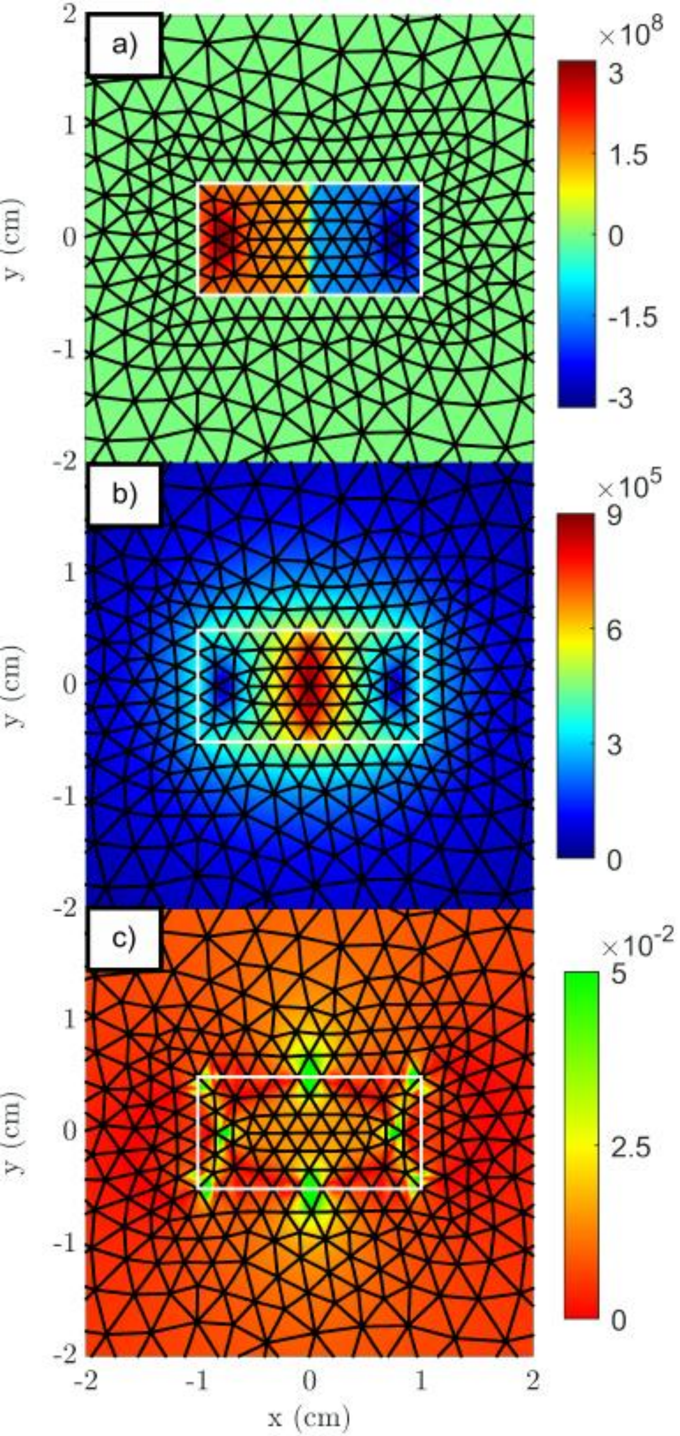

H-ϕ-formulation in COMSOL For Efficient Simulations of Superconductors

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Introduction

Objective:

Present the H-ϕ-formulation implemented in COMSOL Multiphysics to model various superconductor applications

- Comparison of accuracy and computation times with the H-formulation
- The problem of multiply connected domains in the H-ϕ-formulation will be briefly introduced as well as a simple solutions implementable in COMSOL

Outline

1. Formulation definitions and implementation
2. Modelling transport currents in the H-ϕ-formulation

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The H-formulation

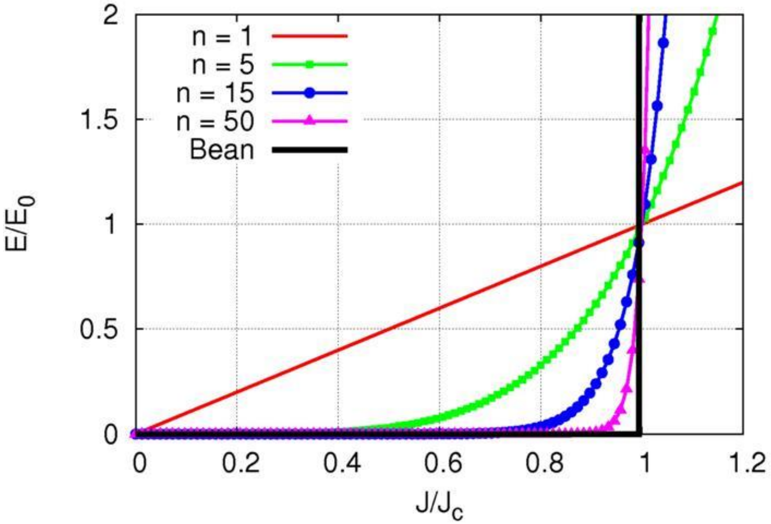
- Uses the magnetic field H as the vector dependent variable
- Simulates Faraday's law combined with Ampere's law:

$$\nabla \times (\rho \nabla \times \boldsymbol{H}) = -\mu \left(\frac{\partial \boldsymbol{H}}{\partial t} \right)$$

- For superconductors, the resistivity (ρ) is generally modelled using the power law model:

$$\rho = \frac{E_c}{J_c} \left(\frac{||\boldsymbol{J}||}{J_c} \right)^{n-1}$$

Where E_c is the electric field criterion, J_c is the critical current density and n is the power law exponent



The H-formulation

- More than 45 international research groups use the H-formulation to model superconductors for the following reasons [1]:
  1. Accurately models an extensive number of applications
  2. Easily implementable in commercial finite element method (FEM) software such as COMSOL
  3. Models can easily be shared when implemented in commercial FEM software

- Problems with the H-formulation:
  1. The resistivity of air must be very high to avoid eddy currents → Degrades matrix conditioning
  2. Currents can still flow in air domains → Inaccurate results for some cases
  3. Vector dependent variables must be used in current-free domains → Unnecessarily adds degrees of freedom

[1] B. Shen, F. Grilli, and T. Coombs, *IEEE Access*, vol. 8, pp. 100403–100414, 2020, doi: 10.1109/ACCESS.2020.2996177.

The magnetic scalar potential ϕ

- In current free domains ($\nabla \times \boldsymbol{H} = 0$), the magnetic field can be written as the gradient of the magnetic scalar potential, $\boldsymbol{H} = -\nabla\phi$
- The constitutive law of the magnetic field for superconductors and air is $\boldsymbol{B} = \mu_0\boldsymbol{H}$
- Gauss' law ($\nabla \cdot \boldsymbol{B} = 0$) can then be used as the governing equation of the field surrounding HTS domains:

$$-\nabla \cdot \nabla\phi = 0$$

- With this formulation, no resistivity needs to be specified and a scalar dependent variable is used in non-conducting domains

Coupling between H and ϕ

<u>H physics</u>

Weak formulation

$$\int_{\Omega_{SC}} \rho(\nabla \times \boldsymbol{H}) \cdot (\nabla \times \boldsymbol{w}) + \partial_t \mu \boldsymbol{H} \cdot \boldsymbol{w}\, d\Omega + \int_{\Gamma} (\boldsymbol{n} \times \boldsymbol{E}) \cdot \boldsymbol{w}\, d\Gamma = 0$$

Coupling to ϕ

Dirichlet boundary condition on **H**

$$\boldsymbol{n} \times \boldsymbol{H} = \boldsymbol{n} \times -\nabla\phi$$

<u>ϕ physics</u>

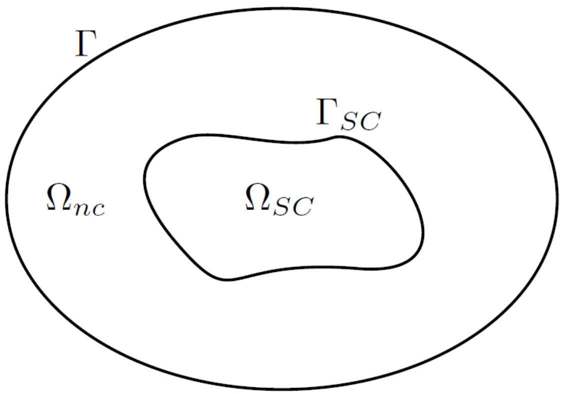
Weak formulation

$$\int_{\Omega_{nc}} \nabla\phi \cdot \nabla v\, d\Omega + \int_{\Gamma_{SC}} \boldsymbol{n} \cdot \nabla\phi\, v\, d\Gamma = 0$$

Coupling to H

Neumann boundary condition in weak form

$$\int_{\Gamma_{SC}} \boldsymbol{n} \cdot \nabla\phi\, v\, d\Gamma = -\int_{\Gamma_{SC}} \boldsymbol{n} \cdot \boldsymbol{H}\, v\, d\Gamma$$



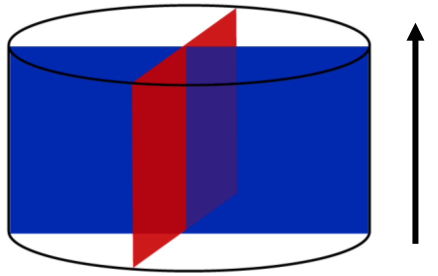
Coupling between H and ϕ in COMSOL

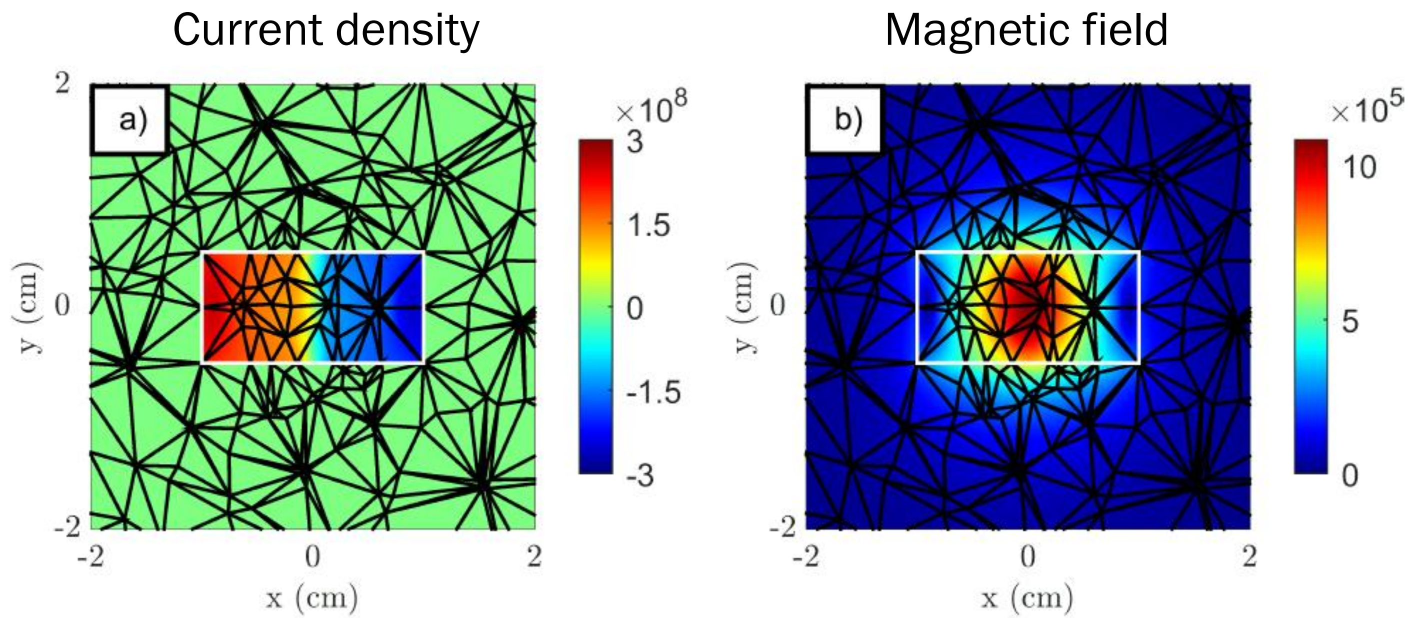
- Both formulations can easily be implemented in COMSOL with the use of the predefined Magnetic Field H (MFH, H physics) module and the Magnetic Field, No Current (MFNC, ϕ physics) module
- The appropriate boundary conditions must be imposed to couple both physics, but these boundary conditions are implemented in both packages
- See [2] for details on the implementation procedure

[2] A. Arsenault, F. Sirois, and F. Grilli, *IEEE Transactions on Applied Superconductivity*, vol. 31, no. 2, pp. 1–11, Mar. 2021, doi: 10.1109/TASC.2020.3033998.

Magnetization of bulk superconductors

- Simulations of the magnetization of bulk superconductors can greatly benefit from the H-ϕ formulation
- In this application, we model a bulk HTS in 3-D with field dependent J_c magnetized by zero field cooling (ZFC) [2]



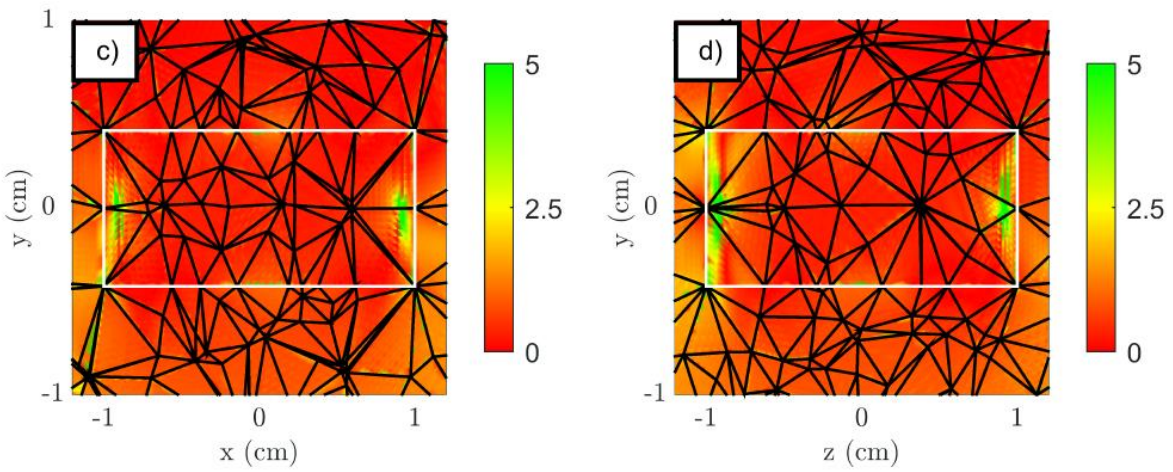


[2] A. Arsenault, F. Sirois, and F. Grilli, *IEEE Transactions on Applied Superconductivity*, vol. 31, no. 2, pp. 1–11, Mar. 2021, doi: 10.1109/TASC.2020.3033998.

Magnetization of bulk superconductors: comparison with the H-formulation

- For 8,325 cubic elements in both the H and H-ϕ formulations, the relative error ($\epsilon = \frac{||H_{H-\phi}|| - ||H_H||}{||H_H||} \times 100$) on the magnetic field remains below 5%

ϵ (%) in x-y plane ϵ (%) in y-z plane



- The H-formulation takes ~10 hours to compute, while the H-ϕ-formulation takes ~2 hours

Outline

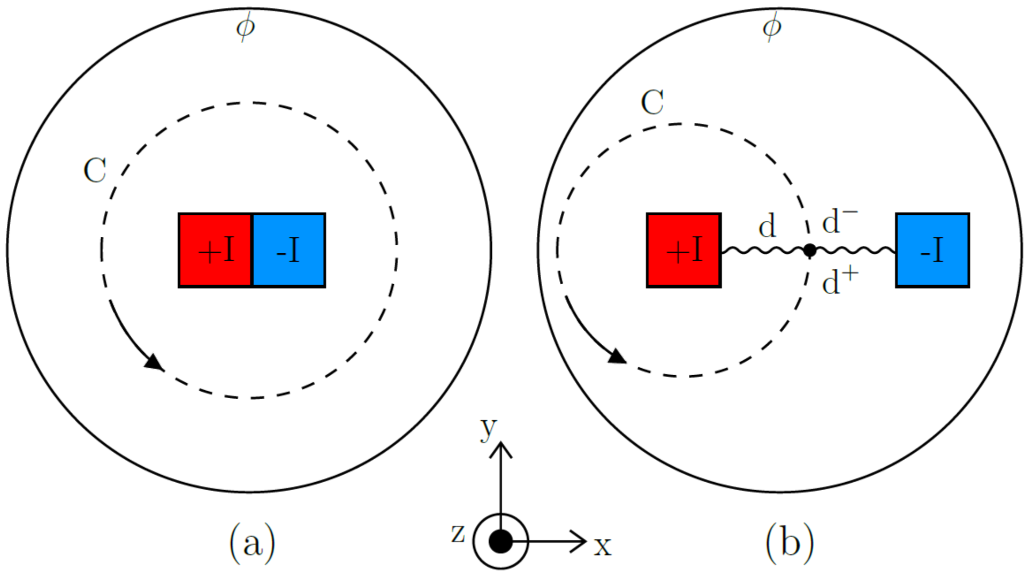
1. Formulation definitions and implementation
2. Modelling transport currents in the H-ϕ-formulation

Modelling transport currents with the H-ϕ-formulation

- Multiply connected conducting domains are problematic in the H-ϕ-formulation since they violate Ampere's law

$$\oint_C \mathbf{H} \cdot \mathrm{d}\mathbf{l} = I_{\text{enc}}$$

- From the definition of the magnetic scalar potential, there should not be any enclosed current in the ϕ physics



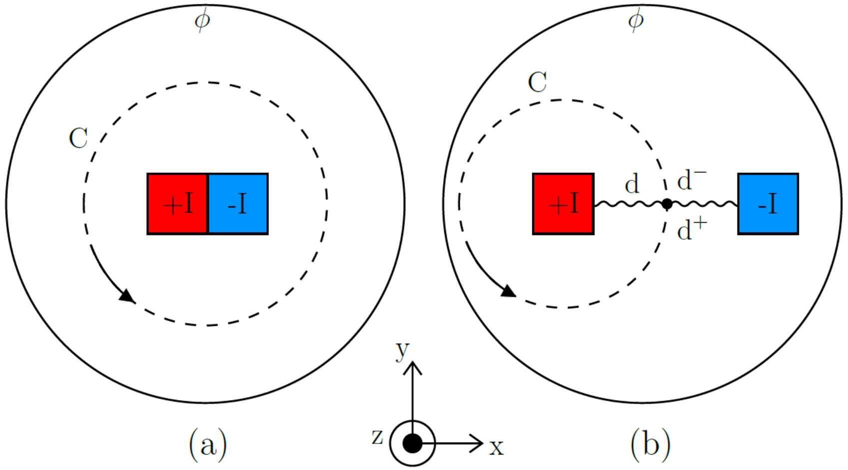
(a) (b)

Modelling transport currents with the H-ϕ-formulation: thin cuts

- To solve this issue, a discontinuity in ϕ can be imposed, such that:

$$\oint_C \mathbf{H} \cdot \mathbf{dl} = -\oint_C \nabla\phi \cdot \mathbf{dl} = \phi(d^-) - \phi(d^+) \equiv [\phi]_d := I_{\text{enc}}$$

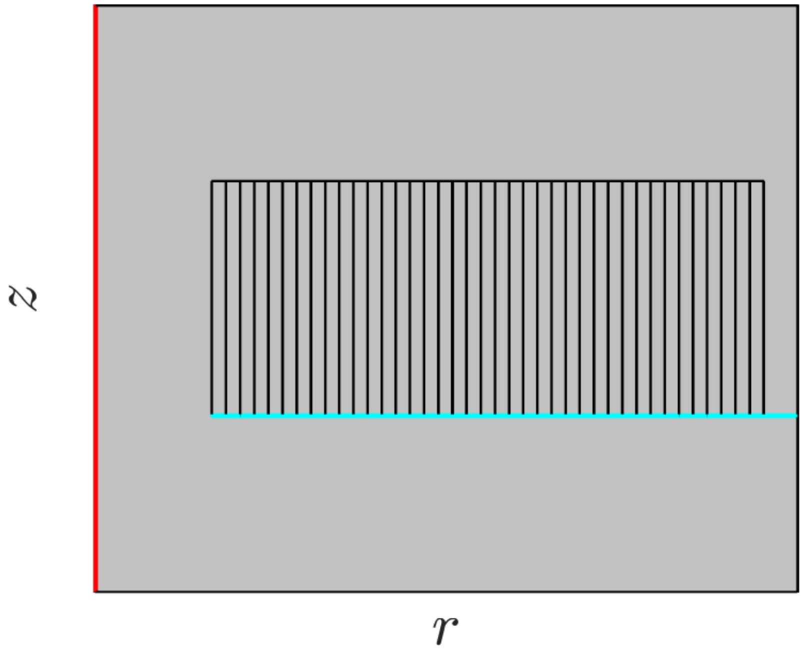
- This can easily be done in the MFNC module of COMSOL with the use of a Magnetic Scalar Potential Discontinuity node
- The discontinuity is imposed on a boundary inside the air domain, which must be manually added
- This is referred to as a “thin cut”
- A more efficient solution is by using cohomology basis functions to impose the discontinuity (thick cuts) [3], but this is not currently available in COMSOL



[3] M. Pellikka, S. Suuriniemi, L. Kettunen, and C. Geuzaine, *SIAM J. Sci. Comput.*, vol. 35, no. 5, pp. B1195-B1214, Jan. 2013, doi: 10.1137/130906556.

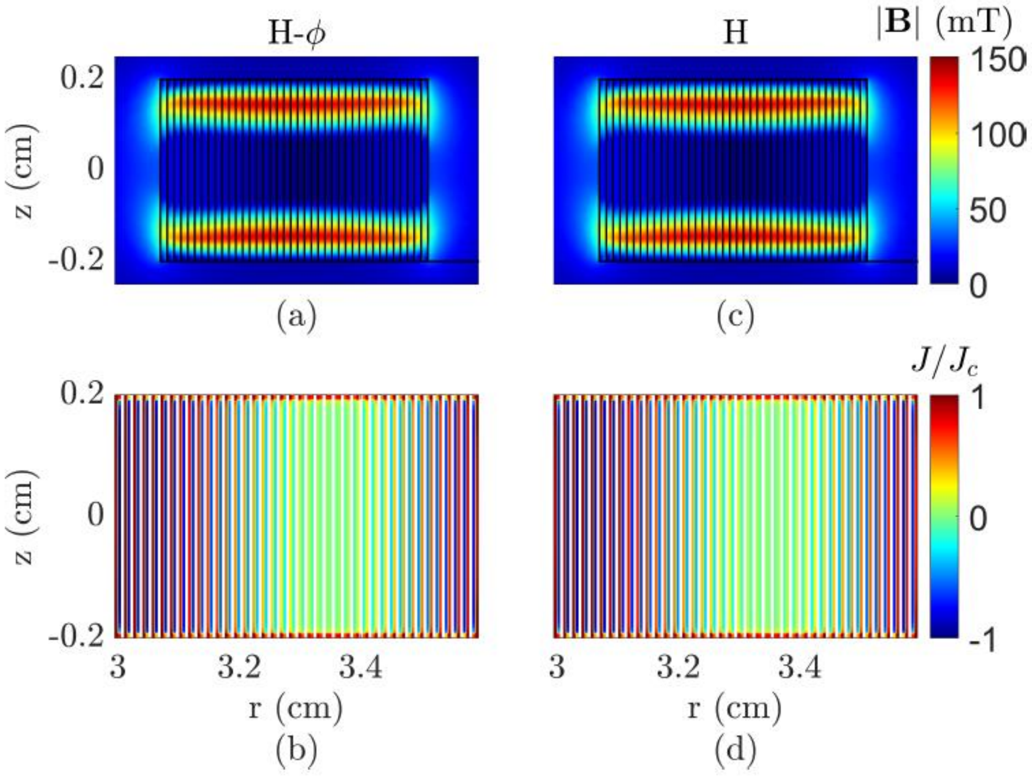
Modelling pancake coils in H-ϕ

- 2-D axisymmetric simulation of 40 turns
- Transport current of 86 A at 50 Hz, corresponding to 80% of critical current
- Only the superconducting layer is modelled for simplicity
- The cyan lines show the thin cuts where the discontinuity in ϕ is applied, while the red line shows the symmetry axis



Modelling pancake coils in H-ϕ

- The magnetic flux density and current density calculated with the H and H-ϕ formulations are nearly identical [4]
- In the H-formulation, the constraint $\int_{\Omega_{SC}} \nabla \times H = I$ is used to impose the current
- However, the computation times are:
  - 20 minutes for the H-ϕ-formulation
  - 2.75 hours for the H-formulation
  - The speed difference comes especially from the constraints used to impose the current

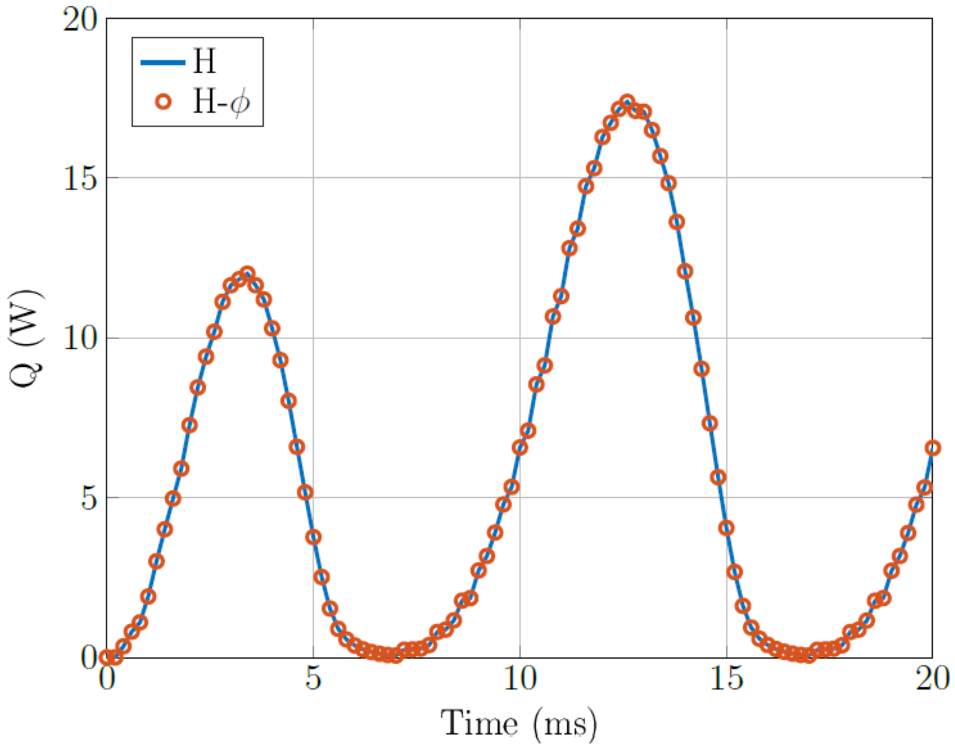


[4] A. Arsenault, B. de Sousa Alves and F. Sirois, Submitted to IEEE TAS, 2021

Modelling pancake coils in H-ϕ

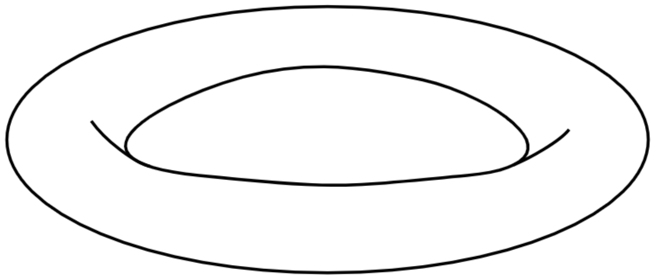
The AC losses are also in agreement between both formulations

$$Q = \int_\Omega \mathbf{E} \cdot \mathbf{J} \, \mathrm{d}\Omega$$



Summary and future work

- We have shown that the H-ϕ-formulation is nearly as versatile and accurate as the H-formulation, while reducing the computation times by a factor of at least 3 in most applications (and up to seven times faster in some cases)
- This formulation is easily implementable in COMSOL and transport currents can be imposed with the use of thin cuts
- Future work:
  - Extend the COMSOL H-ϕ-formulation to model eddy currents in multiply connected conductors
  - Implement circuit relations with thin cuts



Acknowledgements and References

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[1] B. Shen, F. Grilli, and T. Coombs, "Overview of H-Formulation: A Versatile Tool for Modeling Electromagnetics in High-Temperature Superconductor Applications," *IEEE Access*, vol. 8, pp. 100403–100414, 2020, doi: 10.1109/ACCESS.2020.2996177.

[2] A. Arsenault, F. Sirois, and F. Grilli, "Implementation of the H-φ Formulation in COMSOL Multiphysics for Simulating the Magnetization of Bulk Superconductors and Comparison With the H-Formulation," *IEEE Transactions on Applied Superconductivity*, vol. 31, no. 2, pp. 1–11, Mar. 2021, doi: 10.1109/TASC.2020.3033998.

[3] M. Pellikka, S. Suuriniemi, L. Kettunen, and C. Geuzaine, "Homology and Cohomology Computation in Finite Element Modeling," *SIAM J. Sci. Comput.*, vol. 35, no. 5, pp. B1195–B1214, Jan. 2013, doi: 10.1137/130906556.

[4] A. Arsenault, B. de Sousa Alves and F. Sirois, "Modeling of transport currents in superconductors using the H-ϕ formulation," Submitted to IEEE TAS, 2021

[5] F. Grilli, A. Morandi, F. De Silvestri, and R. Brambilla, "Dynamic modeling of levitation of a superconducting bulk by coupled H-magnetic field and Arbitrary Lagrangian-Eulerian formulations," *Superconductor Science and Technology*, vol. 31, no. 12, p. 125003, 2018, doi: https://doi.org/10.1088/1361-6668/aae426.

Superconductors surrounded by magnetic bodies

- In some applications of interest, superconductors interact with other magnetic field sources that have negligible magnetic response (e.g. permanent magnets and coils)

- In such cases, we can separate the field from the independent magnetic source (source field H_s) from the field of the superconductor (reaction field H_r)

$$H = H_r + H_s$$

- The source field can easily be obtained analytically or numerically

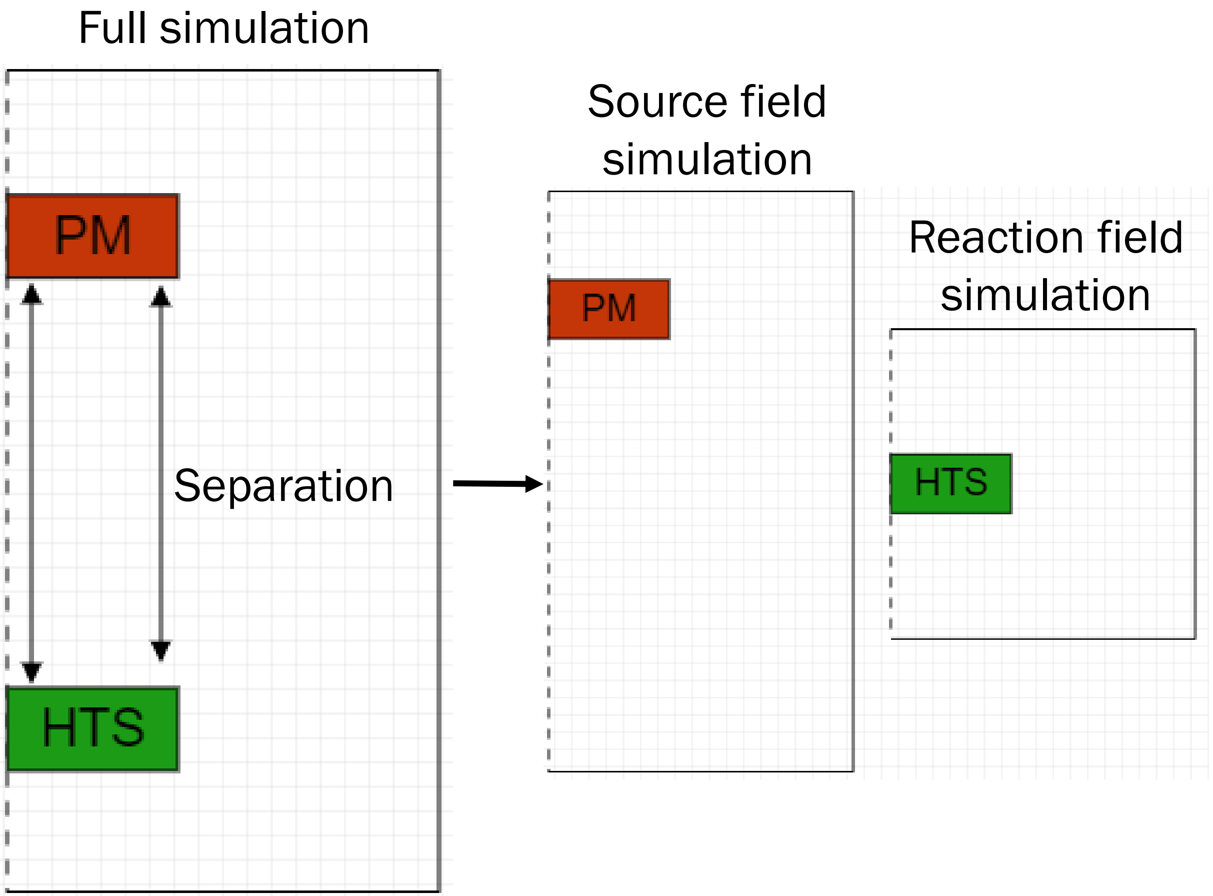
Superconductors surrounded by magnetic bodies

- The superconducting domain can be modelled with the H-formulation with an additional source term:

$$\nabla \times (\rho \nabla \times \mathbf{H}_r) = -\mu_0 \frac{\partial}{\partial t}\left(\mathbf{H}_r + \mathbf{H}_s\right)$$

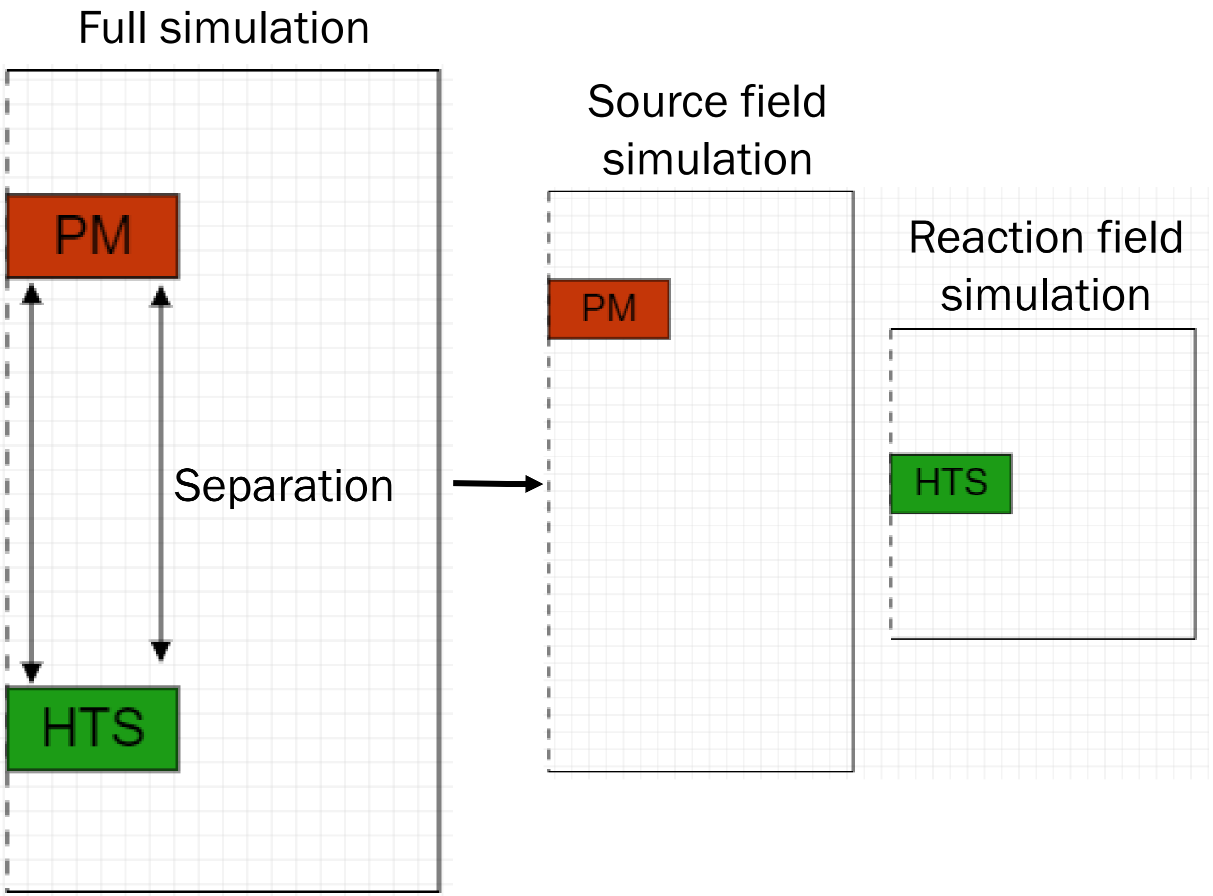
- In this case, the dependent variable is $\mathbf{H}_r$, whereas $\mathbf{H}_s$ is taken as input
- The source field is not associated with any currents inside the superconducting domain, so $\nabla \times H_s = 0$
- The magnetic scalar potential can then easily be coupled to this modified H-formulation, as previously done, to obtain H_r in the non-conducting domains

Levitation of a permanent magnet over a superconducting bulk



- ❑We can model the levitation of a permanent magnet (PM) over a superconducting bulk without simulating any movement with this method
- ❑The field of the moving PM can act as a source for the reaction field of the superconductor
- ❑ The simulation is then separated in two: the source field from the permanent magnet is calculated separately from the reaction field of the superconductor

Levitation of a permanent magnet over a superconducting bulk



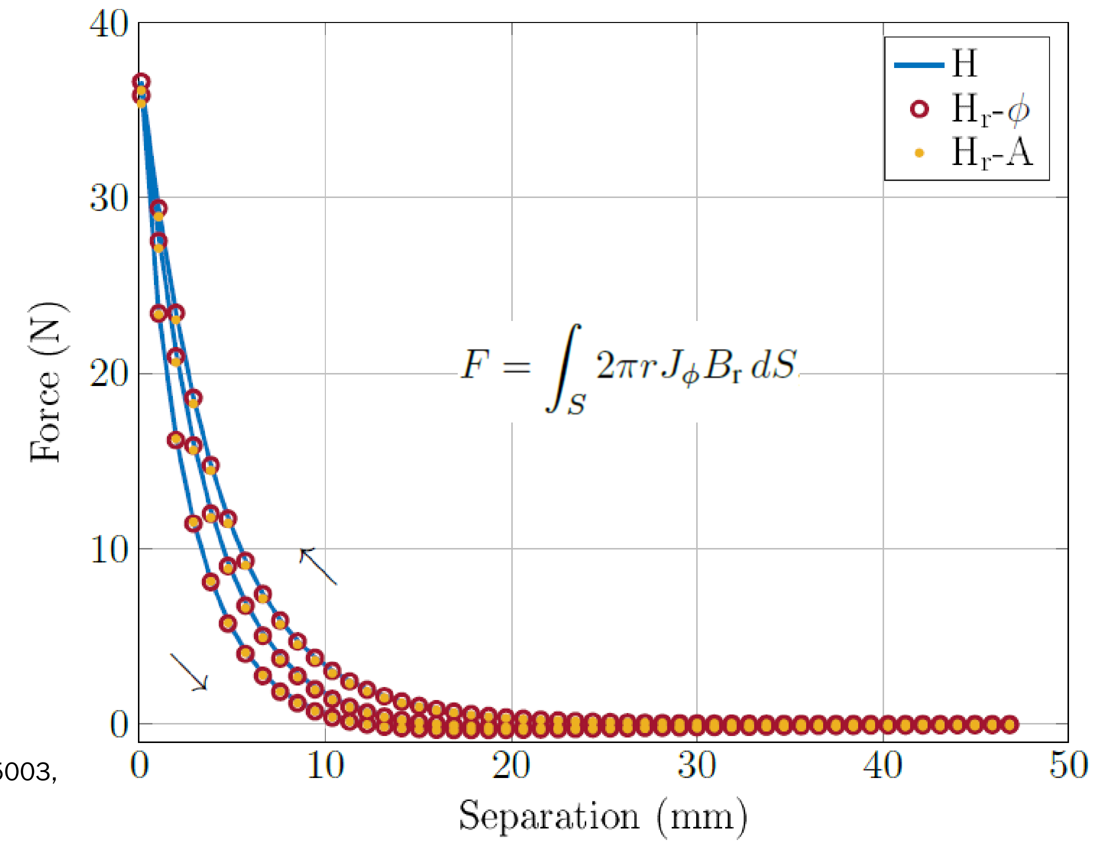
- In this case, both simulations can be static, with the motion of the permanent magnet given through the modified H-formulation:

$$\nabla \times (\rho \nabla \times \mathbf{H}_r) = -\mu_0 \left(\frac{\mathrm{d}\mathbf{H}_r}{\mathrm{d}t} + \frac{\mathrm{d}\mathbf{H}_s}{\mathrm{d}z} \frac{\mathrm{d}z}{\mathrm{d}t} \right)$$

where $\frac{dz}{dt}$ is the velocity of the PM

Levitation of a permanent magnet over a superconducting bulk

- We compare this formulation with the dynamic H-formulation [5], simulating the whole domain with a moving mesh
- We also replace the magnetic scalar potential with the magnetic vector potential (scalar in 2-D) to compare with the H-A formulation
- The force in all formulations are in agreement, with the H-A formulation slightly underestimating the force for smaller separation values



[5] F. Grilli, A. Morandi, F. De Silvestri, and R. Brambilla, *Superconductor Science and Technology*, vol. 31, no. 12, p. 125003, 2018, doi: https://doi.org/10.1088/1361-6668/aae426.

Levitation of a permanent magnet over a superconducting bulk

- The degrees of freedom are reduced by nearly a factor of five between the dynamic H-formulation and the H-ϕ and H-A formulations due to the absence of a moving mesh and a scalar dependent variable in non-conducting domains
- The computation times are accordingly ~3 times faster in the mixed formulations

Formulation	Degrees of Freedom	Computation time (s)
H	10,731	462
H-ϕ	2,273	149
H-A	2,273	165