A new E-field finite element formulation for the numerical modelling of high temperature superconductors

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Features:

- **Electric field is the main unknown**, which allows the **direct calculation** of the induced **eddy currents** and other magnitudes of interest (e.g. **Joule heating**) **without resorting to derivatives.**
- **It does not require the addition of extra unknowns** (e.g. Lagrange multipliers or scalar potentials) **to be stable**.
- It can account for **capacitive and inductive effects simultaneously**, even at low frequencies.
- Same formulation can be used at **low and high frequencies.**
- It uses **1st order nodal elements** enriched with inner and surface bubbles (**easier interpolation in multiphysics problems)**.
- **Iterative solver friendly** (resultant matrix is well conditioned and it can be solved easily with lightly preconditioned iterative solvers).

Formulation:

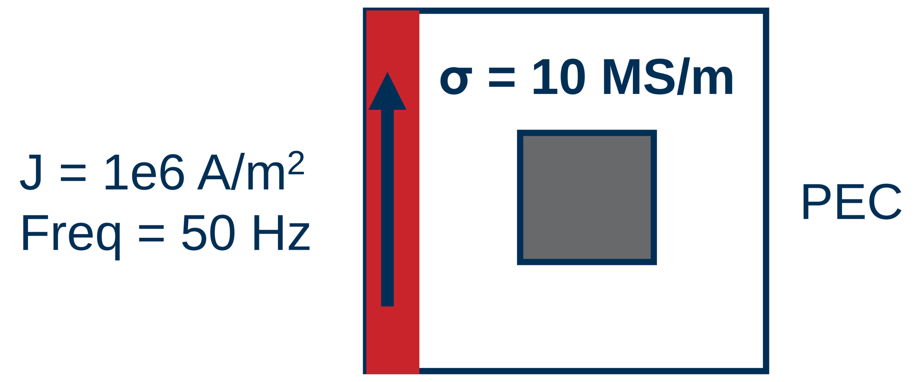
Find $\mathbf{E}_h \in U_h$ such that $\forall \mathbf{V}_h \in U_h$ it is satisfied:

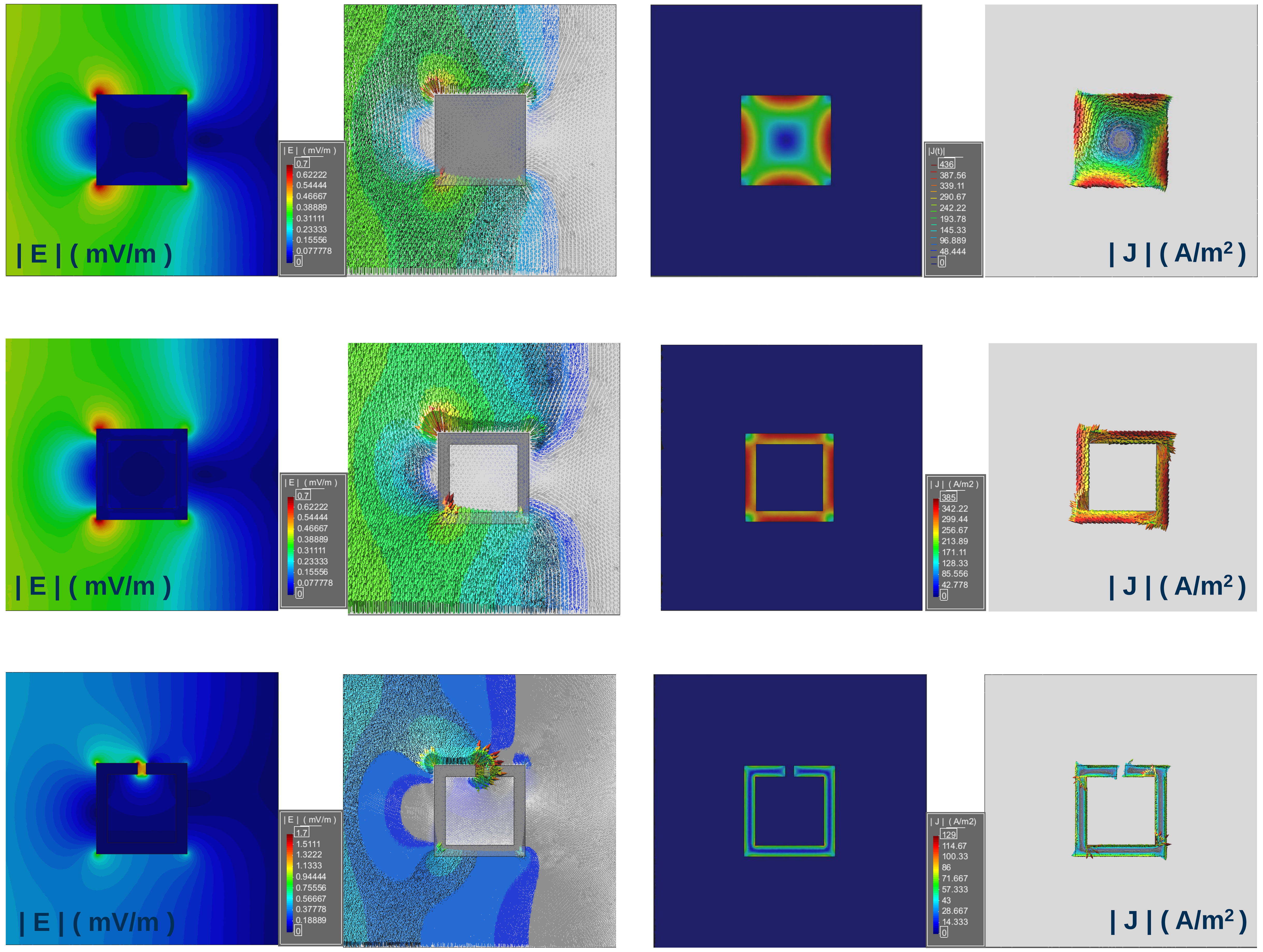
$$(L(\nabla \times \mathbf{E}_h), \mu^{-1} L(\nabla \times \mathbf{V}_h)) + (\hat{L}(\nabla \cdot \varepsilon \mathbf{E}_h), (\mu\varepsilon^2)^{-1} \hat{L}(\nabla \cdot \varepsilon \mathbf{V}_h)) - \omega^2 (\varepsilon \mathbf{E}_h, \mathbf{V}_h)_\Omega + \mathbf{R.B.C.} + h^2 \mathbf{S}_h = j\omega (\mathbf{J}, \mathbf{V}_h)_\Omega$$

where:
U_h is the functional space composed of 1st order nodal elements P_1 enriched with elemental and surface bubbles [1]
R.B.C. are the boundary conditions as given in [2]
$\mathbf{S_h}$ is the stabilization factor: $(\nabla \times \mathbf{E}_h, \mu^{-1} \nabla \times \mathbf{V}_h) + (\nabla \cdot \varepsilon \mathbf{E}_h, (\mu\varepsilon^2)^{-1} \nabla \cdot \varepsilon \mathbf{V}_h)$
$L(\cdot)$ and $\hat{L}(\cdot)$ are the L2 projections defined as:

$$(L(\nabla \times \mathbf{u}), \mathbf{q}) = (\nabla \times \mathbf{u}, \mathbf{q}) \quad \forall \mathbf{q} \in (P_1)^3$$
$$(\hat{L}(\nabla \cdot \varepsilon \mathbf{u}), q) = (\varepsilon \nabla \mathbf{u}, q) \quad \forall q \in P_1.$$

Results:





References:

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