Numerical investigation of critical states in superposed superconducting films

L. Burger[1], I. S. Veshchunov[2], T. Tamegai[2], A. V. Silhanek[3], S. Nagasawa[4], M. Hidaka[4] and B. Vanderheyden[1]

[1] Electrical Engineering and Computer Science Department, Université de Liège, Liège, Belgium
[2] Department of Applied Physics, University of Tokyo, Tokyo, Japan
[3] Experimental Physics of Nanostructured Materials, Q-MAT, CESAM, Université de Liège, Liège, Belgium
[4] National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba, Japan

HTS 2020 Modelling

Problem statement

We consider the assembly made of a $L \times L$ square niobium film and $W \times L$ niobium rectangular strips. The thickness of all films is denoted by d. The long sides of the rectangular strips are placed parallel to one of the side of the square, and two consecutive films are separated by an insulating SiO_2 layer of thickness t_{SiO_2}, as shown in Figure 1d. A uniform magnetic field is applied perpendicular to the assembly. This field is ramped up from 0 to a given value, H_a, at a constant rate, $\dot{H}_a$, and then ramped back to 0 at the same rate.

The critical state of these kind of assemblies are non-trivial [1]. For instance, in a two-layers assembly, the observed discontinuity lines (d-lines), which highlight the sharp changes of direction of the current density, cannot be obtained from a simple composition of the d-lines of individual films, as depicted in Figure 1a and Figure 1b. In particular, **in the remanent state, an additional horizontal line of length is ℓ_h, appears at the center of the assembly**, as shown in Figure 1c.

In a previous work [2], it was shown how the magnetic-field dependence of the critical current density in the films is a crucial attribute that contributes to the observation of a critical states like those of Figure 1c in two-layers superimpositions. In what follows, we recall the most salient results for two-layers assemblies and then proceed with an investigation of the geometrical parameters involved in the two-layers assembly. The results are also extended to the case of three-layers systems. **These investigations are carried out by numerical means.**

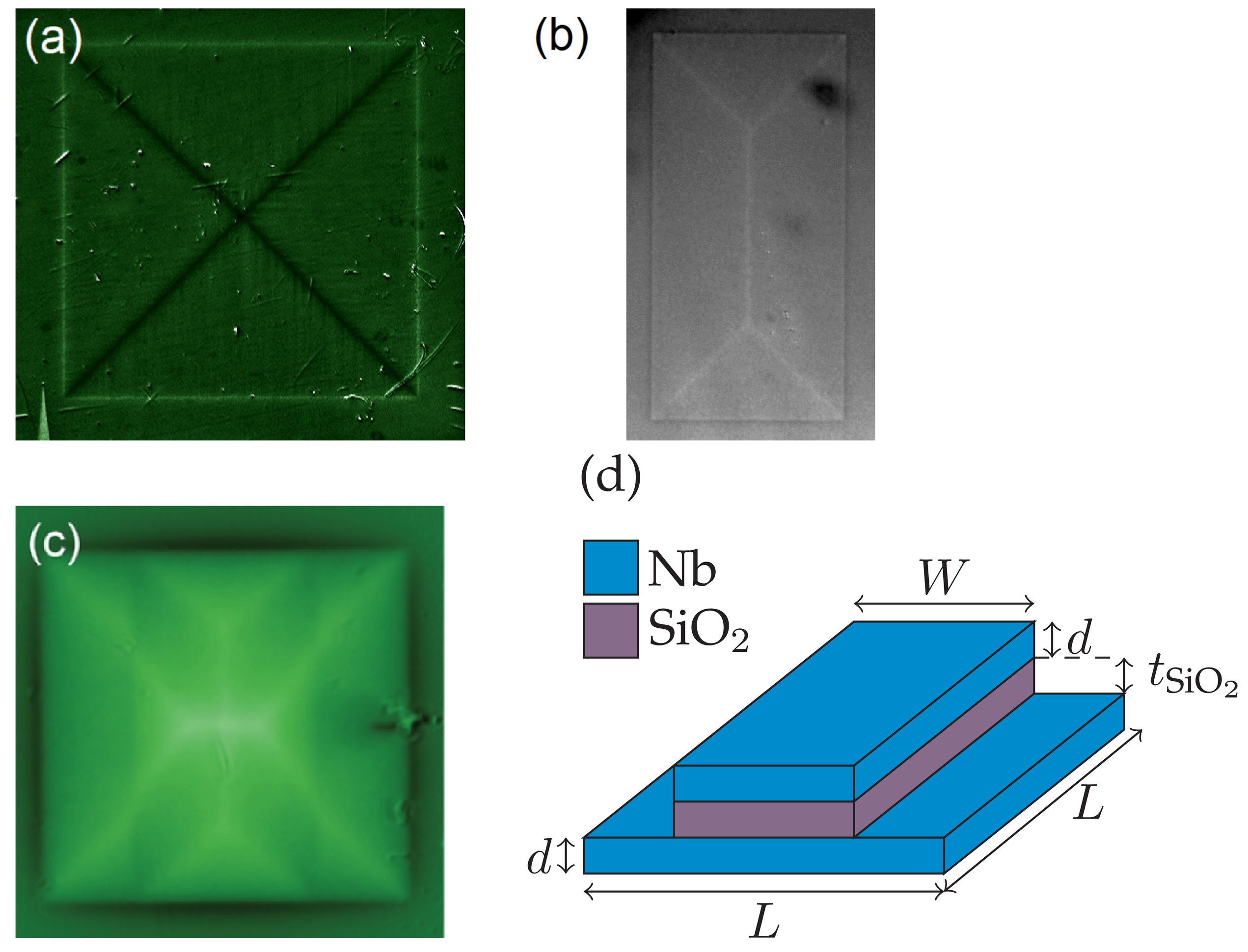

Figure 1: Magneto-optical images of d-lines in (a), a single square film, (b) a single rectangular strip, (c) an assembly of a square film and a rectangular film. (d) Sketch of the superconducting two-layers assembly. If not stated otherwise, it is assumed that $L = 200\ \mu$m, $W = L/2$, $d = 300$ nm and $t_{SiO_2} = 300$nm.

Numerical modelling

In order to model the penetration of magnetic field in two-layers superconducting structures, a finite element (FE) $H-\phi$ formulation was used. The equation to solve is :

$$\int_\Omega \mu_0\, \dot{\mathbf{h}} \cdot \psi\, d\Omega + \int_\Omega \mu_0\, \dot{\mathbf{H}}_a \cdot \psi\, d\Omega + \int_{\Omega_c} \rho(|\nabla\times\mathbf{h}|)\nabla\times\mathbf{h}\cdot\nabla\times\psi\, d\Omega_c = 0$$

The magnetic field, $\mathbf{H} = \mathbf{h} + \mathbf{H_a}$, where $\mathbf{h}$ and $\mathbf{H_a}$ are the reaction field and the applied field, respectively. $\mu_0 = 4\pi\times10^{-7}$ H/m is the vacuum magnetic permeability, $\rho = E_c/J_c(|\mathbf{B}|)(|\mathbf{J}|/J_c(|\mathbf{B}|))^{n-1}$ is the electrical resistivity in the superconducting films, where $E_c = 1\ \mu$V/cm is the critical electric field and $J_c(|\mathbf{B}|) = J_{c0}/(1+|\mathbf{B}|/B_0)^\alpha$ is the magnetic field dependent critical current density. If not stated otherwise, it is assumed that $n = 19$, $\dot{H}_a = 1$ kA/m.s, $J_{c0} = 3.4$ MA/cm^2, $B_0 = 1.25$ mT, $\alpha = 0.42$ in the square film and $J_{c0} = 5.4$ MA/cm^2, $B_0 = 4.9$ mT, $\alpha = 0.51$ in the strips. Ω is the total domain, Ω_c is the conducting domain. ψ are linear edge test-functions, and ϕ are linear nodal test-functions. The mesh is produced in Gmsh, while the FE equation is solved in GetDP [3].

The crucial role of $J_c(|\mathrm{B}|)$

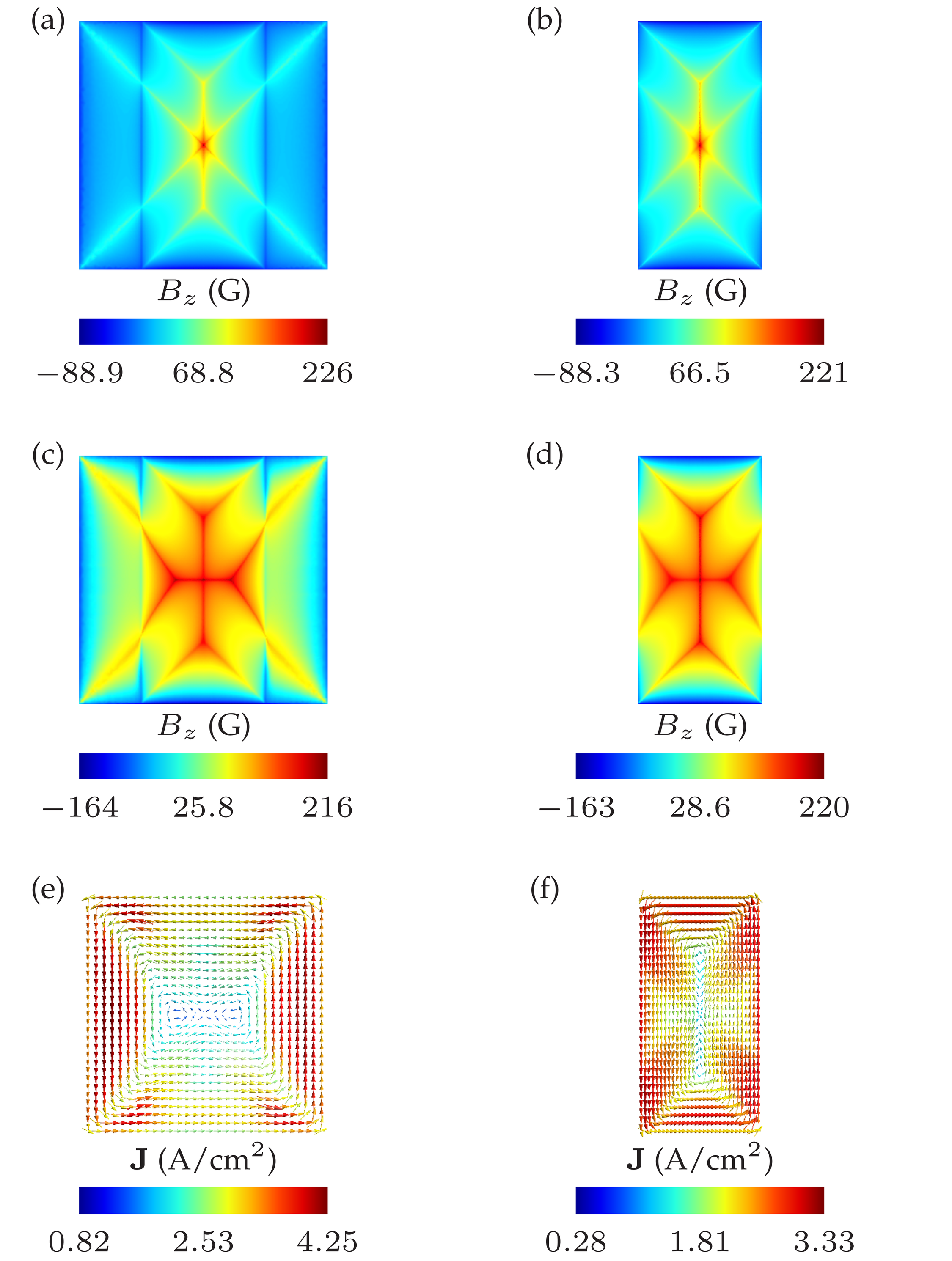

Figure 2: (a)-(b): $J_c = 2$ MA/cm^2 in both films. (c)-(f): $J_c(|\mathbf{B}|)$, with $J_{c0} = 12$ MA/cm^2 and $B_0 = 5$ mT in both films. (a)-(d): B_z in the remanent state. (e)-(f): Current lines in the remanent state.

Two-layers system: ℓ_h vs W

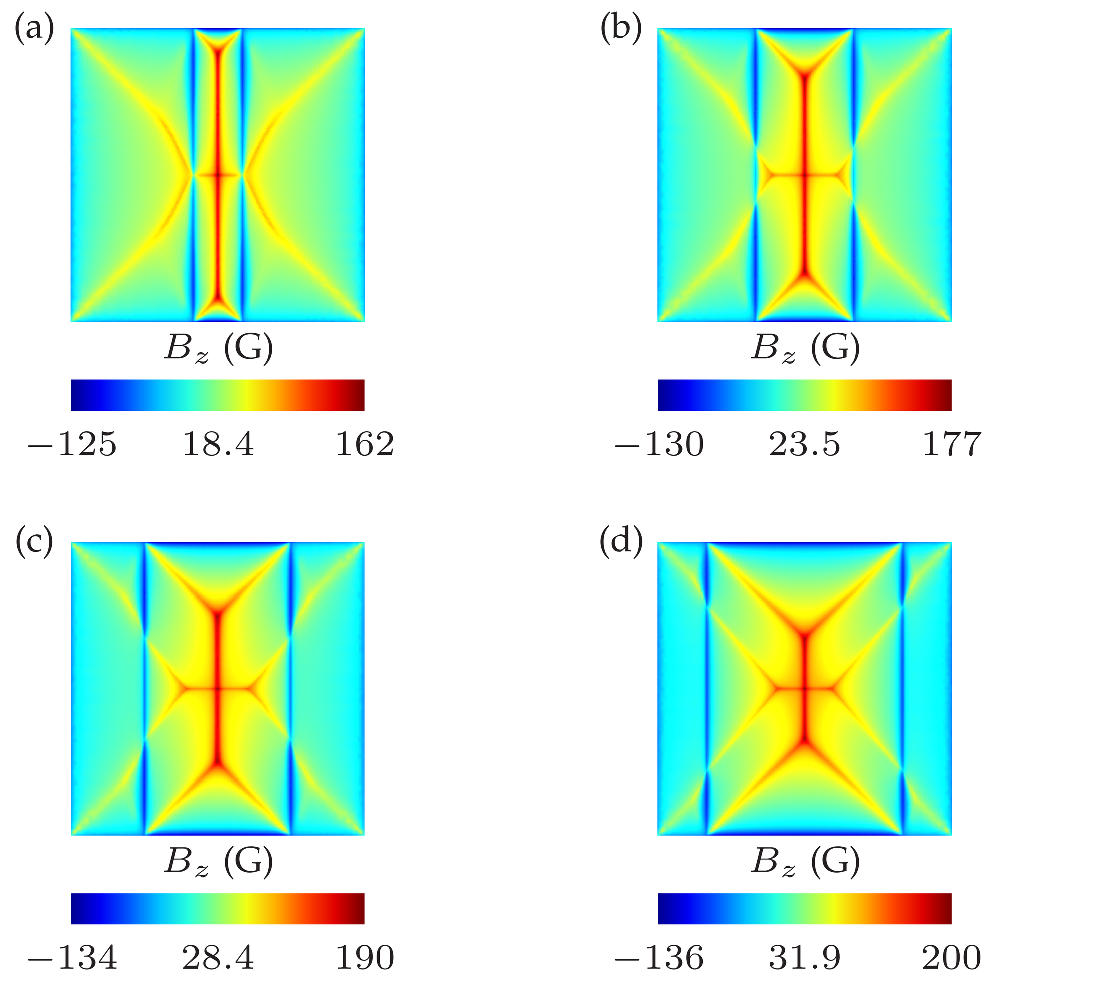

Figure 3: B_z in the square film for (a) $W = L/6$, (b) $W = L/3$, (c) $W = L/2$ and (d) $W = 2L/3$.

Two-layers system: off-centred strip

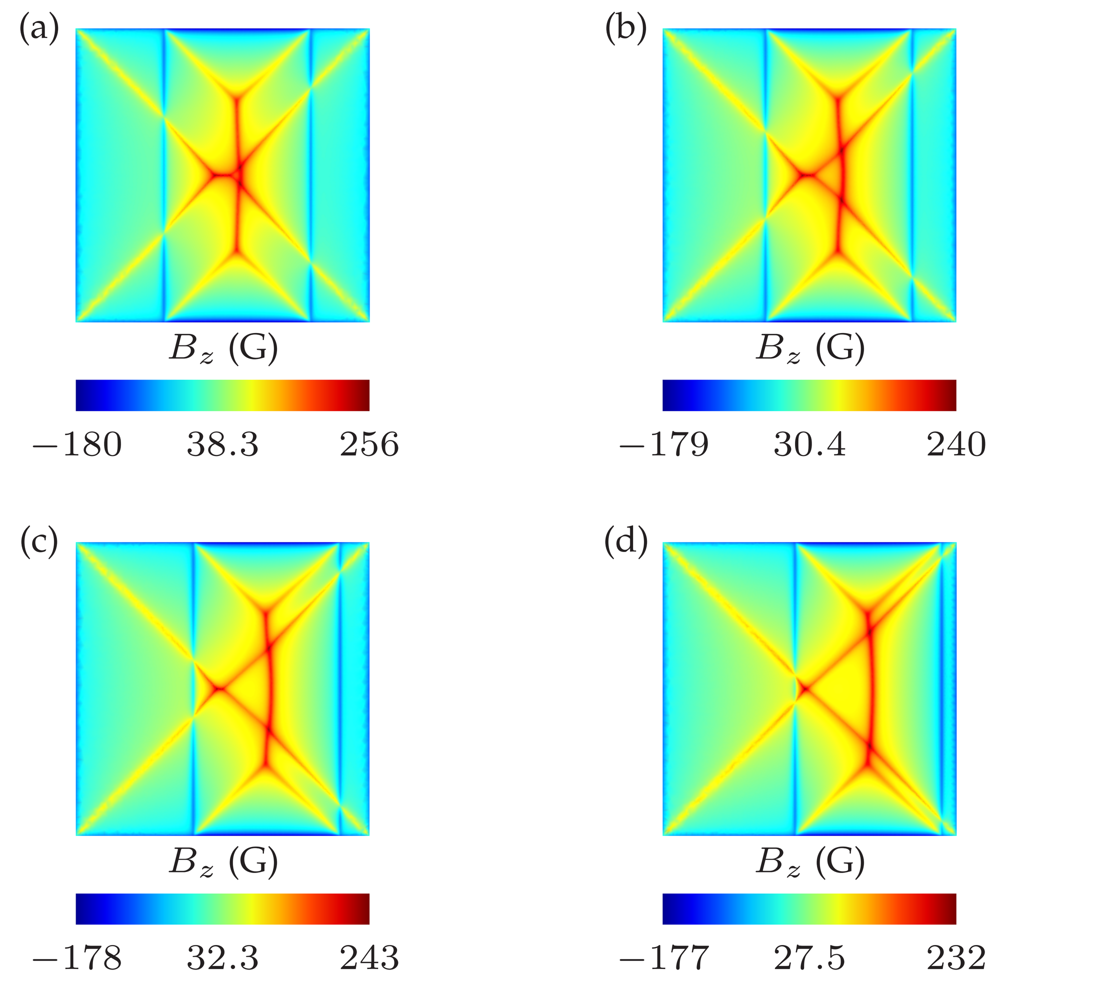

Figure 4: B_z in the square film for a strip is off-centred with respect to the centre of the square film by a distance, X_c, equal to (a) $X_c = 10\ \mu$m, (b) $X_c = 20\ \mu$m, (c) $X_c = 30\ \mu$m and (d) $X_c = 40\ \mu$m.

Two-layers system: ℓ_h vs. t_{SiO_2}

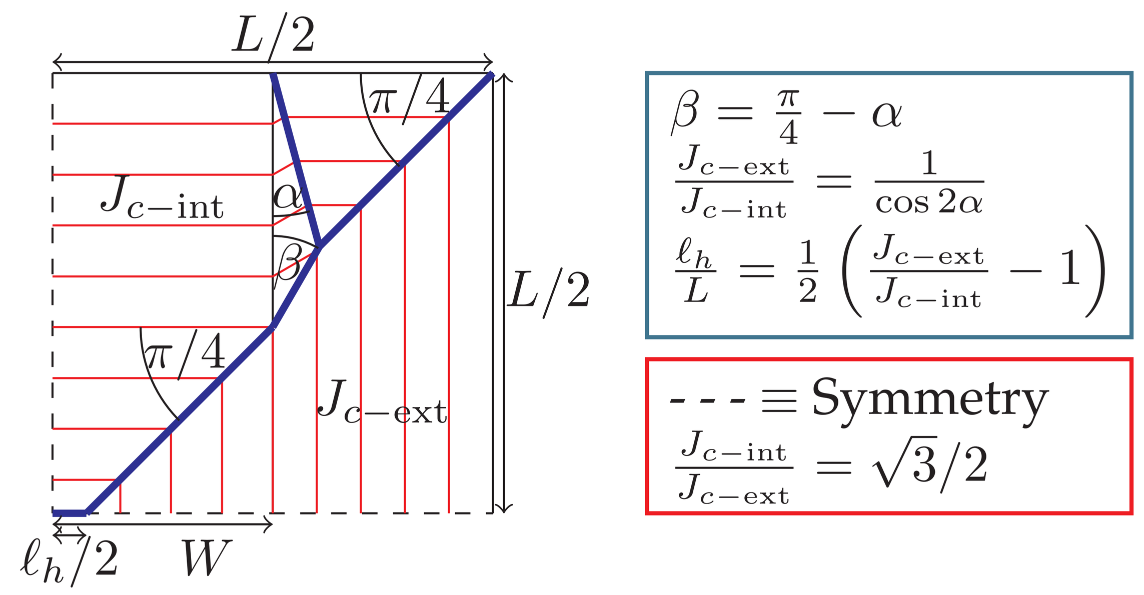

Figure 5: Current lines in a single film with non-uniform J_c, according to the critical state. In the central strip of width $W = L/2$, $J_c = J_{c-\mathrm{int}}$, while $J_c = J_{c-\mathrm{ext}} > J_{c-\mathrm{int}}$ elsewhere. Blue lines represent the d-lines, while thin red lines show the current lines in the square film.

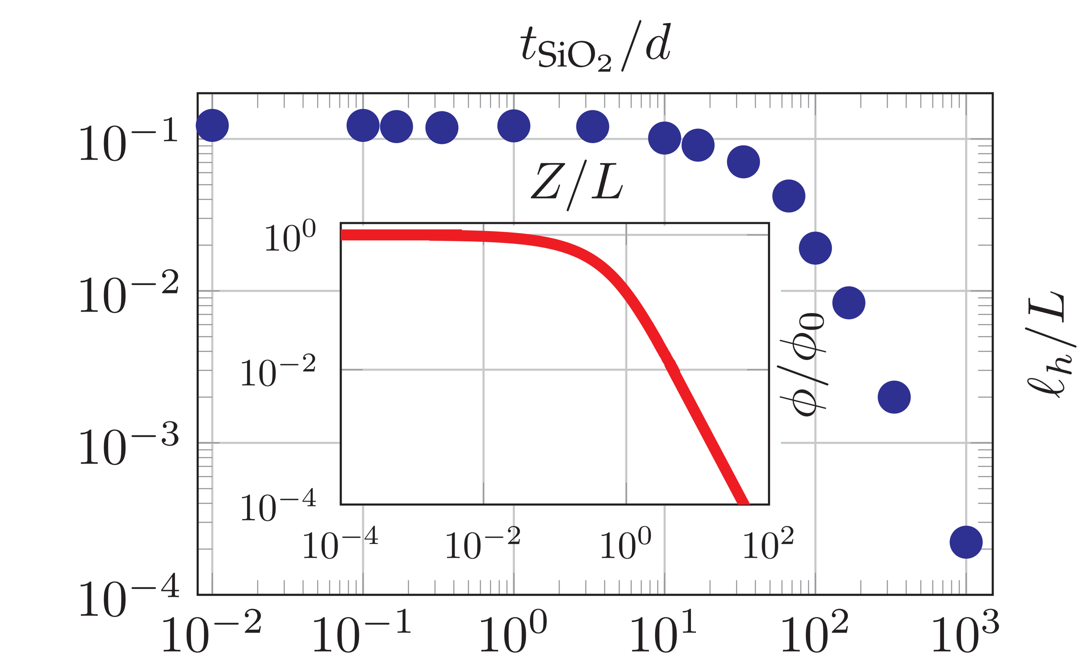

Figure 6: ℓ_h/L as a function of t_{SiO_2}/d. The inset represents the decay of the scalar magnetic potential, ϕ/ϕ_0, as a function of the distance away from the centre of the surface of a single square film, Z/L, where ϕ equals ϕ_0.

Three-layers systems

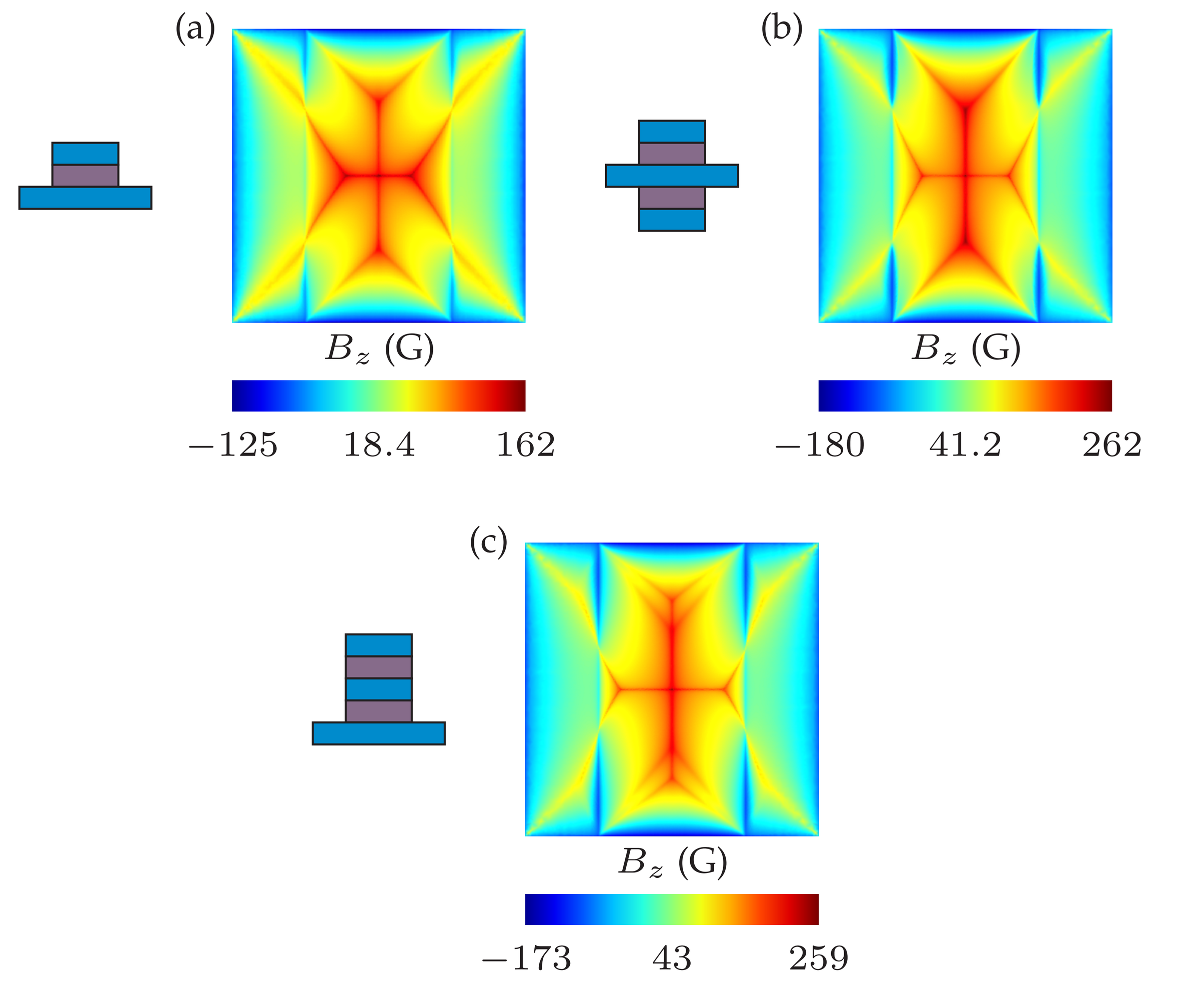

Figure 7: B_z in the square film for diverse assemblies of films. $J_{c0} = 12$ MA/cm^2, $B_0 = 5$ mT, $\alpha = 1$ in each film.

Conclusion

Numerical modelling was used to highlight how the geometric parameters of the assembly impacts the distribution of magnetic field inside three-dimensional assemblies made of a square film and several rectangular strips. In particular, it is illustrated how the distance between the strip and the film modulates the magnetic coupling between the films and influences ℓ_h.

Adding a second strip on the other side of the square film, the total reaction field from the strips is strengthened, the critical current density non-uniformities are even more exaggerated, and, hence, ℓ_h increases. If the second strip is placed on top of the first one, the critical structures becomes more complex, because of the two distances that separate each strip from the film are uneven. This illustrates **how complex the critical states in superposed films with different cross-sections can become**, and how much the current patterns may become non-trivial.

References/Acknowledgement

[1] T. Tamegai *et al.*, Physica C **533**, 74-79 (2017).
[2] L. Burger *et al.*, SUST **32**, 125010 (2019).
[3] C. Geuzaine *et al.*, International Journal for Numerical Methods in Engineering **79**, 1309-1331 (2009).