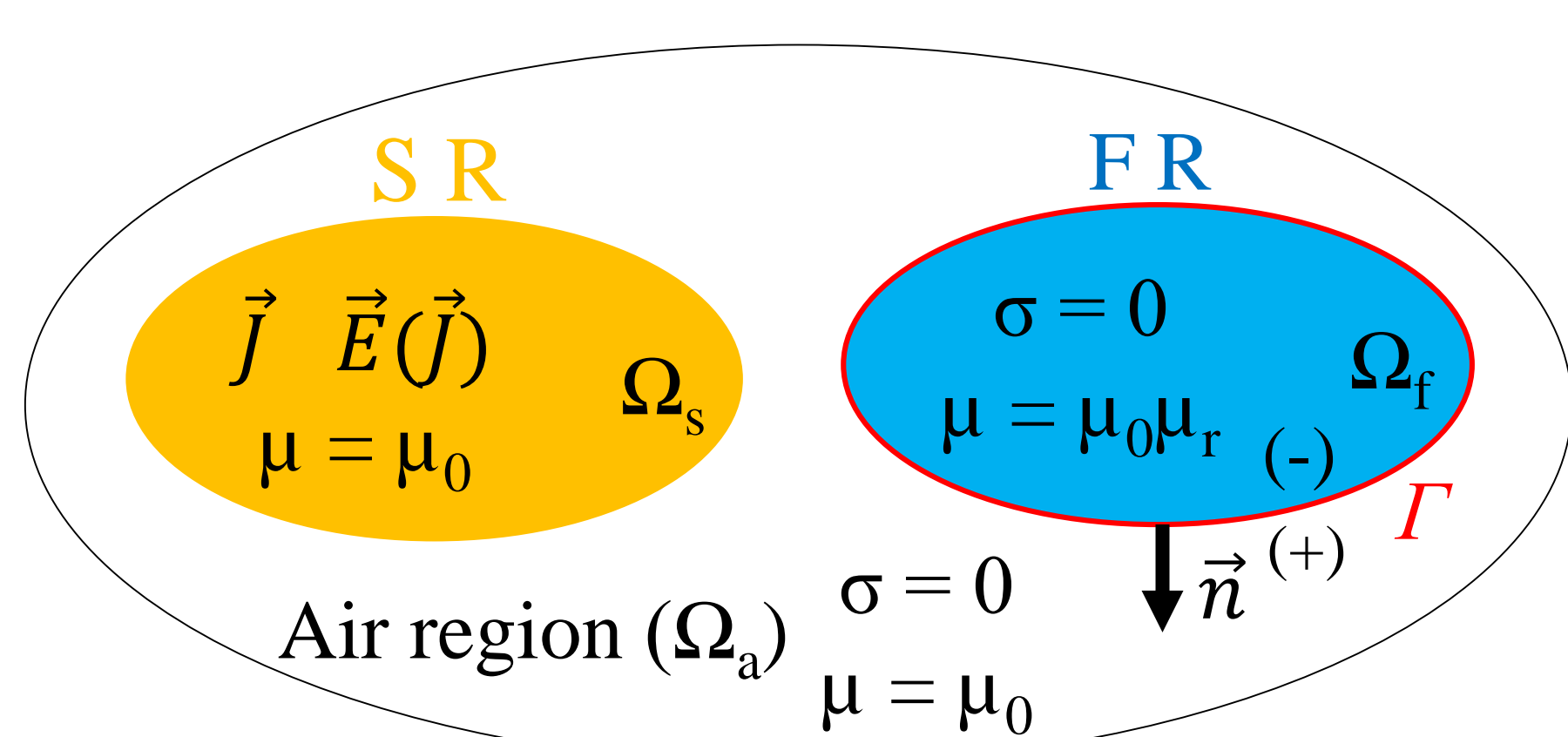


Abstract

This work presents a fast integral modeling approach for computing AC loss in high temperature superconducting (HTS) tapes and stacks. The A-V formulation is combined with the equivalent current model to take into account of the proximity of ferromagnetic materials with finite relative permeability [1]. The obtained results are compared with finite element analysis, both on local and global quantities.

The modeling approach

The modeled system



Superconducting region (S R)

$$E(J, B) = E_c |J_c^{-1}(B) J|^n$$

$$J_c(B) = \frac{J_{c0}}{\left(1 + B_0^{-1} \sqrt{\alpha^2 |B_x|^2 + |B_y|^2}\right)^\beta}$$

Ferromagnetic region (F R)

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{M} = \frac{(\mu_r - 1)}{\mu_0 \mu_r} \vec{B}$$

Equivalent surface current density : \vec{k}

$$\vec{k}(\vec{r}) = \vec{M} \times \vec{n} = \frac{(\mu_r - 1)}{\mu_0 \mu_r} \vec{B}(\vec{r} \in \Gamma^{(-)}) \times \vec{n} \quad \text{With: } \vec{B}(\vec{r}) = \vec{B}_j(\vec{r}) + \vec{B}_{\vec{k}}(\vec{r})$$

$$\text{When } \vec{r} \in \Gamma, \text{ we have: } \vec{B}(\vec{r} \in \Gamma^{(\pm)}) \times \vec{n} = \mp \frac{\mu_0}{2} \vec{k}(\vec{r}) + \left[\vec{B}_{\vec{k}(\vec{r}' \neq \vec{r})}(\vec{r}) + \vec{B}_j(\vec{r}) \right] \times \vec{n}$$

$$\text{The jump of } \vec{B} \text{ across } \Gamma \text{ is: } \left[\vec{B}(\vec{r} \in \Gamma^{(+)}) - \vec{B}(\vec{r} \in \Gamma^{(-)}) \right] \times \vec{n} = -\mu_0 \vec{k}(\vec{r})$$

$$\text{The final relation between } \vec{k} \text{ \& } \vec{j} \text{ is: } \vec{k}(\vec{r}) = \frac{2(\mu_r - 1)}{\mu_0 (\mu_r + 1)} \left[\vec{B}_{\vec{k}(\vec{r}' \neq \vec{r})}(\vec{r}) - \vec{B}_j(\vec{r}) \right] \times \vec{n}$$

$$\text{The matrix form of } \vec{k} \text{ is then: } \left\{ \vec{I} + \overline{\overline{M_{FF}}} \right\} \vec{k} = \overline{\overline{M_{FS}}} \vec{j} \Rightarrow \vec{k} = \left\{ \vec{I} + \overline{\overline{M_{FF}}} \right\}^{-1} \overline{\overline{M_{FS}}} \vec{j}$$

Formulation

$$\text{From the Maxwell's equations, we have: } \partial_t \vec{A} + \vec{E} + \nabla V = \vec{0}$$

$$\text{With: } \vec{A} = \vec{A}_j + \vec{A}_k = \overline{\overline{L_{SS}}} \vec{j} + \overline{\overline{L_{SF}}} \left(\vec{I} + \overline{\overline{M_{FF}}} \right)^{-1} \overline{\overline{M_{FS}}} \vec{j}$$

$$\text{We have to solve: } \left[\overline{\overline{L_{SF}}} \left(\vec{I} + \overline{\overline{M_{FF}}} \right)^{-1} \overline{\overline{M_{FS}}} + \overline{\overline{L_{SS}}} \right] \partial_t \vec{j} + \vec{E} + \nabla V = \vec{0}$$

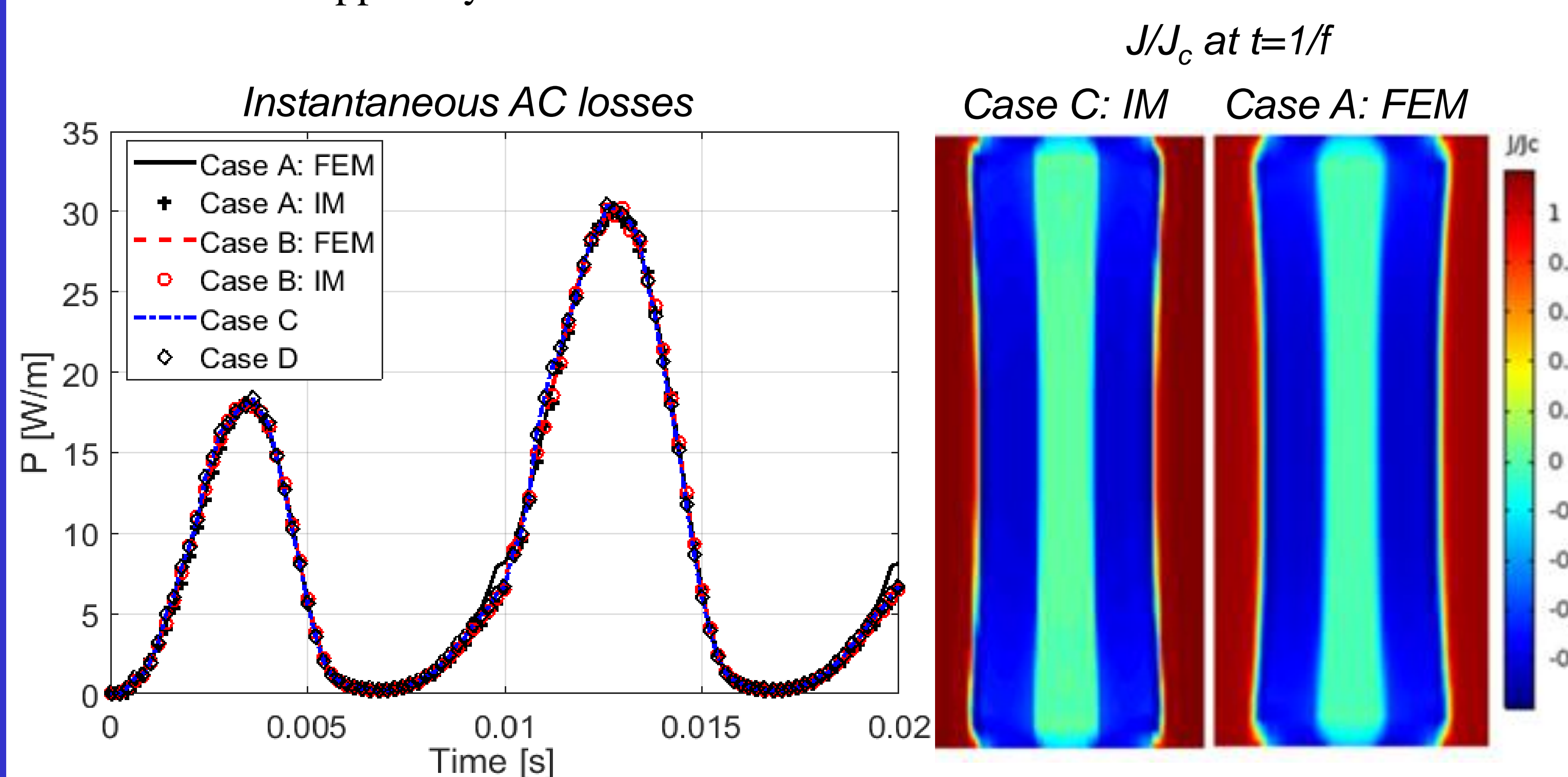
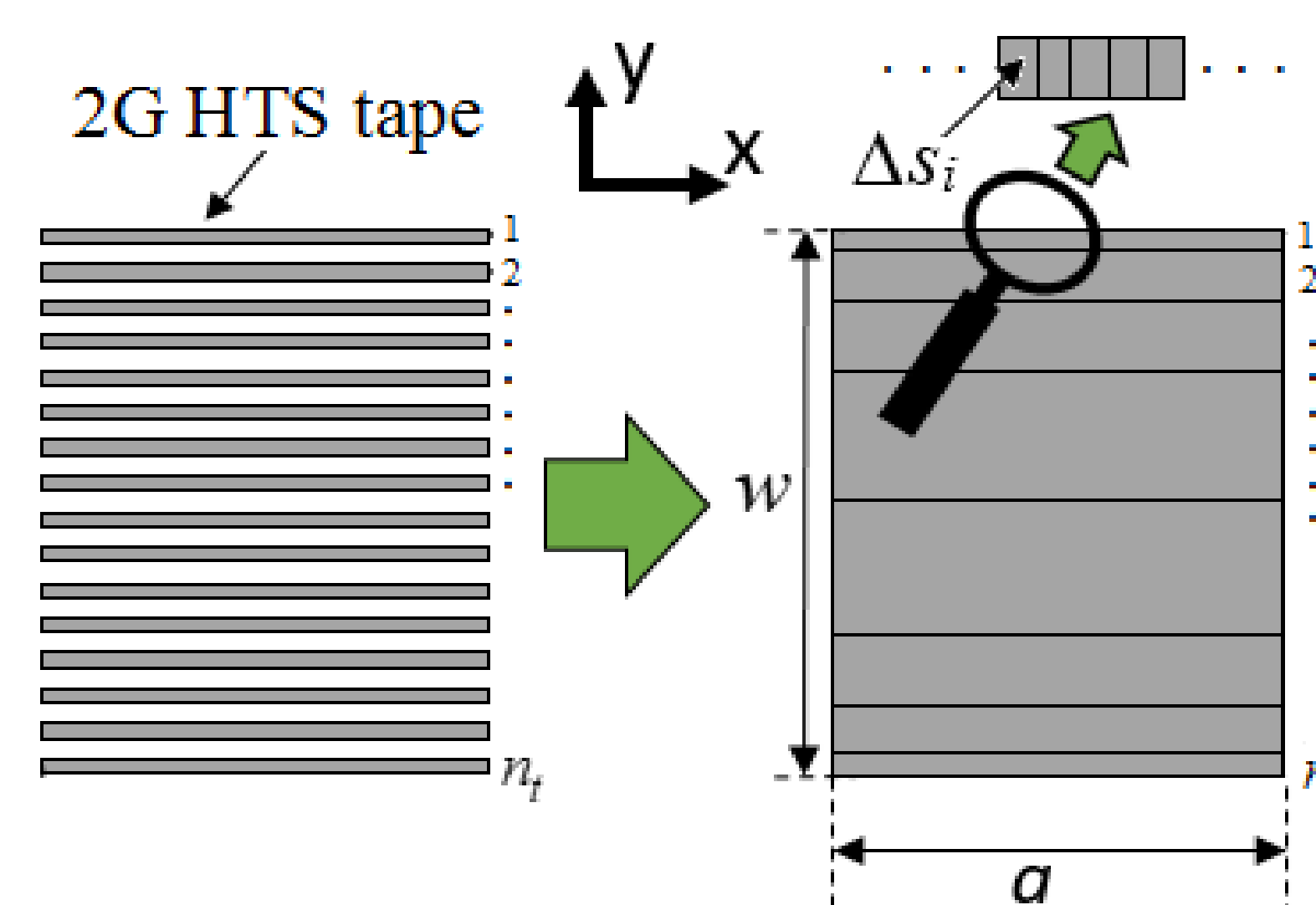
Results and Comparisons

The used parameters: A sinusoidal current of amplitude (I_{app}) was applied

Parameter	J_{c0}	α	B_0	β	n	f	a	h_y	h_s	h_c	μ_r	n_t	n_l	w
Value	28 GA/m ²	0.29515	42,65 mT	0.7	38	50 Hz	4 mm	1 μm	50 μm	293 μm	100	32	18	$n_t \times h_c$

2G HTS stack

Case A: Original stack.
Case B: Homogenized stack.
Case C: Homogenized stack, reducing the integral matrices by using the fast multipole method (FMM).
Case D: The same as in Case C where the losses are evaluated only on the half of the layers, skipping one layer between two layers. An interpolation method is then used to predict the losses on the skipped layers.

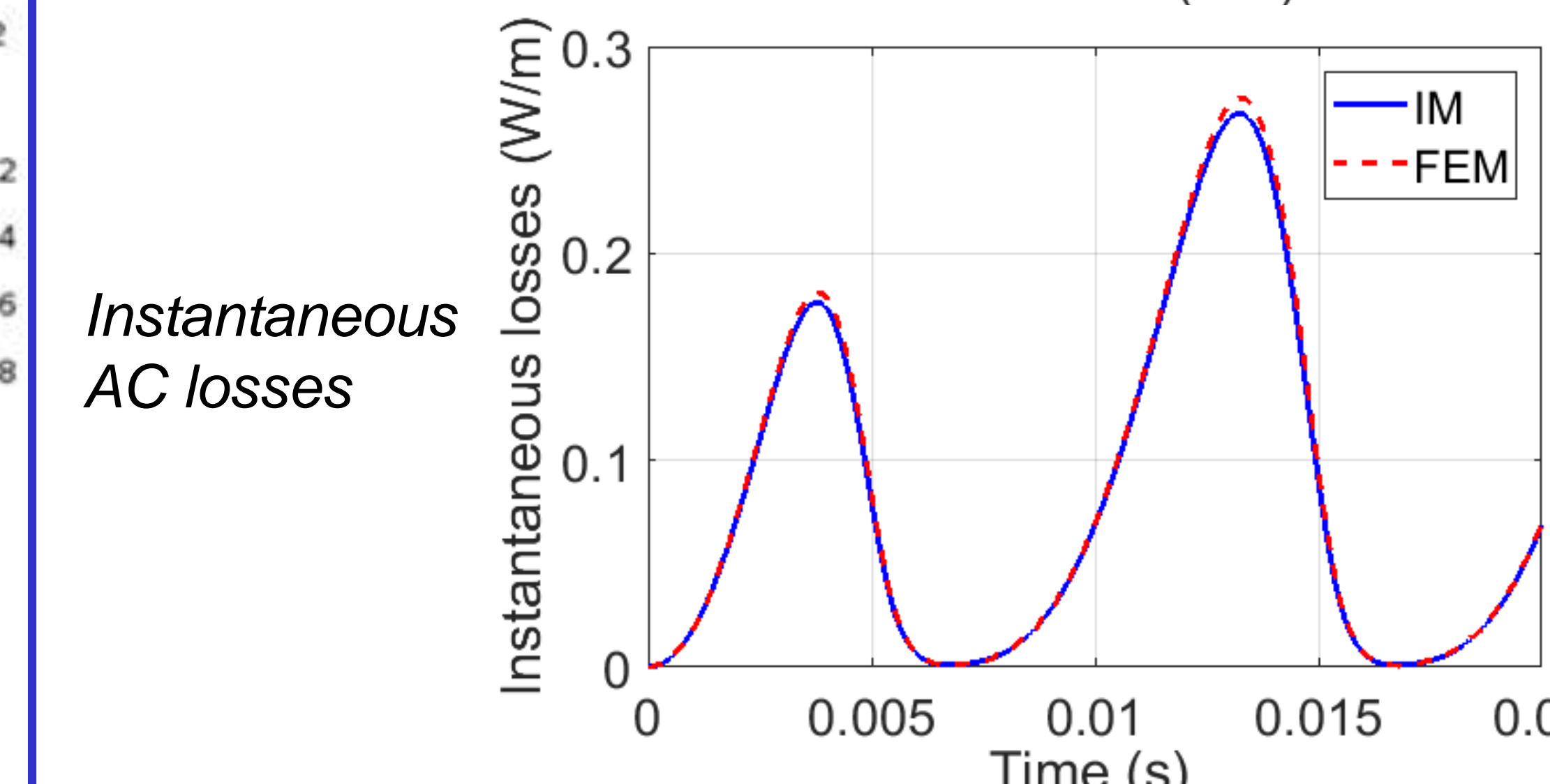
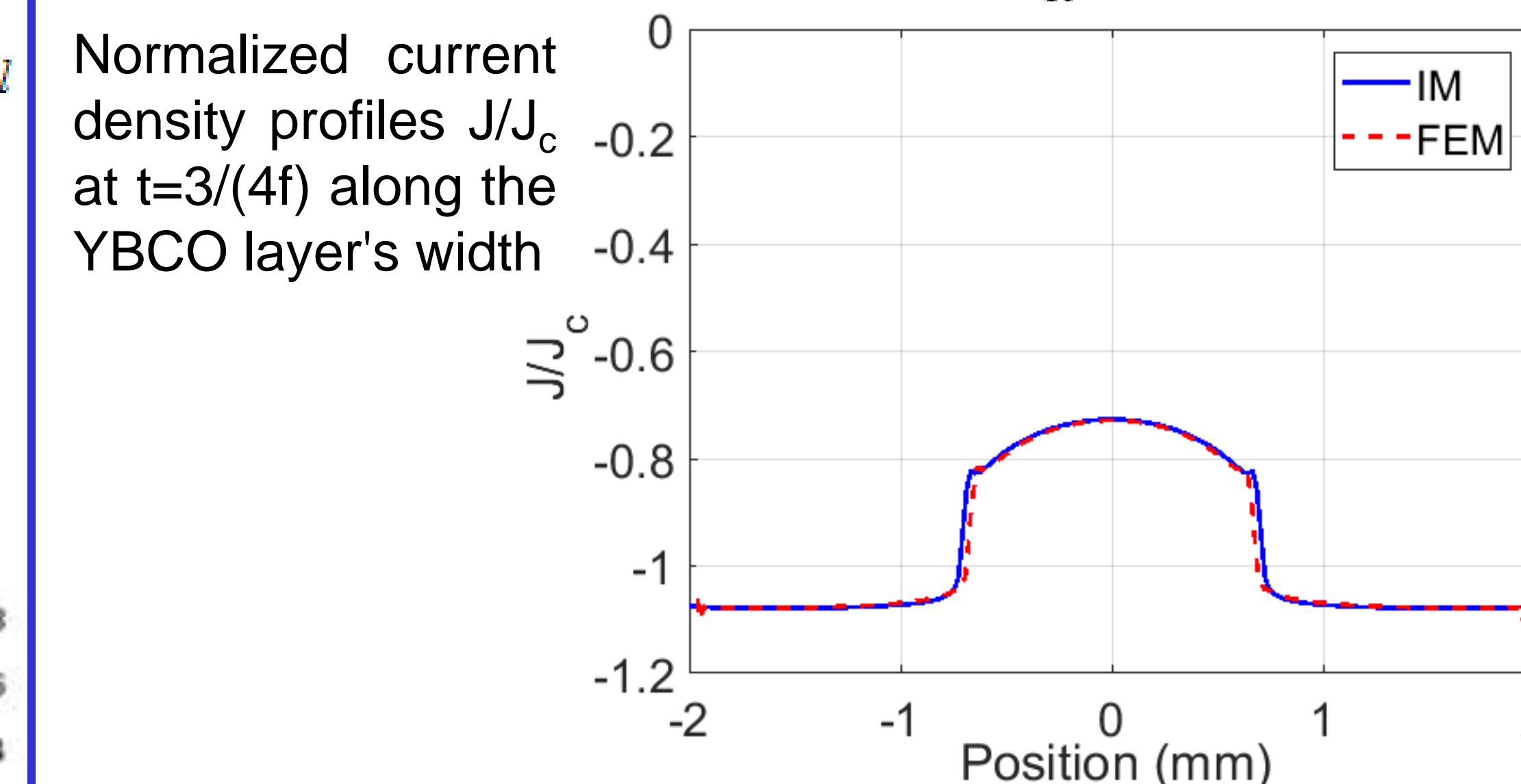
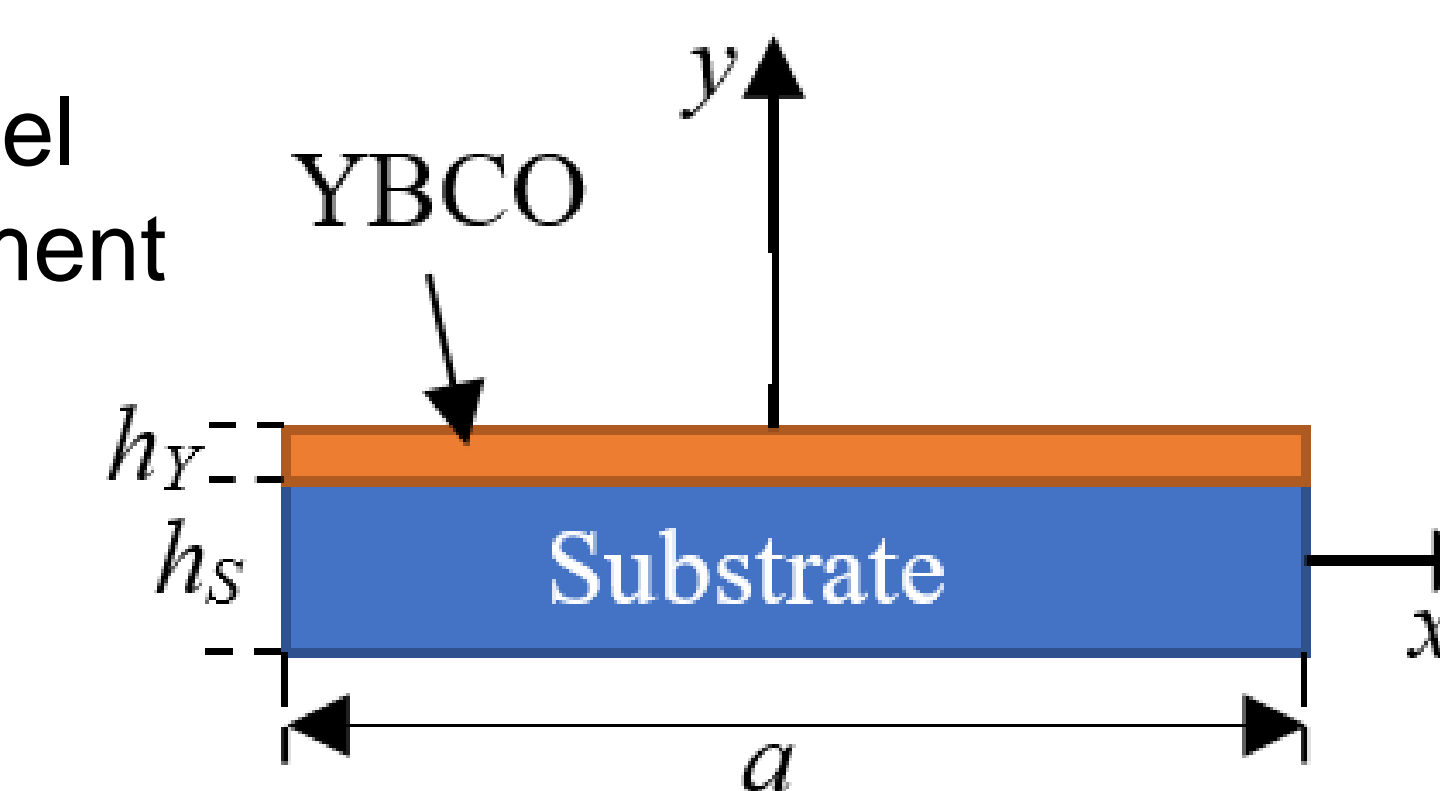


I_{app}	Model	Case A	Case B	Case C	Case D
60 A	IM	11,23	607	11.30	208
	FEM	11.42	3478	11.40	449

Benchmark # 02 : 2G HTS tape with ferromagnetic substrate

IM: Integral model
FEM: Finite element method

$$I_{app} = 90 \text{ A}$$



[1] Y. Statra, H. Menana and B. Douine, «Semianalytical Modeling of AC Losses in HTS Stacks Near Ferromagnetic Parts», IEEE Trans. Appl. Supercond., vol. 31, no. 1, pp. 1-6, 2021.