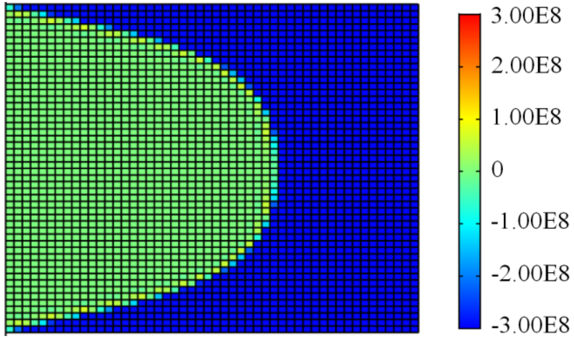
PAUL SCHERRER INSTITUT

A: RAA - Cylindrical bulk superconductor

Trapped J_Z after ZFC magnetization from 0 to 1T, J_e=3e8 A/m^2



ANSYS(RAA), critical state



COMSOL(***E-J*** power law), n = 100

Inconsistent critical state magnetization currents are found in ANSYS and COMSOL models, further examination with backward computation method proves the COMSOL result is correct.

It is still unclear why the RAA method fails in modelling the bulk superconductor ? Only feasible for transport current cases ? More research studies are required to address this problem …

B: Direct iteration method

① Initial resistivity ρ_0 of all HTS elements is set to a low value;

② The whole magnetization process is divided into N steps;

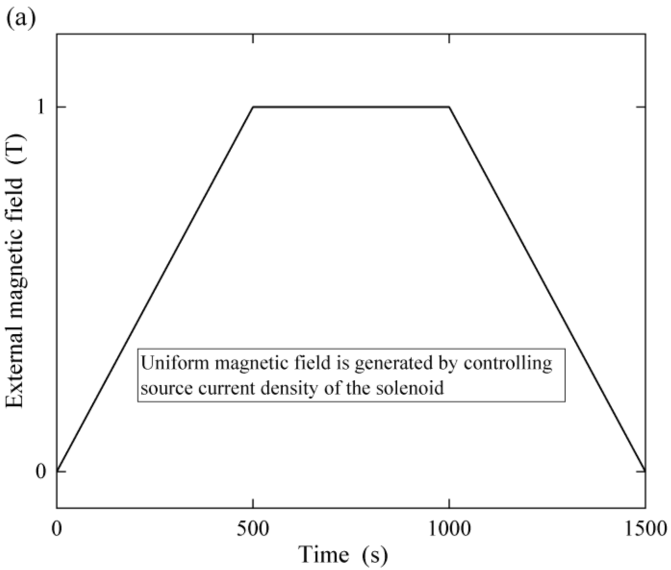
For simulating the **flux creep model**, resistivity of each HTS element is updated after every iteration

$$\rho_m^{i+1} = k\rho_m^i + (1-k)\cdot max\left\{\rho_0, \frac{E_c}{J_c}\cdot\left(\frac{\left|\boldsymbol{J}_{Tm}^i\right|}{J_c}\right)^{n-1}\right\}$$

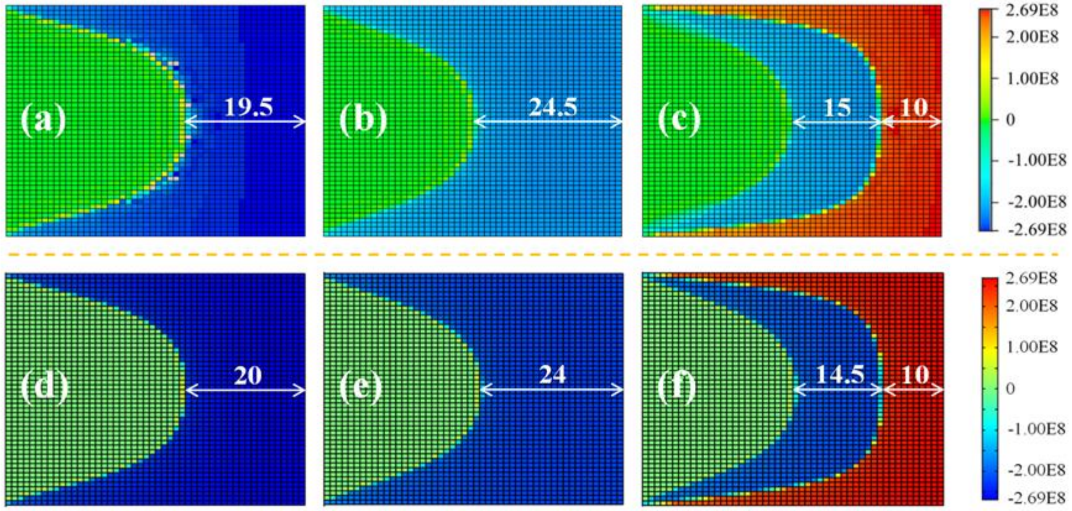
reservation coefficient, usually quite large

For simulating **critical state model**, each penetrated HTS element is forced with the latest J_c after every iteration

B: Direct iteration method (*E-J* power law, 2D)



Applied magnetic field versus time



J_z in the magnetized bulk HTS (n = 20) at (a) t = 500 s, (b) t = 1000 s and (c) t = 1500 s from using ANSYS ***A***-*V* formulation;

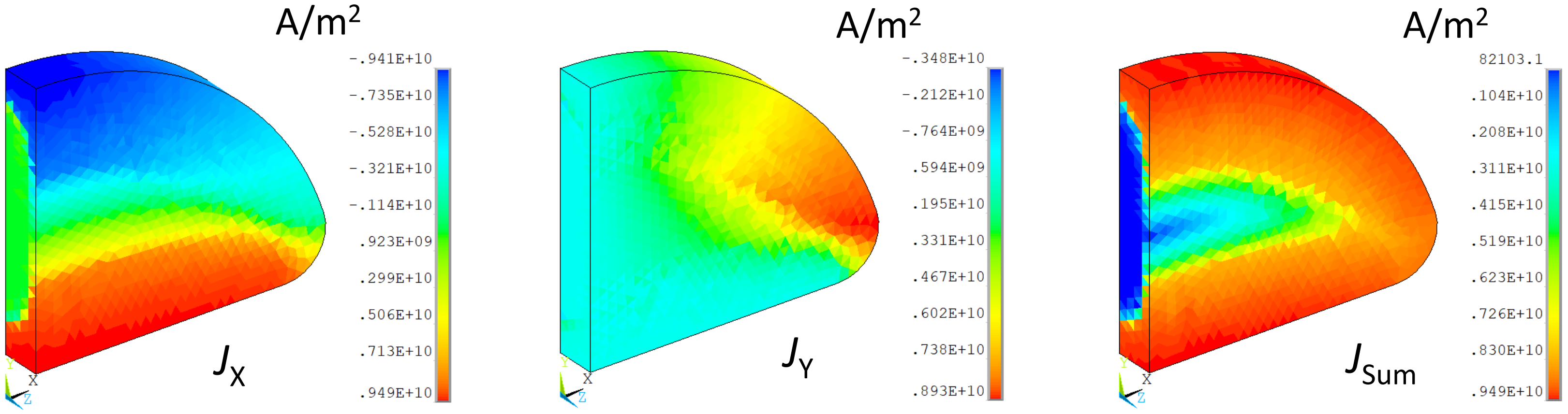
J_z in the magnetized bulk HTS at (d) t = 500 s, (e) t = 1000 s and (f) t = 1500 s from using COMSOL ***H***-formulation.

Zhang K et al 2021 IEEE Trans. Appl. Supercond. 31 6800206

Related APDL codes are shared in

https://www.researchgate.net/profile/Kai-Zhang-32

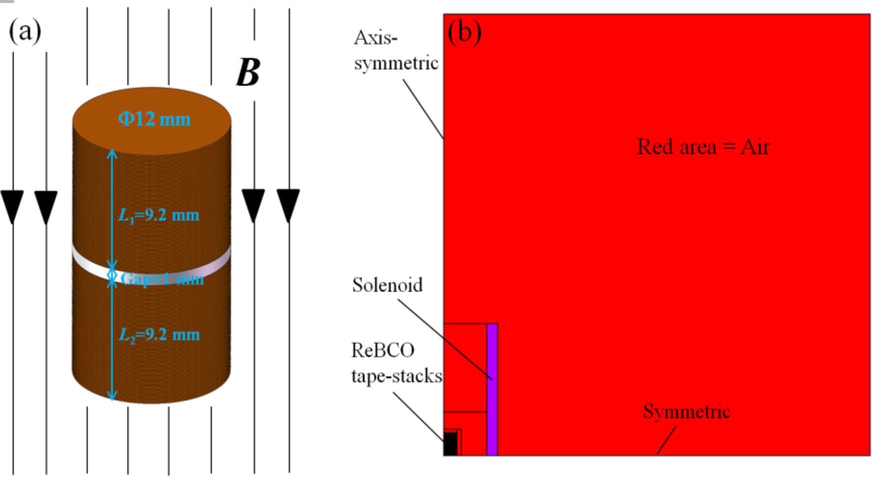
B: Direct iteration method (*E-J* power law, 3D)



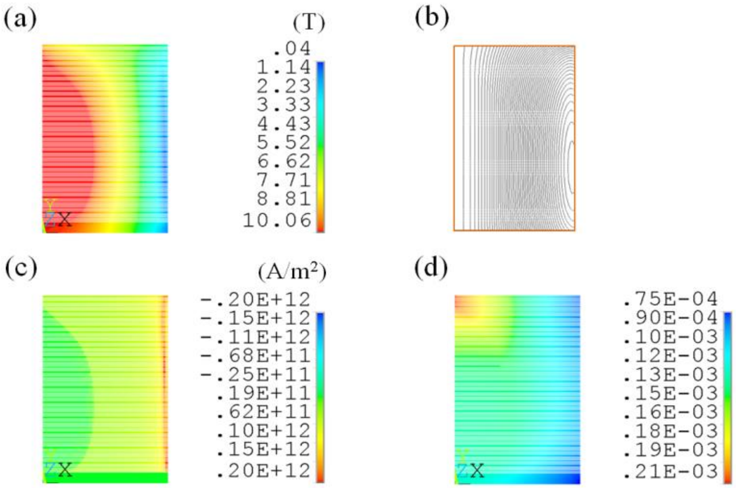
Trapped current in a ¼ half-moon shaped bulk superconductor model after FC magnetization from 8T to zero (J_e = 1e10 A/m^2, n = 20)

Problems: a number of iteration steps (usually >200) are essential to obtain smooth ***E-J*** power law based simulation results, this might result in a large amount of computation time for complex 3D FEA model.

B: Direct iteration method (critical state model, 2D)



(a) FC magnetization of the ReBCO tape stack; (b) 2D axis-symmetric half FEA model.



(a) Trapped B_s, (b) flux lines, (c) J_z, and (d) hoop strain in the ReBCO tape stack after FCM from 10 T.

Zhang K et al 2020 IEEE Trans. Appl. Supercond. 30 4700805

Related APDL codes are shared in

http://www.htsmodelling.com/ (model #23)

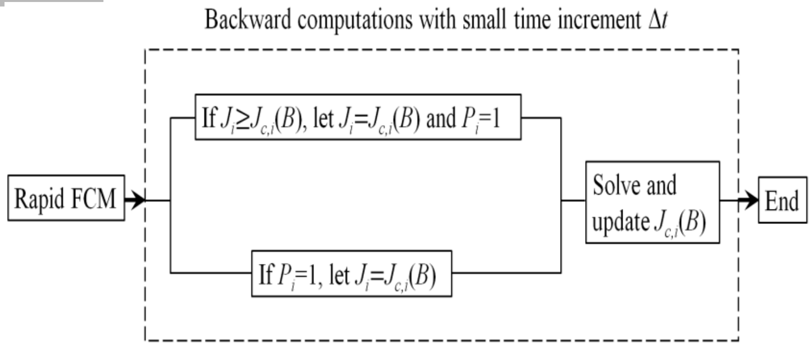
C: Backward computation method

- ✓ Ideal: HTS bulk acts as a permanent magnet after FC magnetization.
- ✓ Reality: this situation can never be realized since the flux pinning force is always limited.
- ✓ Assuming HTS bulk is FC-magnetized under isothermal conditions,

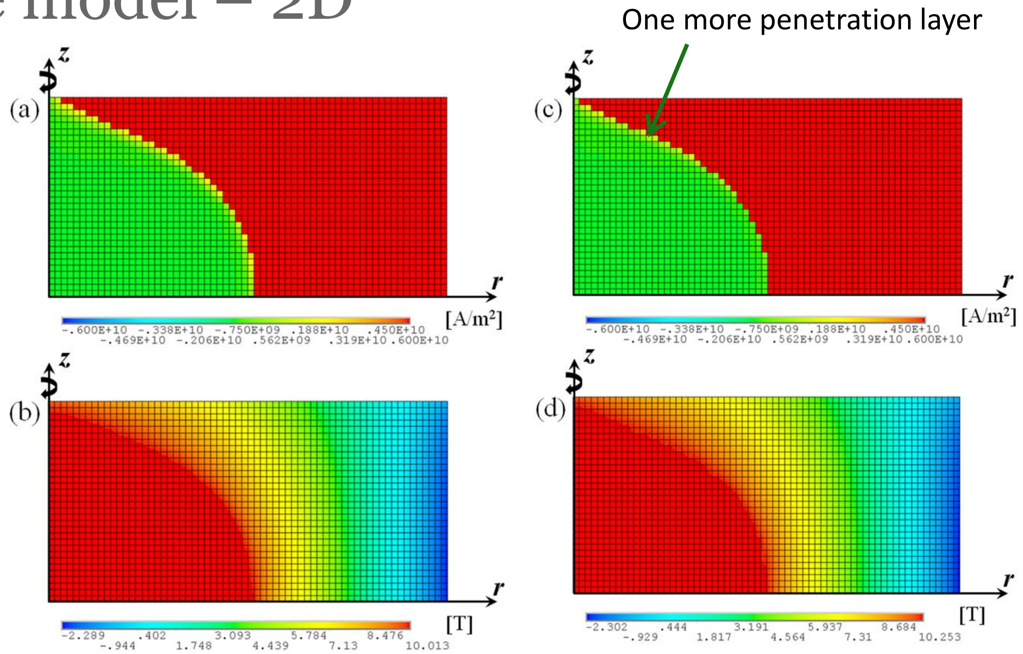
  eddy currents will gradually penetrate inwards

  following a quasi-static critical state model.

C: Backward computation method
Critical state model – 2D



Algorithm for the backward computation method for computing the critical state in a field-cooled magnetized bulk superconductor.

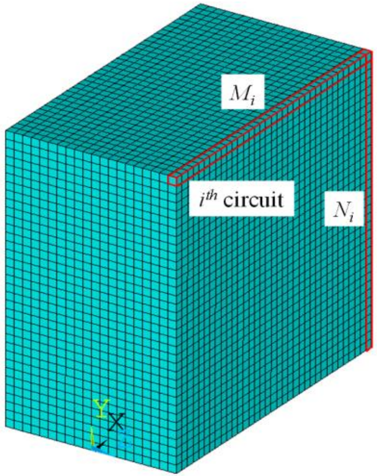


Critical state after FCM from 10T

(with minimum electro-magnetic entropy production [Pardo E 2017 J. Comput. Phys 344 339-63])

Trap >10T field after introducing an additional penetration layer

C: Backward computation method
Critical state model – 3D



Schematic of 1/4 cuboid bulk

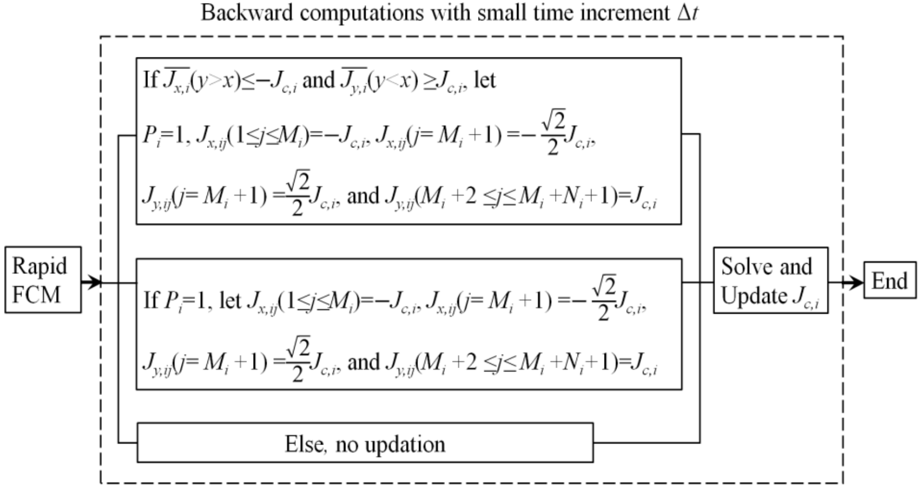
$$\boldsymbol{AZ}|_{x=0} = 0$$

$$\boldsymbol{AZ}|_{y=0} = 0$$

$$V|_{x=0} = 0$$

$$V|_{y=0} = 0$$

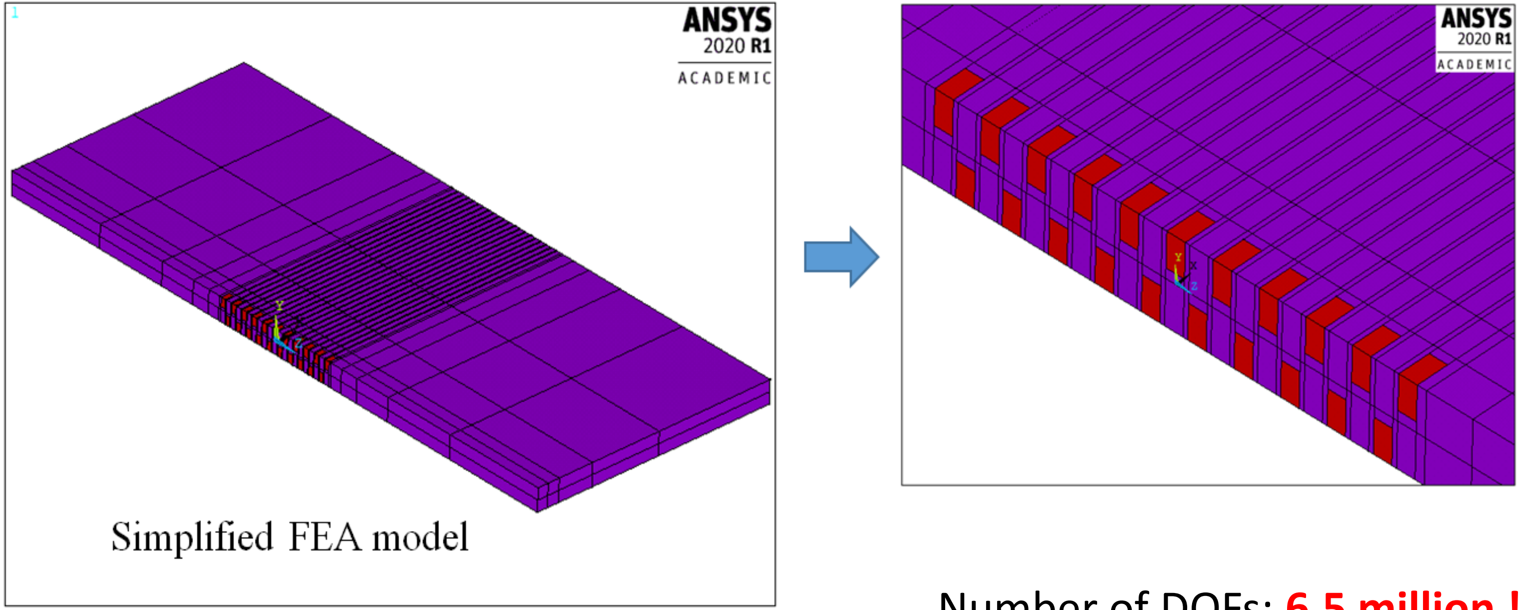
$$\begin{cases} \overline{J_{x,i}}(y>x) = \dfrac{1}{M_i}\displaystyle\sum_{j=1}^{M_i} J_{x,ij} \\ \overline{J_{y,i}}(y<x) = \dfrac{1}{N_i}\displaystyle\sum_{j=M_i+2}^{M_i+N_i+1} J_{y,ij} \end{cases}$$



$$\begin{cases} J_c(B) = J_{c1}\exp\left(-\dfrac{B}{B_L}\right) + J_{c2}\dfrac{B}{B_{max}}\exp\left[\dfrac{1}{y}\left(1 - \left(\dfrac{B}{B_{max}}\right)^{y}\right)\right] \\ J_{c,i} = \dfrac{1}{M_i + N_i + 1}\displaystyle\sum_{j=1}^{M_i+N_i+1} J_{c,ij}(B) \end{cases}$$

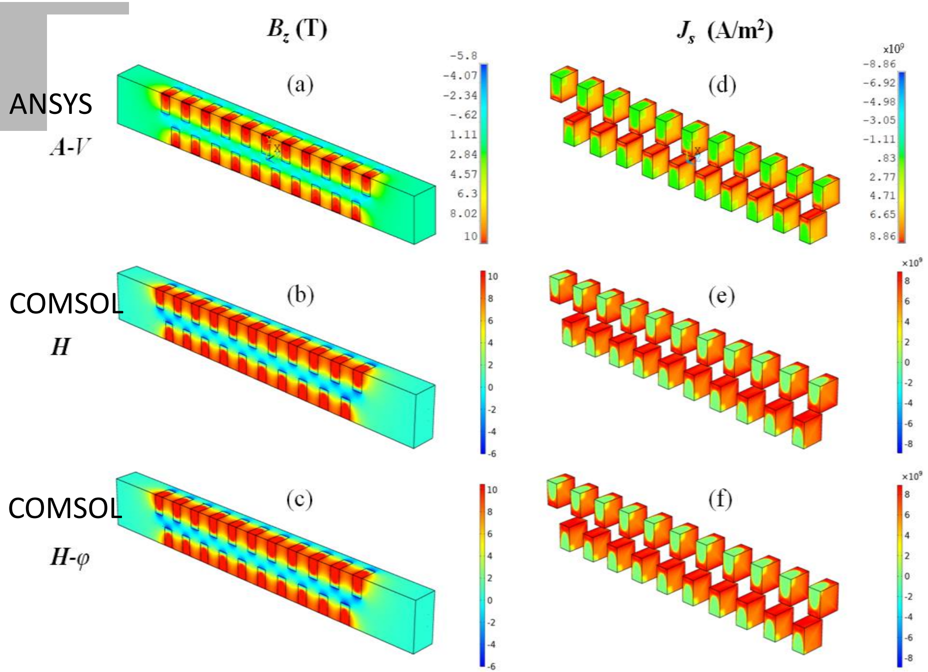
C: Backward computation method
FEA model of 3D bulk HTS undulator

(d)

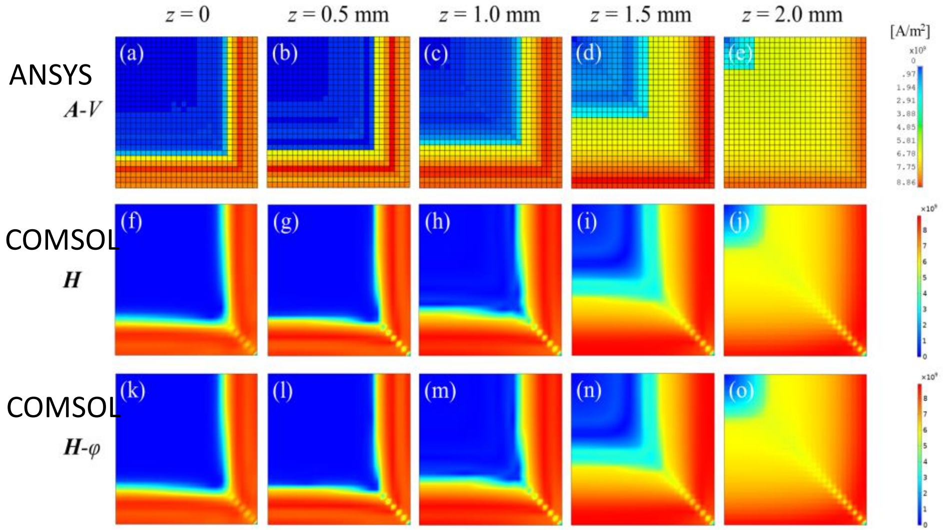


Number of DOFs: **6.5 million !**

C: Backward computation method
Modelling of 3D bulk HTS undulator

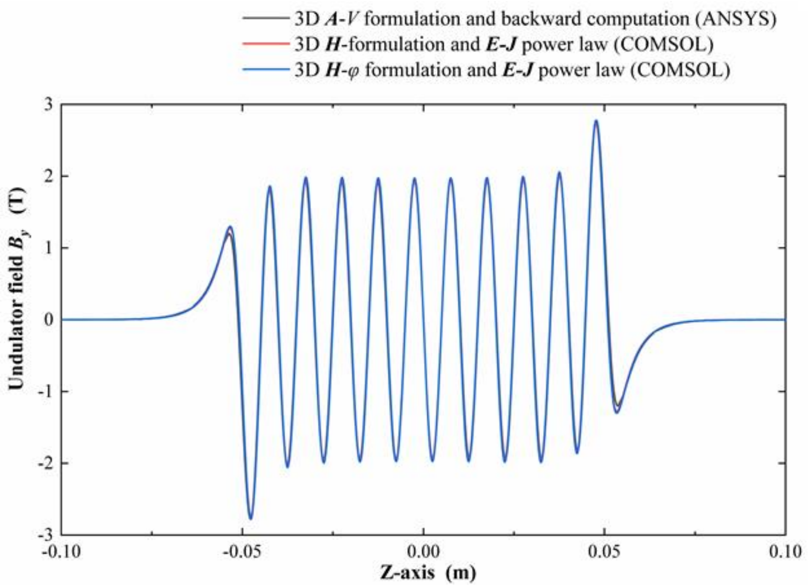


Magnetic field component B_z and the magnetization current density J_s in the BHTSU obtained using the ***A**-V*, ***H*** and ***H**-φ* [see A. Arsenault's talk] formulations



Magnitude of J_s in the central HTS bulk in the *xy*-plane. "*z* = 0" refers to the mid-plane of the HTS bulk; "*z* = 2 mm" refers to the outer surface of the HTS bulk.

C: Backward computation method Modelling of 3D bulk HTS undulator



Comparison of the calculated on-axis undulator field obtained using the ***A***-*V*, ***H*** and ***H***-*φ* formulation models

Summary of computation times for the ten-period bulk HTS undulator

		No. of DOFs (million)	Computation time (hour)
ANSYS	*A*-*V* (backward computation)	6.5	12.5
COMSOL	*H*	3.2	151
COMSOL	*H*-*φ*	1.3	23

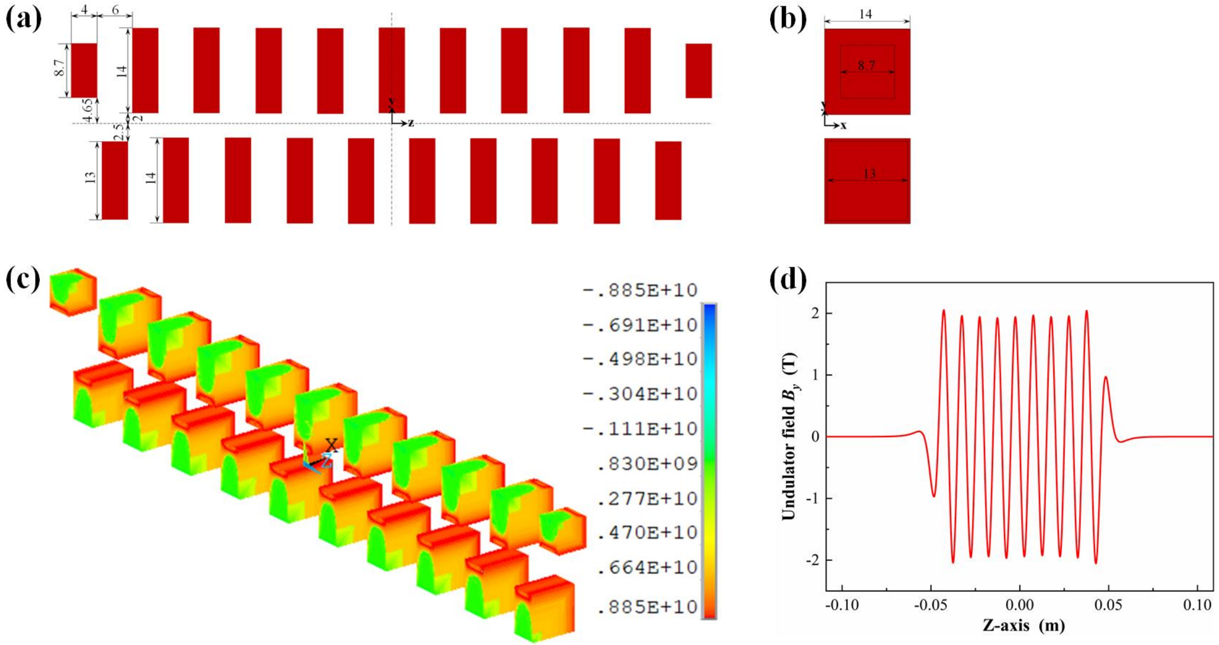
Zhang K et al 2021 arXiv:2105.01761

Related APDL codes and COMSOL models will be shared soon in

http://www.htsmodelling.com/

PAUL SCHERRER INSTITUT

C: Backward computation method
Optimal design of bulk HTS undulator

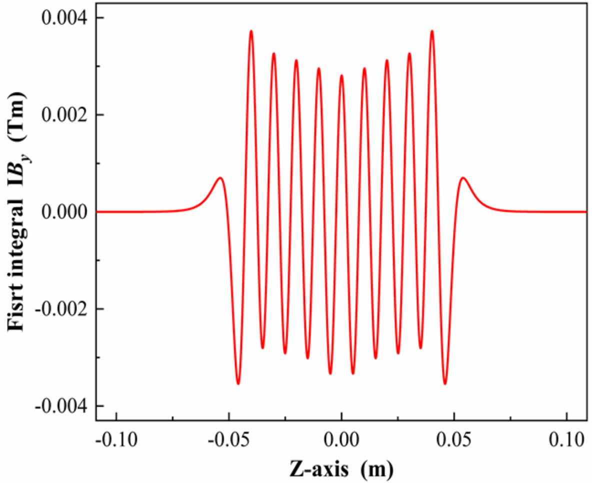


(a) Side view of the optimized BHTSU; (b) End view of the optimized BHTSU; (c) Vector sum of the magnetization current density in the simplified BHTSU model; (d) Undulator field B_y along *z*-axis;

C: Backward computation method
Optimal design of bulk HTS undulator

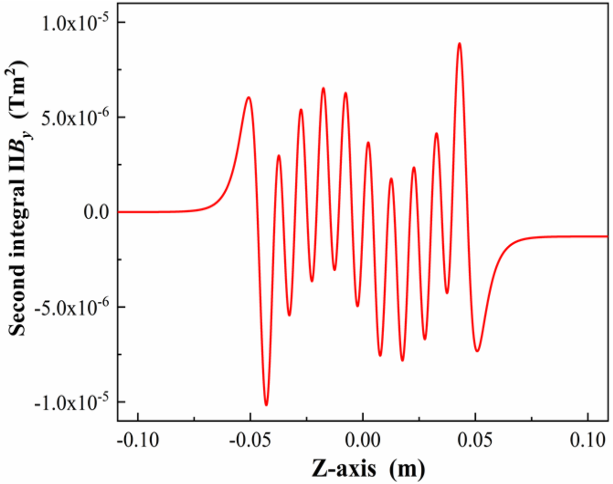
(e)

$$IB_y(z) = \int_{-\infty}^{z} B_y(z')\,dz'$$



(f)

$$IIB_y(z) = \int_{-\infty}^{z} IB_y(z')\,dz'$$



(e) First integral of the undulator field IB_y along z-axis; (f) Second integral of the undulator field IIB_y along z-axis.

Conclusion

- ✓ Three numerical algorithms implemented in ANSYS are benchmarked with COMSOL ***H/H***-*φ* formulation.
- ✓ The **RAA method** shows **fast computation** of the screen currents in HTS coils charged with **transport current**; problems are met for simulating bulk superconductors.
- ✓ The **direct iteration method** can solve magnetization problems for both **flux creep model** (***E-J*** power law) and **critical state model**, and for both **2D** & **3D**; the critical state solution is **fast** while the flux creep solution is **slow** (a large amount of iteration steps are required).
- ✓ The **backward computation method** shows extremely **fast** computation speed in modelling the critical state in large-scale **(6.5 million** DOFs**)** bulk HTS undulator model for both 2D & 3D (important for optimal design).
- ✓ ANSYS is quite flexible for secondary development; most of the HTS magnetization or multi-physics coupled problems can be solved efficiently by using the above mentioned numerical algorithms.

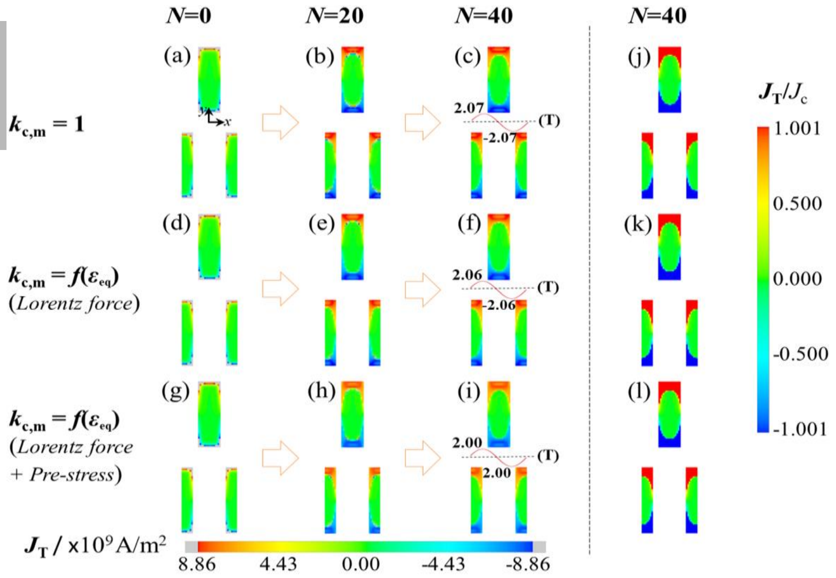
PAUL SCHERRER INSTITUT

The APDL codes for both the RAA and the direct iteration method have been shared on the HTS modelling workgroup (#19, #23); the APDL codes for the backward computation will also be shared on the webpage soon.

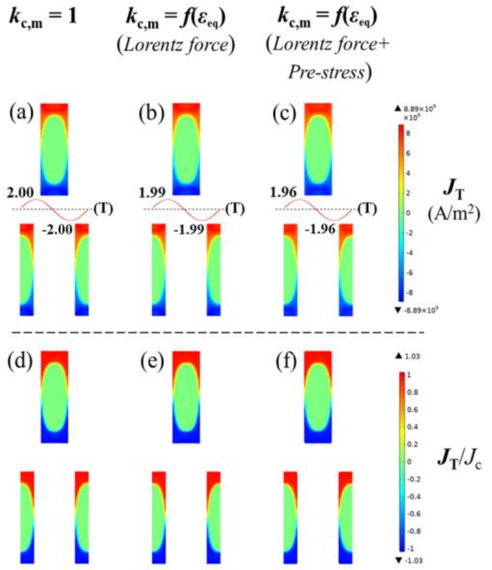
We are delighted to share our APDL codes and stimulate the ANSYS community in HTS modelling !

Contact : kai.zhang@psi.ch

Extra - Backward computation method



Magnetization current J_T in the periodical HTS bulk undulator during the backward iterations



Magnetization current J_T in the periodical HTS bulk undulator solved using COMSOL ***H***-formulation

[Ainslie M et al 2016]

[Trillaud F et al 2018]

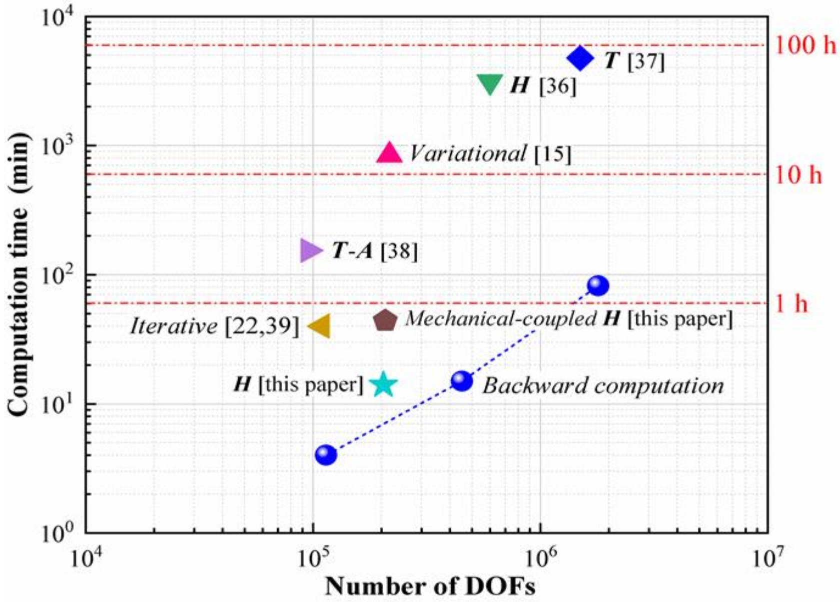
$$J_c(B,\varepsilon_{eq}) = k_{c,m}\left\{J_{c1}\exp\left(-\frac{B}{B_L}\right) + J_{c2}\frac{B}{B_{max}}\exp\left[\frac{1}{y}\left(1-\left(\frac{B}{B_{max}}\right)^{y}\right)\right]\right\}; \quad k_{c,m} = \left(1-\gamma\left(\frac{\varepsilon_{eq}}{\varepsilon_c}\right)^2\right)\times\left[\alpha+\frac{1-\alpha}{1+\exp\left(\left(|\varepsilon_{eq}/\varepsilon_c|-1\right)/\beta\right)}\right]$$

Zhang K et al 2020 SUST 33 114007

Ainslie M et al 2016 SUST 29 074003

Trillaud F et al 2018 IEEE TASC 28 6800805

Extra - Backward computation method



Comparison of computation times reported in the literature for other state-of-the-art techniques for the electromagnetic analysis of HTS materials.

Zhang K et al 2020 Supercond. Sci. Technol. 33 114007

Note: the listed ***H***-, ***T***- and ***T-A*** formulation were implemented for other applications (e.g., AC loss or SCIF) and that benchmarking this particular problem would provide a true comparison.