Modelling Pulsed Field Magnetization of Ring-Shaped High-Temperature Superconductors

M. Beck[1], V. Cientanni[1], M.D. Ainslie[1]

[1]Bulk Superconductivity Group, Department of Engineering, University of Cambridge

UNIVERSITY OF CAMBRIDGE

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1. Background

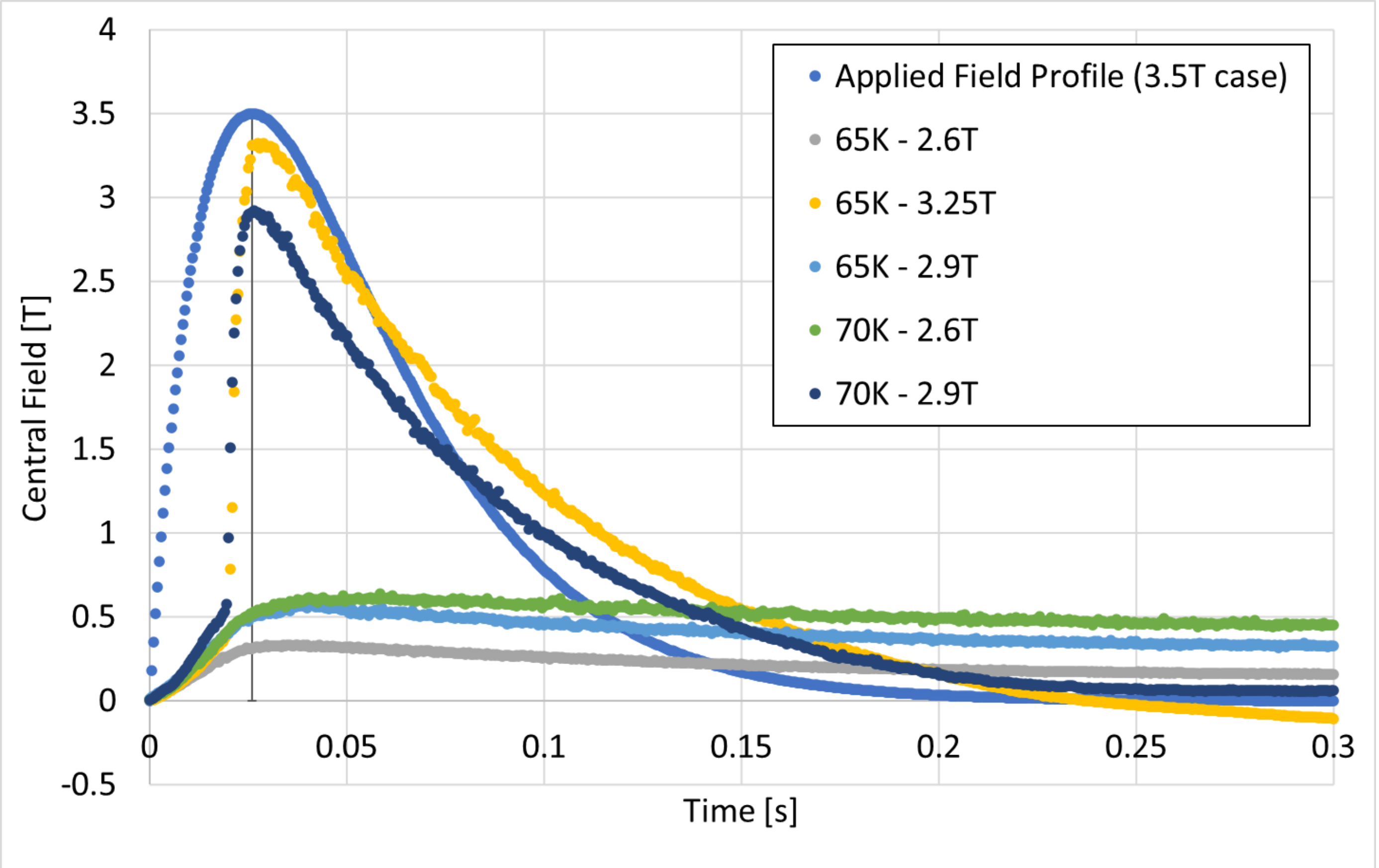
To avoid thermal oscillations in coupled finite element models (FEM), the maximum allowable element size (e_{max}) is a function of the size of the timestep taken (Δt), thermal conductivity (κ), mass density (ρ) and specific heat (C) [1]

$$e_{max} = \sqrt{\frac{6\kappa\Delta t}{\rho C}}$$

For high temperature superconductors (HTS) during pulsed-field magnetization (PFM) it is common for spatial thermal oscillations to occur due to low conductivity and small timesteps to solve within the required error tolerances.

In finite ring-shaped samples, flux propagates from both the inner and outer edges [2] which induces a numerical singularity at the point where these collide, after which the flux density within the bore of the ring suddenly increases. FEM solvers necessarily reduce the timestep taken to compensate for the singularity, breaching the stability condition unless very fine meshes are used. The required mesh sizes for stability often render the problem computationally intractable.

Here we investigate options to accelerate the modelling of PFM in ring-shaped HTS using COMSOL Multiphysics whilst avoiding thermal oscillations within the solution.



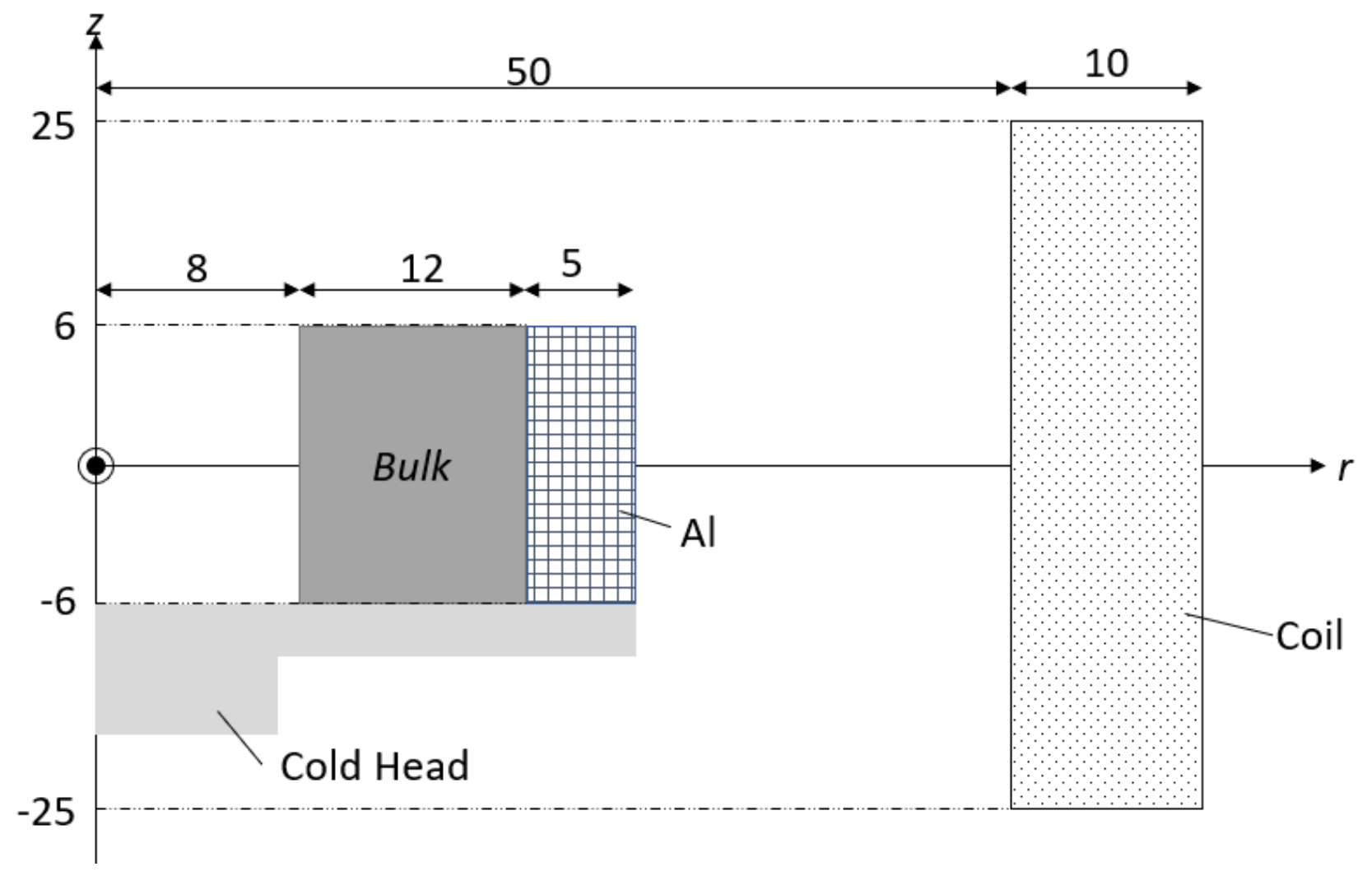
Measured central field dynamics during PFM of a ring-shaped GdBCO sample.

2. Model Set-up

A fully-coupled 2D axisymmetric model was built in COMSOL 5.5 using the H-formulation to model the superconducting (SC) domain. Non-SC domains were modelled using either the H- or A-formulations. The mesh density within the SC domain is adjusted, as is the discretization order for the magnetic and thermal physics.

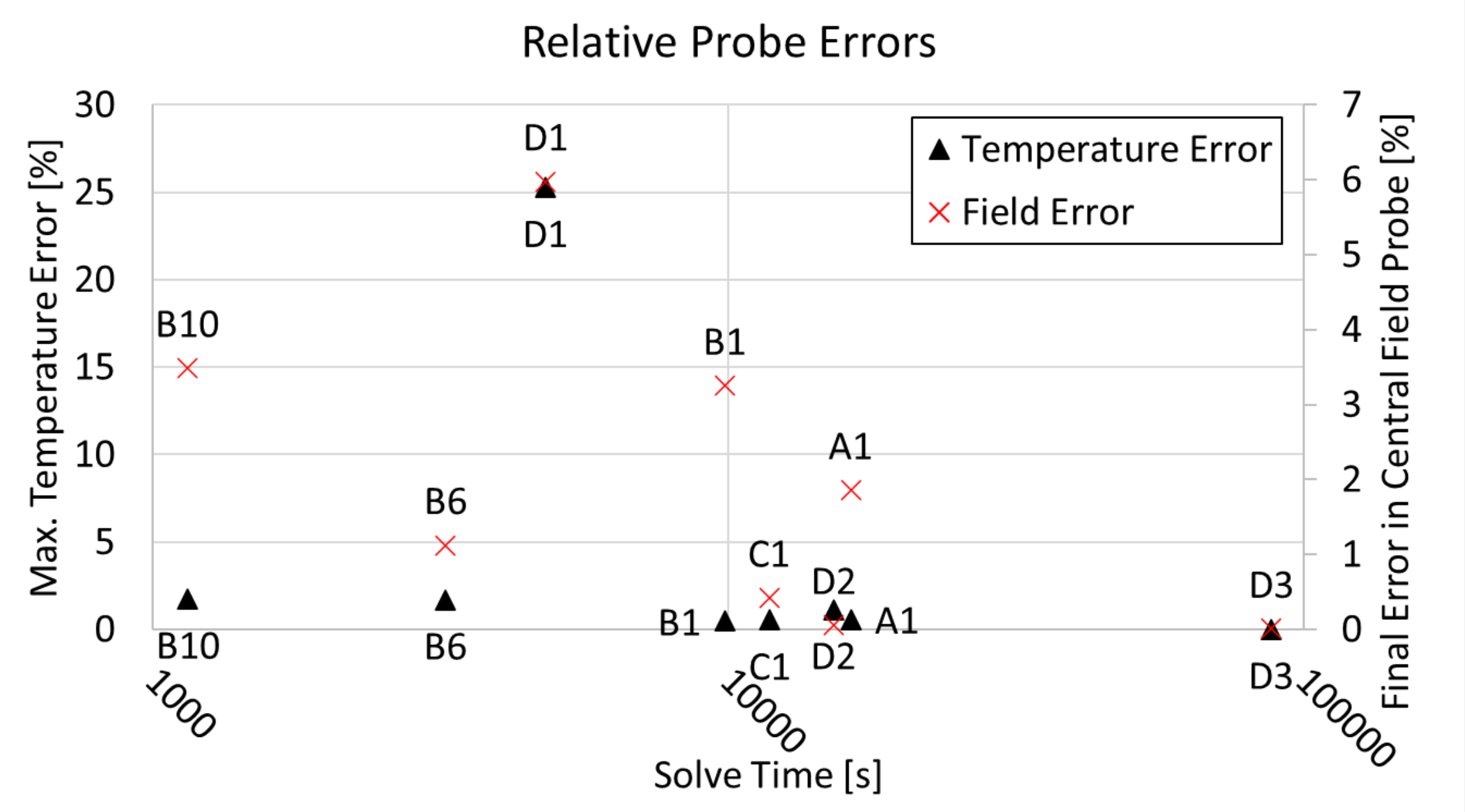
A measured J_c(T,B) and specific heat capacity are used, whilst the thermal properties are assumed $\kappa_{ab} = 20, \kappa_c = 4$ W/m.K. A finite coil is used to apply the magnetising pulse of 4.5T with a 13ms rise time. Cooling is applied via an infinite sink at constant temperature from the bottom surface, with all other faces perfectly thermally isolated from the surrounding domains.

Results are compared to those for the linearly discretized case with a mesh size of 62.5/50µm, which is assumed closest to the true solution.



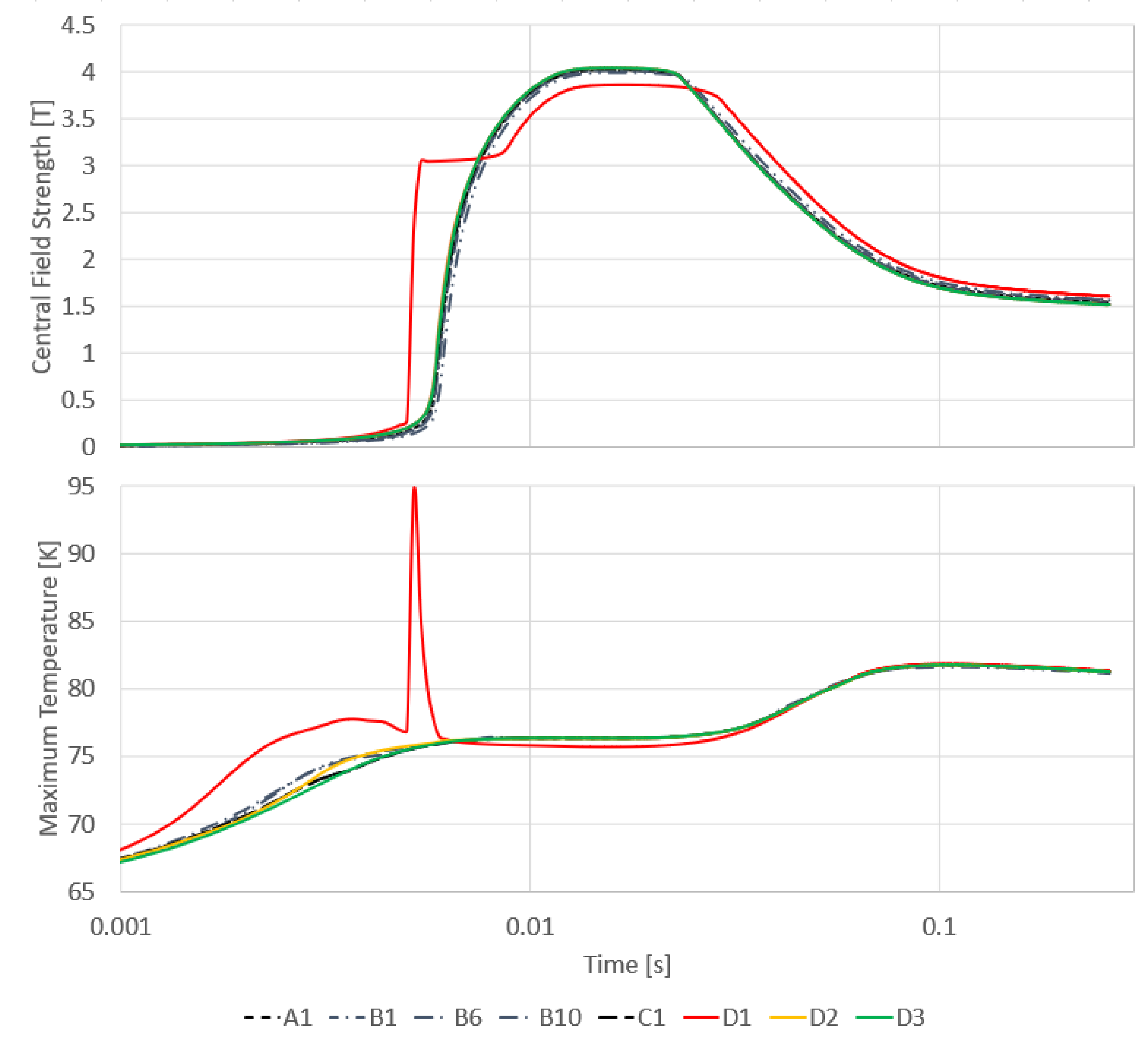
2D axisymmetric model configuration

3. Results and Discussion



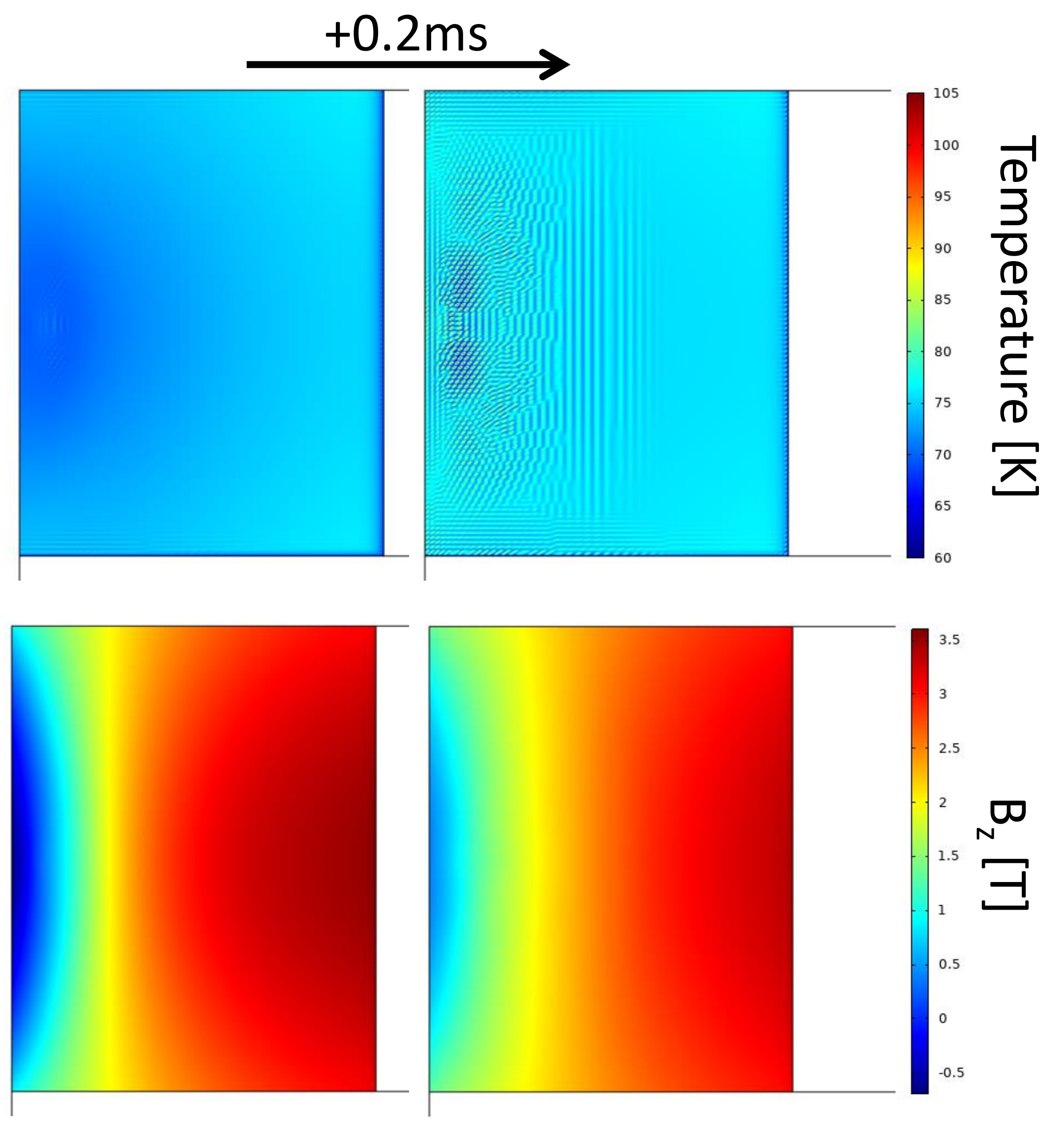
For the tested linear meshes with elements greater than 100µm, small spatial thermal oscillations (~1K) were seen as early in the simulation, which grew to 30K as the z-component of the magnetic field at the inner edge becomes positive. Small oscillations (~1K) were also seen for meshes of 50µm, but induce field and temperature errors of 0.06 and 1.1% respectively.

Quadratic heat elements worsened the instability, with peak oscillations of 85K observed. Quadratic magnetic elements, with linear heat elements, allows the mesh density to be relaxed by up to a factor of 8 without thermal oscillation. Case C1 allows a 7.5x improvement in run-time for < 0.5% error in the solution. With the exception of case D1, all configurations qualitatively reproduced the measured propagation profile, which may allow for rapid initial studies.



Simulated field and temperature dynamics during PFM of a ring-shaped GdBCO sample

Model	Discretization		Element Size (mm)	
	H	A	r-	z-
A1	Quad	-	0.25	0.2
B1	Quad	Lin	0.25	0.2
B6	Quad	Lin	0.5	0.4
B10	Quad	Lin	1	0.8
C1	Quad	Quad	0.25	0.2
D1	Lin	Lin	0.25	0.2
D2	Lin	Lin	0.125	0.1
D3	Lin	Lin	0.063	0.05



Temperature and Magnetic field profiles around field reversal at the inner edge

References

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