A volume integral equation based equivalent circuit for 3D calculation of the levitation force

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Outline

- The volume integral equation method
    - ✓ mathematical formulation
    - ✓ finite element model
        - ➢ Search of the independent loops
        - ➢ The distributed parameters equivalent circuit
- 3D modeling of the levitation between PMs and SC bulks
    - ✓ Experimental apparatus
    - ✓ Numerical results and validation – 2D axisymmetric and 3D cases
- Conclusion

Volume integral equation method – (some) essential references

[1] J. van Welij, "Calculation of Eddy currents in terms of H on hexahedra," in IEEE Transactions on Magnetics, vol. 21, no. 6, pp. 2239-2241, Nov 1985.
[2] R. Albanese and G. Rubinacci, “Integral formulation for 3D eddy-current computation using edge elements,” in IEE Proc.-A, Sci. Meas. Technol. , Sep. 1988, vol. 135, pp. 457–462.
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[4] P. Dular, J. Y. Hody, A. Nicolet, A. Genon and W. Legros, "Mixed finite elements associated with a collection of tetrahedra, hexahedra and prisms," in *IEEE Transactions on Magnetics*, vol. 30, no. 5, pp. 2980-2983, Sept. 1994.
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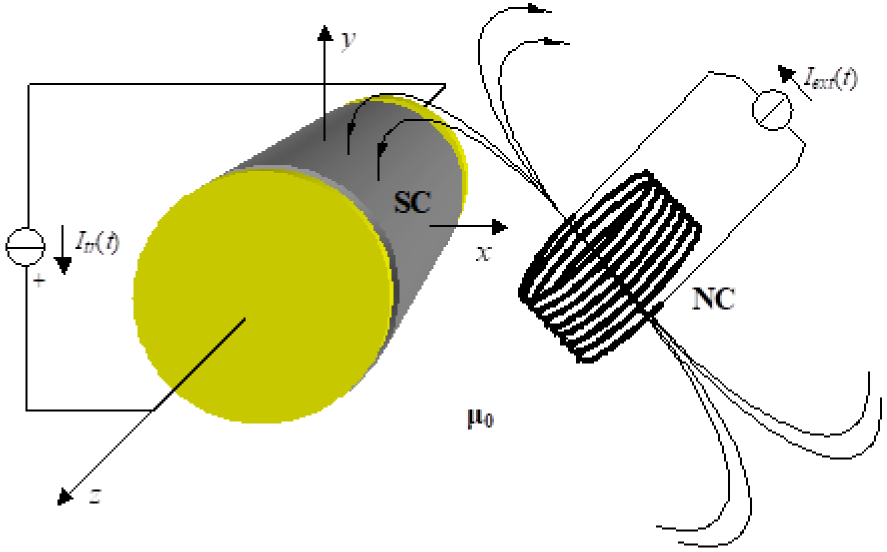
- A. Morandi, “Circuit Methods for three-dimensional field analysis in Large Scale Superconducting Systems,” Ph.D. dissertation, University of Bologna, Italy, 2004.
- A. Morandi, M. Fabbri and P. L. Ribani, "Loops and Meshes Formulations for 3-D Eddy-Current Computation in Topologically Non-Trivial Domains With Volume Integral Equations," in IEEE Transactions on Magnetics, vol. 56, no. 10, pp. 1-14, Oct. 2020, Art no. 7401114, doi: 10.1109/TMAG.2020.3012632.

Volume integral equation method for 3D eddy current computation – mathematical formulation



- Conducting domain τ_c exposed to the electric force produced by external sources.

$$\mathbf{E}^{\mathrm{ext}} = -\frac{\partial \mathbf{A}^{\mathrm{ext}}}{\partial t} + \mathbf{v} \times \mathbf{B}^{\mathrm{ext}}$$

- The domain can be connected to an external circuit by means of two (or more) electrodes.

- Faraday's law + power law model

$$\rho \mathbf{J} = -\frac{\mu_0}{4\pi} \frac{\partial}{\partial t} \int_{\tau_c} \frac{\mathbf{J}(\mathbf{r}')}{\mathbf{r}-\mathbf{r}'} d^3 r' - \mathbf{E}^{\mathrm{ext}} - \nabla \varphi$$

$$\rho = \frac{E_0}{J_c} \left(\frac{J}{J_c} \right)^{N-1} \frac{\mathbf{J}}{J_c}$$

- Boundary conditions

$$\mathbf{J} \cdot \mathbf{n} = 0 \quad \text{on} \quad \partial \tau_c - \cup \Sigma_k$$

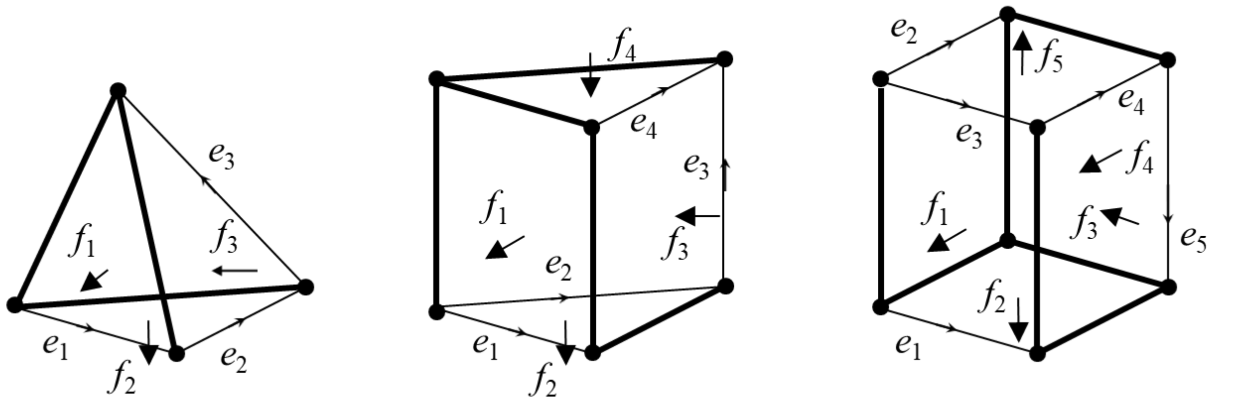
$$\int_{\Sigma_k} \mathbf{J} \cdot \mathbf{n} \, d^2 r = I_k(V_k) \quad \text{on} \quad \Sigma_k$$

where Σ_k is the surface of k-th electrode

Volume integral equation method for 3D eddy current computation – finite element model

The conducting domain is subdivided in a finite number of volume elements

Any set of **three-edges-per-node** elements can be used



$\mathbf{T} = T_1\mathbf{N}_1 + T_2\mathbf{N}_2 + T_3\mathbf{N}_3$
$\mathbf{J} = \nabla\times\mathbf{T} = I_1(\mathbf{S}_1 - \mathbf{S}_4) + I_2(\mathbf{S}_2 - \mathbf{S}_4) +$
$+ I_3(\mathbf{S}_3 + \mathbf{S}_4)$

$\mathbf{T} = T_1\mathbf{N}_1 + T_2\mathbf{N}_2 + T_3\mathbf{N}_3 + T_4\mathbf{N}_4$
$\mathbf{J} = \nabla\times\mathbf{T} = I_1(\mathbf{S}_1 - \mathbf{S}_5) + I_2(\mathbf{S}_2 - \mathbf{S}_5) +$
$+ I_3(\mathbf{S}_3 + \mathbf{S}_5) + I_4(\mathbf{S}_4 + \mathbf{S}_5)$

$\mathbf{T} = T_1\mathbf{N}_1 + T_2\mathbf{N}_2 + T_3\mathbf{N}_3 + T_4\mathbf{N}_4 + T_5\mathbf{N}_5$
$\mathbf{J} = \nabla\times\mathbf{T} = I_1(\mathbf{S}_1 - \mathbf{S}_6) + I_2(\mathbf{S}_2 - \mathbf{S}_6) +$
$+ I_3(\mathbf{S}_3 + \mathbf{S}_6) + I_4(\mathbf{S}_4 + \mathbf{S}_6) + I_5(\mathbf{S}_5 - \mathbf{S}_6)$

The vector potential **T** is (implicitly) used for expressing the div-conforming current density and expanded in terms of edge elements shape functions

$$\mathbf{J} = \nabla\times\mathbf{T} \qquad \mathbf{J}_h = \sum_{i=1}^{e_h - n_h + 1} T_i \nabla\times\mathbf{N}_i$$

Uniqueness is obtained by selecting the unique vector potential with null projection onto any vector field **w** with streamlines not forming closed loops (two-component gauge).

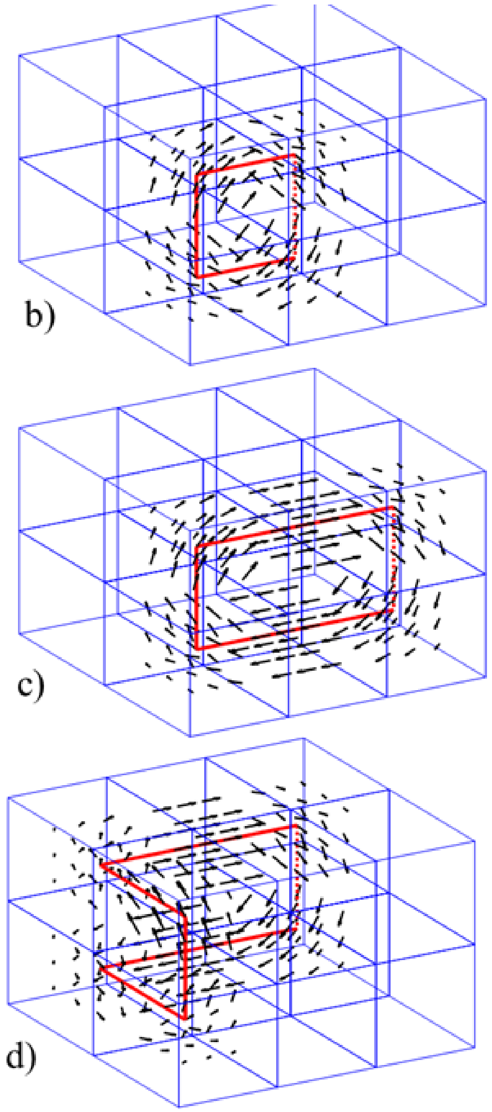
$$\mathbf{T}\cdot\mathbf{w} = 0$$

Weak form – eliminating the scalar potential

A div-conforming loop shape function is associated to a closed chain of elements via facet element shape functions (obtained from edge elements shape functions)

$$\mathbf{J}_k^l = I_k^l\,\mathbf{U}_k^l \rightarrow \begin{cases} \mathbf{U}_k^l(\mathbf{r}) = \sum_{i=1}^{f_h-1} \delta_i\,(\mathbf{S}_i(\mathbf{r}) \pm \mathbf{S}_{f_h}(\mathbf{r})) \\ \nabla \cdot \mathbf{U}_k^l = 0 \quad \text{on} \quad \tau_c \\ \mathbf{U}_k^l \cdot \mathbf{n} = 0 \quad \text{on} \quad \partial\tau_c \end{cases}$$

$$\left(\mathbf{S}_i(\mathbf{r}) \pm \mathbf{S}_j(\mathbf{r}) = \nabla \times \mathbf{N}_k\right)$$



Solenoidality of loop shape function is the key for elimination of the scalar electric potential form the weak solution

$$\int_{\tau_c} \mathbf{U}_k^l \cdot \left(\rho\,\mathbf{J} + \frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_{\tau_c}\frac{\mathbf{J}(\mathbf{r}')}{\mathbf{r}-\mathbf{r}'}\,d^3r' + \frac{\partial \mathbf{A}^{ext}}{\partial t} + \nabla\varphi \right) dV = 0$$

$$k = 1, \ \dots \ , N_{LOOPS}$$

$$\int_{\tau_c} \mathbf{U}_i \cdot \nabla\varphi\, dV = \oint_{\partial\tau_c} \varphi\mathbf{U}_i \cdot \mathbf{n}\, dS - \int_{\tau_c} \varphi\nabla\cdot\mathbf{U}_i\, dV = 0$$

$$\mathbf{M}^l \frac{d}{dt}\mathbf{I}^l = -\mathbf{R}^l\,\mathbf{I}^l + \mathbf{V}^{l\,ext}$$

with

$$M_{ij}^l = \int_{\tau_i}\int_{\tau_j} \frac{\mathbf{U}_i^l(\mathbf{r})\cdot\mathbf{U}_j^l(\mathbf{r}')}{\mathbf{r}-\mathbf{r}'}\, d^3r'\, d^3r$$

$$R_{ij}^l = \int_{\tau_i} \rho\,\mathbf{U}_i^l(\mathbf{r})\cdot\mathbf{U}_j^l(\mathbf{r})\, d^3r$$

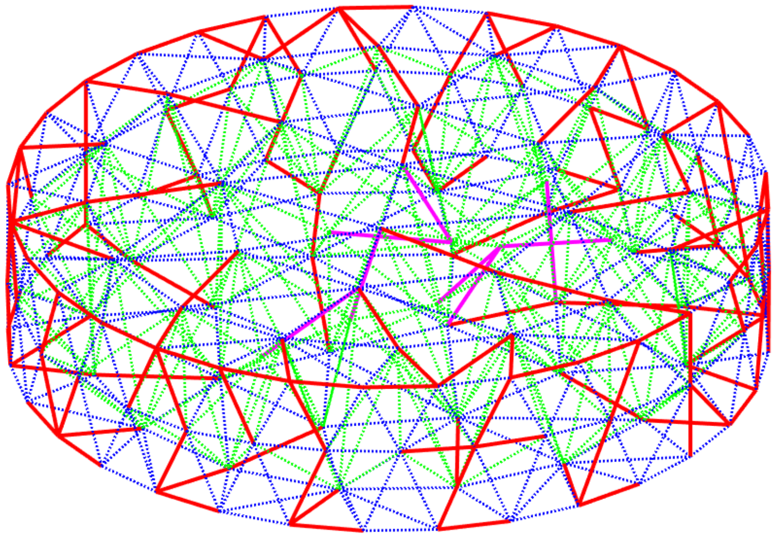
$$V_i^{l\,ext} = \int_{\tau_i} \mathbf{U}_i^l(\mathbf{r})\cdot\frac{\partial\mathbf{A}^{ext}(\mathbf{r})}{\partial t}\, d^3r$$

How to select the independent loops?

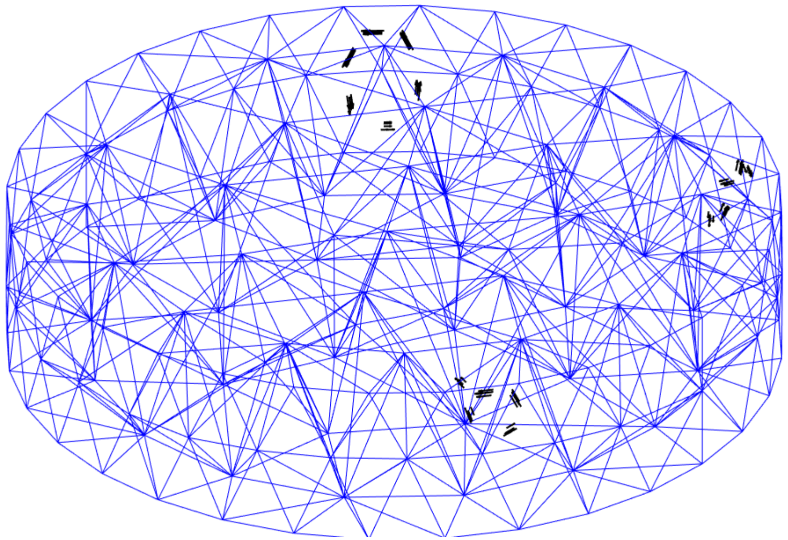
- In principle, a set of *R*-(*N*-1) independent loops can be arbitrarily selected by using the **dual graph**
  - Easy to implement, also in case of topologically nontrivial domain
  - Non-minimal long-range loops are selected (severe CPU storage requirement)

- More commonly, a set of ***R*-(*N*-1) independent cordless loops** is obtained by selecting the co-tree edges of the primal graph not lying on the boundary
  - **Requires special treatment of multiply connected domains**

Tree-cotree decomposition of the primal graph



Inner cotree edges (in green) generating a minimal loop

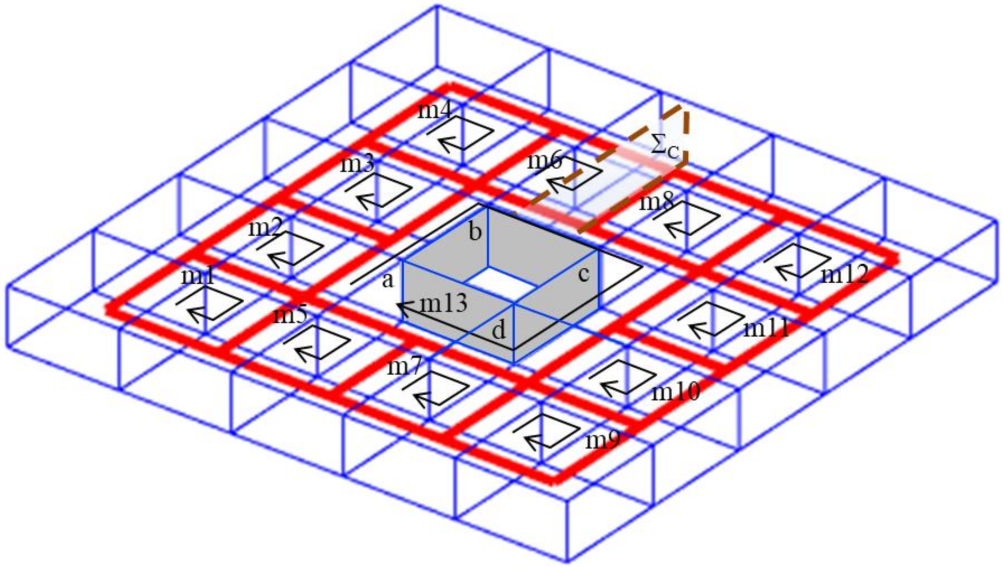


loop shape functions

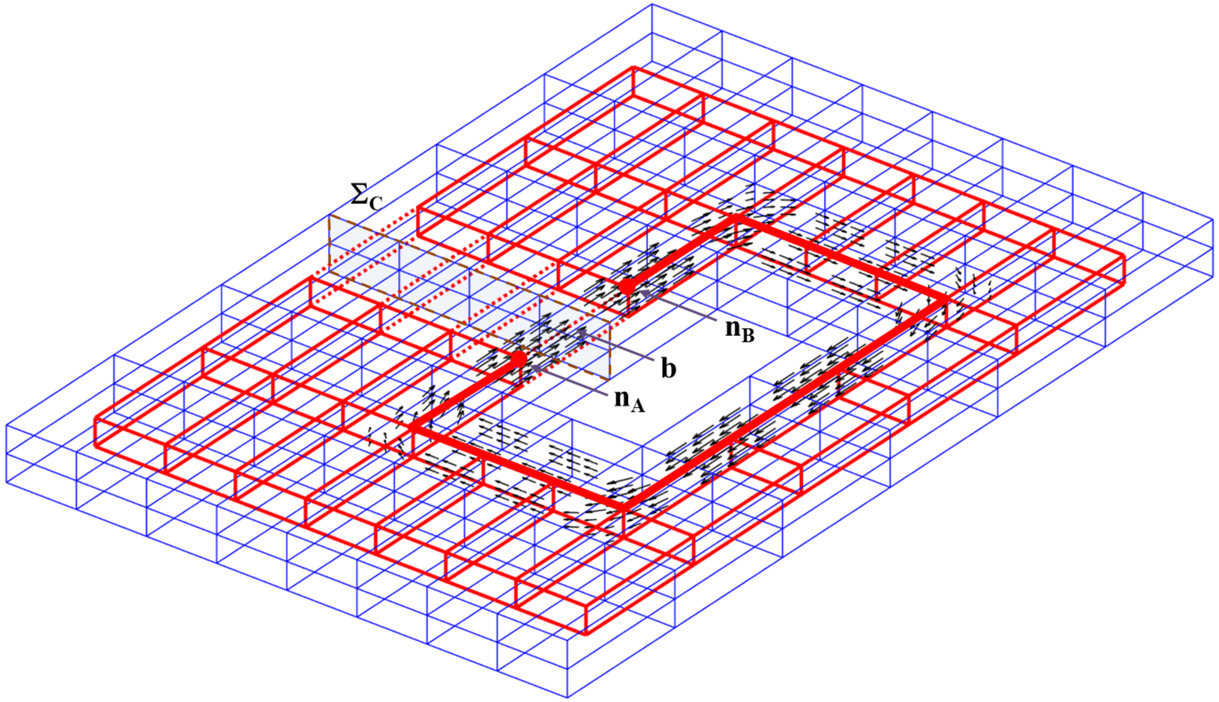
Selecting independent loops in topological non-trivial domains

- Loops associated to the co-tree branches lying in the interior of the domain produces zero current through any cutting surface that makes the domain simply connected.

- to allow a net current circulating in the domain additional loops must be added. This additional meshes and can be built by selecting one closed loop linked with the cutting surface of the domain

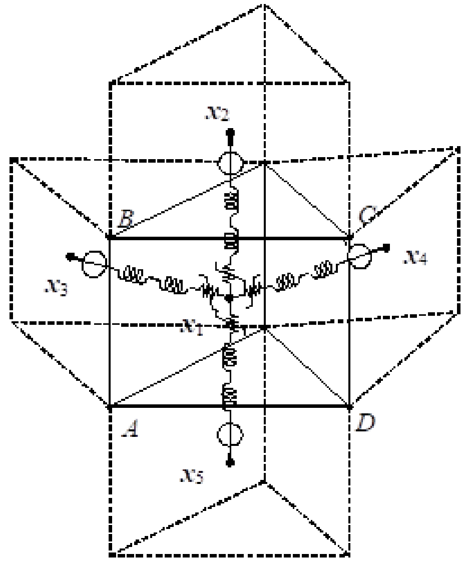


Examples of meshes for a multiply connected slab domain



Identification of the additional mesh by using the dual graph.

The distributed parameters equivalent circuit

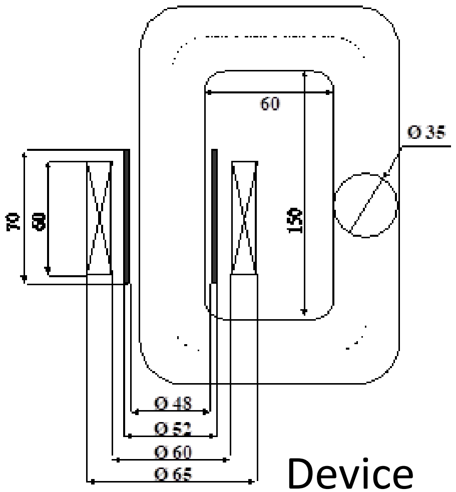


The discretized FEM problem corresponds to a distributed parameters equivalent circuit

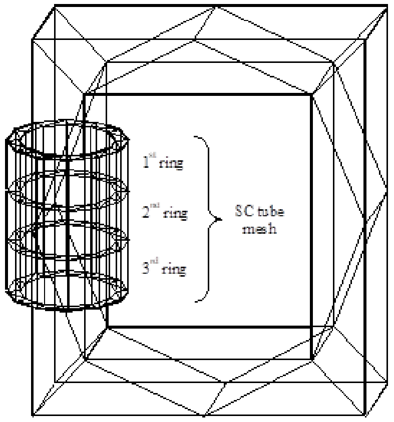
The equivalent circuit naturally includes possible coupling with external circuit hosting the device

$$\mathbf{M}^{l}\frac{d}{dt}\mathbf{I}^{l} = -\mathbf{R}^{l}\mathbf{I}^{l} + \mathbf{V}^{l\,\mathrm{ext}} + \mathbf{c}V_{s}$$

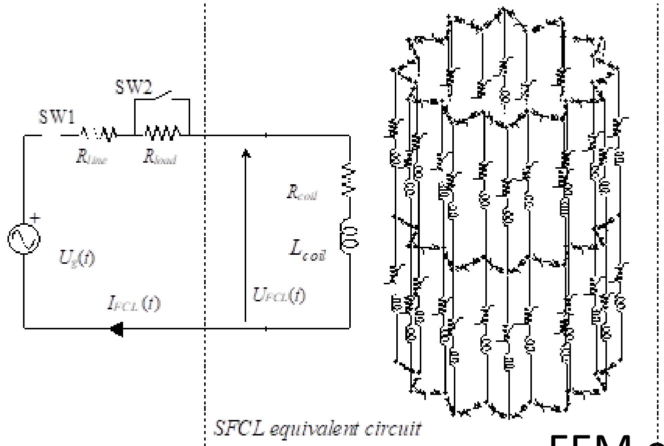
An example: FEM based equivalent circuit of shielded type SFCL



Device



Geometric model



FEM equivalent circuit coupled with power grid

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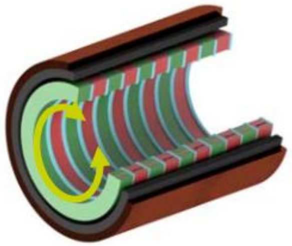
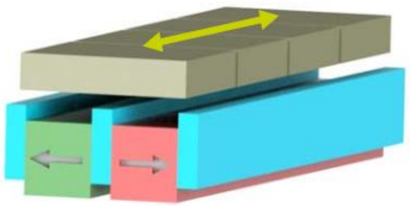
Superconducting levitation - flywheels and maglev

Inherently stable levitation is obtained between permanent magnets and HTS bulks allowing obtaining passive (fail-safe) axial and linear bearings

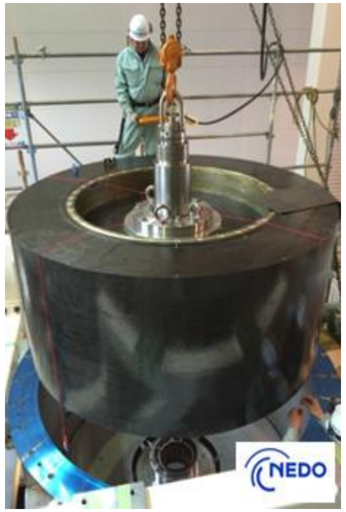


axial bearing



linear bearing

Superconducting flywheel



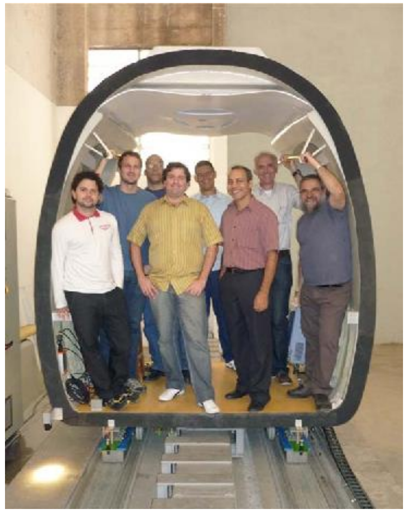
NEDO FlyWheel, 2015

- 100 kWh energy
- 300 KW power
- 6000 rpm speed
- 4 tons rotating mass

GdBCO bulks + DP REBCO coils

NEDO

Superconducting MAGLEV



YBCO bulks

Maglev-Cobra, UFRJ 2014

- 1.5-m-long wagons
- 200 meters test line

The levitation facility

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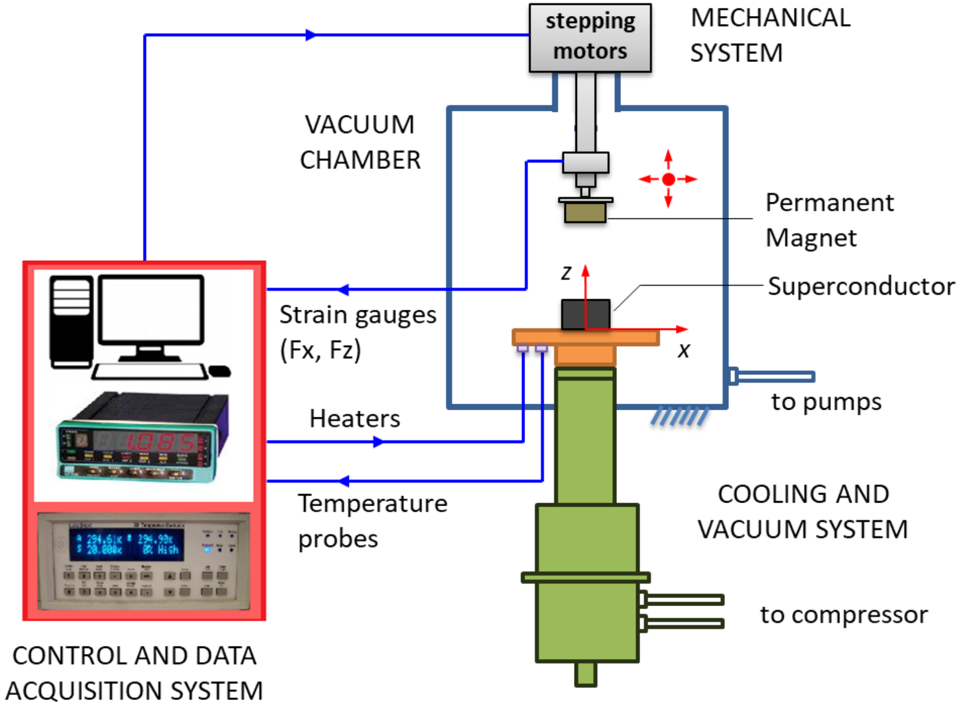
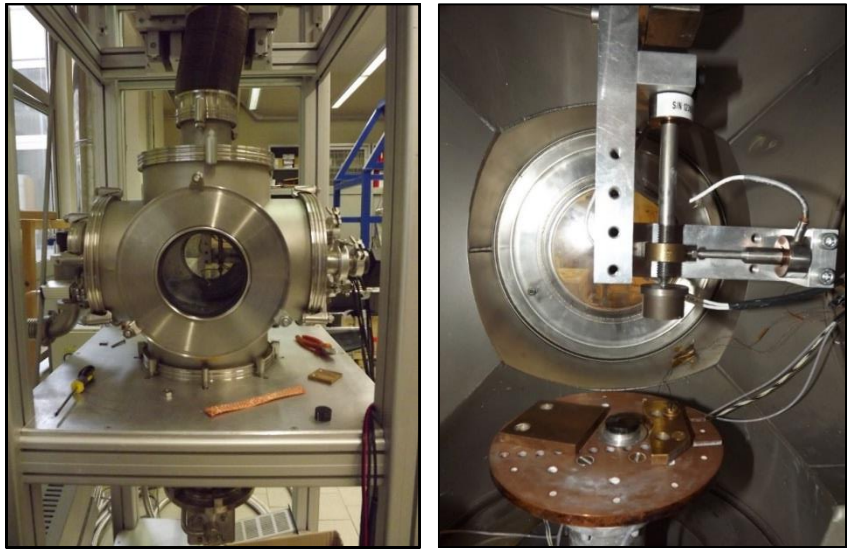
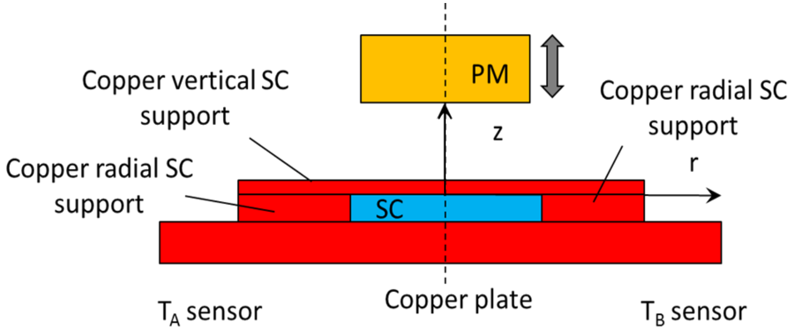




Bulk pressed with a copper plate for good thermal contact



→ limit on the minimum distance (zmin) between PM and bulk

Operating temperature range	**20 K – 90 K**
Maximum excursion of the PM (x,z)	**55.9 mm**
Minimum step (x,z)	**0.1 mm**
Maximum velocity	**60 mm/s**
Maximum measurable force (x,z)	**500 N**
Max size of sample	**200 mm**

MgB_2 Bulks and test procedure

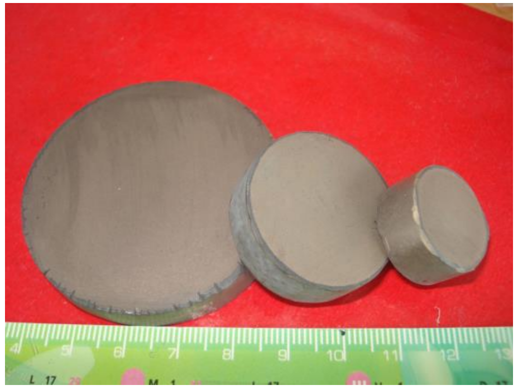
Four cylindrical MgB_2 bulks produced by SPS at CRISMAT-CAEN

D = 70.33 mm - H = 9.38 mm
Bulk 1

D = 49.66 mm - H = 10.78 mm
Bulk 2

D = 39.52 mm - H = 13.00 mm
Bulk 3

D = 29.62 mm - H = 5.18 mm
Bulk 4



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CRISMAT, ENSICAEN
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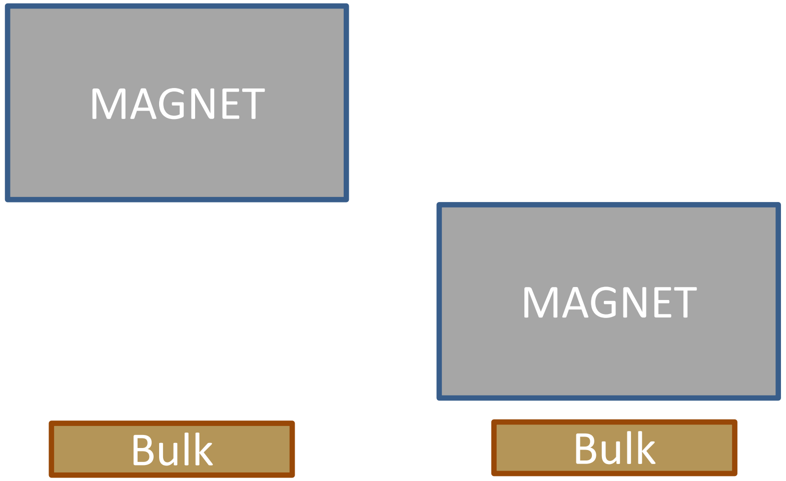
Caen, France

Zero Field Cooling | **Field Cooling (FC)**

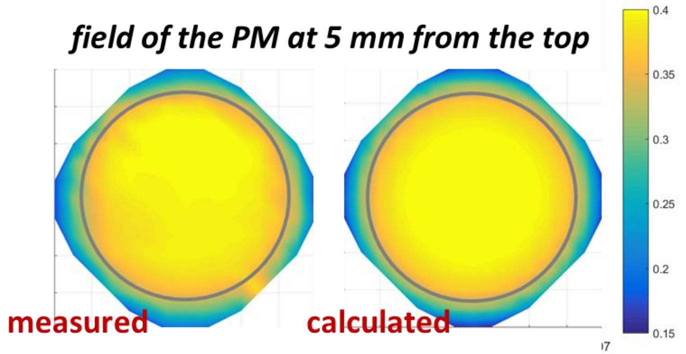


	z_{min}	z_{max}
Bulk 1	**3.8**	**43.8**
Bulk 2	**5.0**	**39.4**
Bulk 3	**5.0**	**37.6**
Bulk 4	**5.0**	**42.2**

values in mm

four back and fort runs

Accurate PM model is crucial for the accuracy of the numerical results



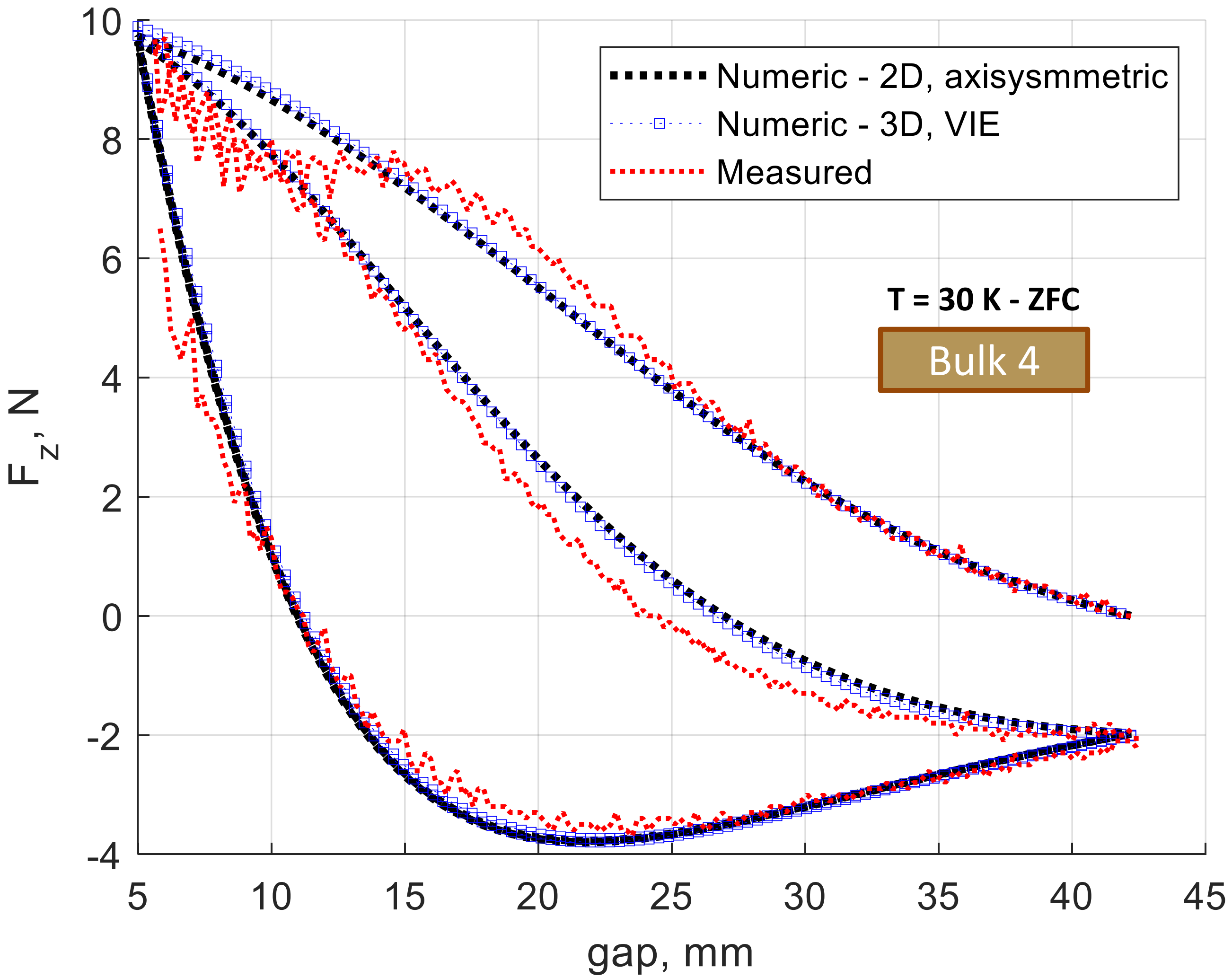
Cylindrical NdFeB Permanet Magnet

D = 70 mm
H = 40 mm
M = 1.02 MA/m

Numerical results

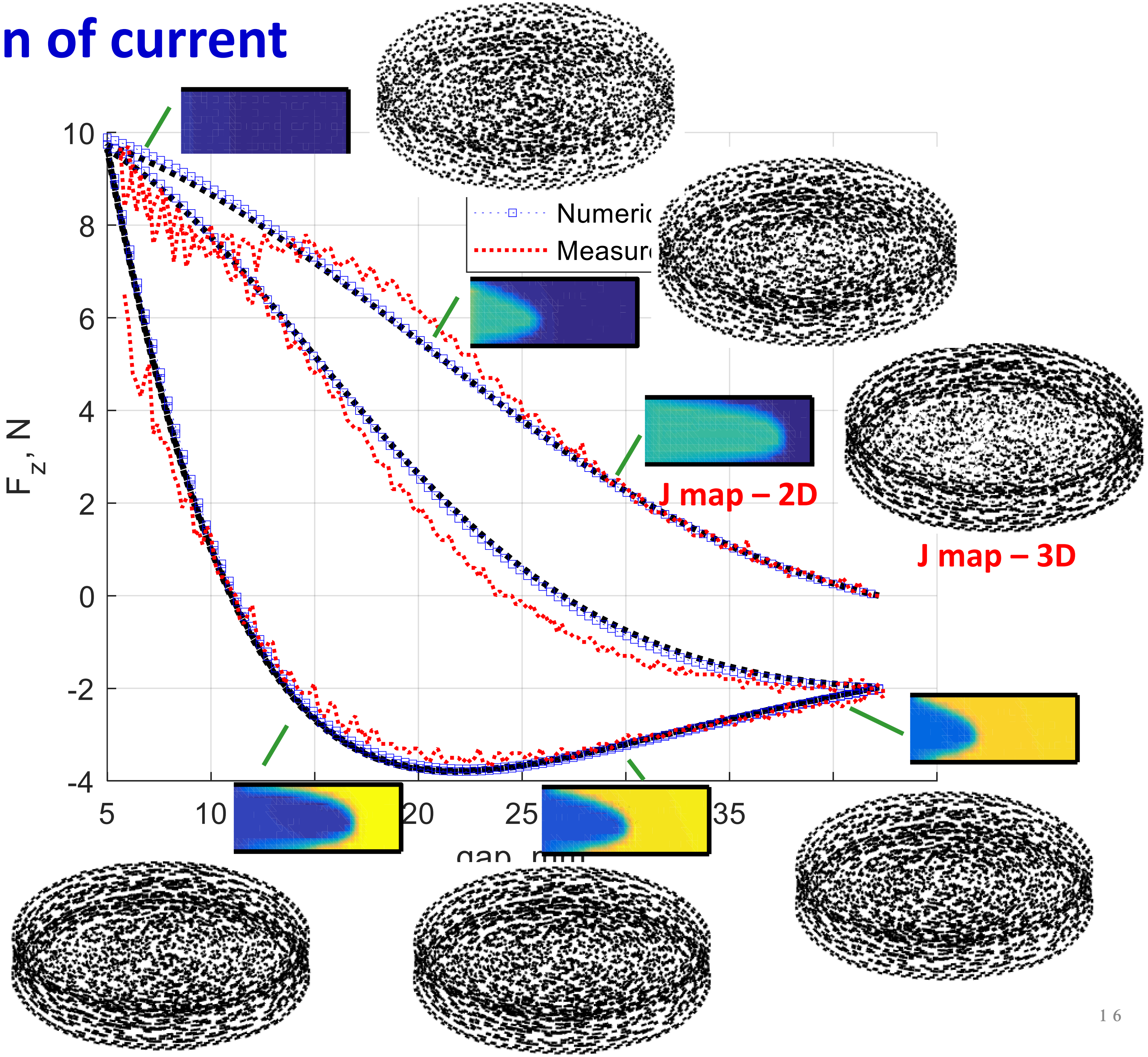
Best fitting of the experimental data obtained with

Jc = 4.2·10^7 A/m^2

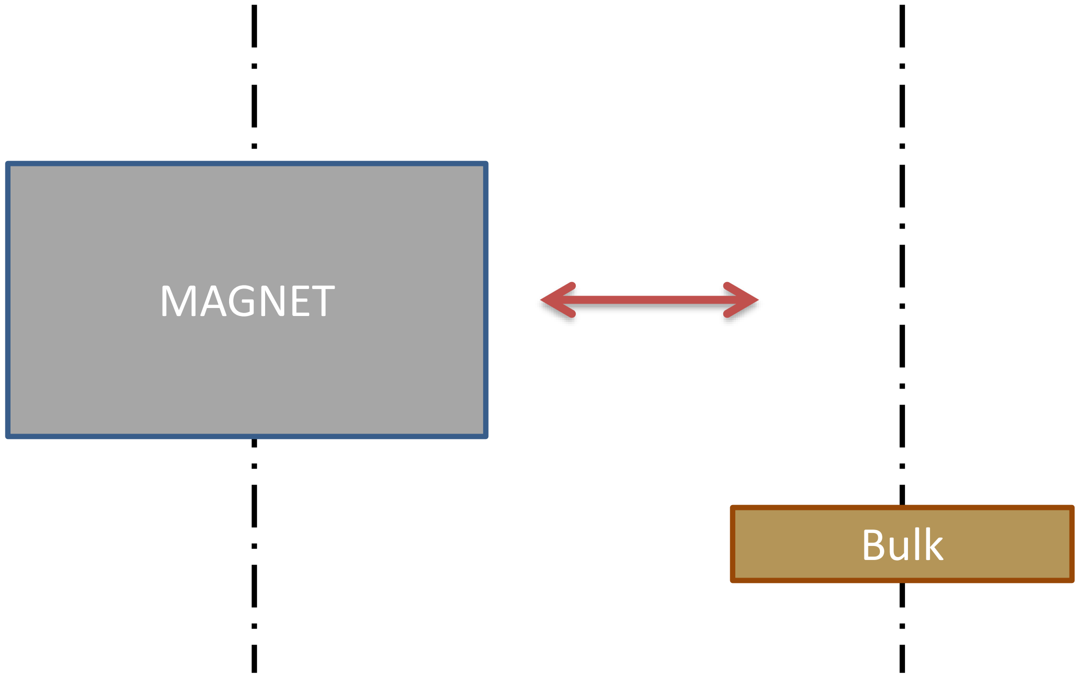


- **An excellent matching exist between “D axisymmetric and 3 D results**
- **A good agreement exists between numerical and experimental data**

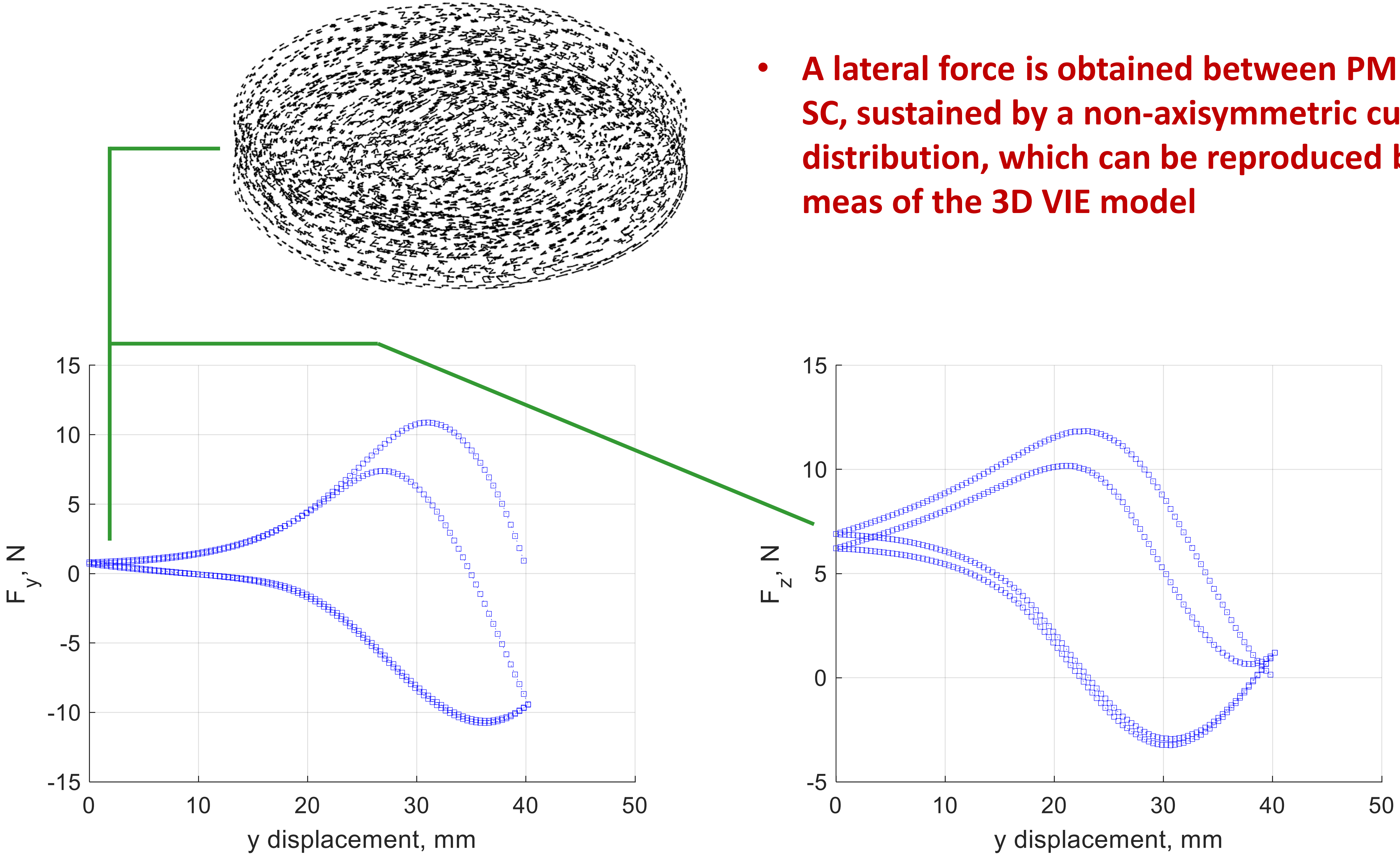
Diffusion of current



A true 3D case - lateral movement of the PM



- **A lateral force is obtained between PM and SC, sustained by a non-axisymmetric current distribution, which can be reproduced by meas of the 3D VIE model**



Conclusion and future work

- The distributed parameters equivalent circuit model for 3D eddy current computation in superconductor was developed based on volume integral equations
- The model was successfully applied to investigating 3D levitation problems
- Any on problem, including grid connected devices, can be investigated by means of the model

- Future work:
  - ✓ Measurement of lateral levitation force for model validation in 3D opeartion
  - ✓ Include anisotropic current density in the model