HTS 2020 Modelling

POLYTECHNIQUE MONTRÉAL
UNIVERSITÉ D'INGÉNIERIE

H-ϕ Finite Element Formulation for Modeling Thin Superconducting Layers

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- Alexandre Arsenault

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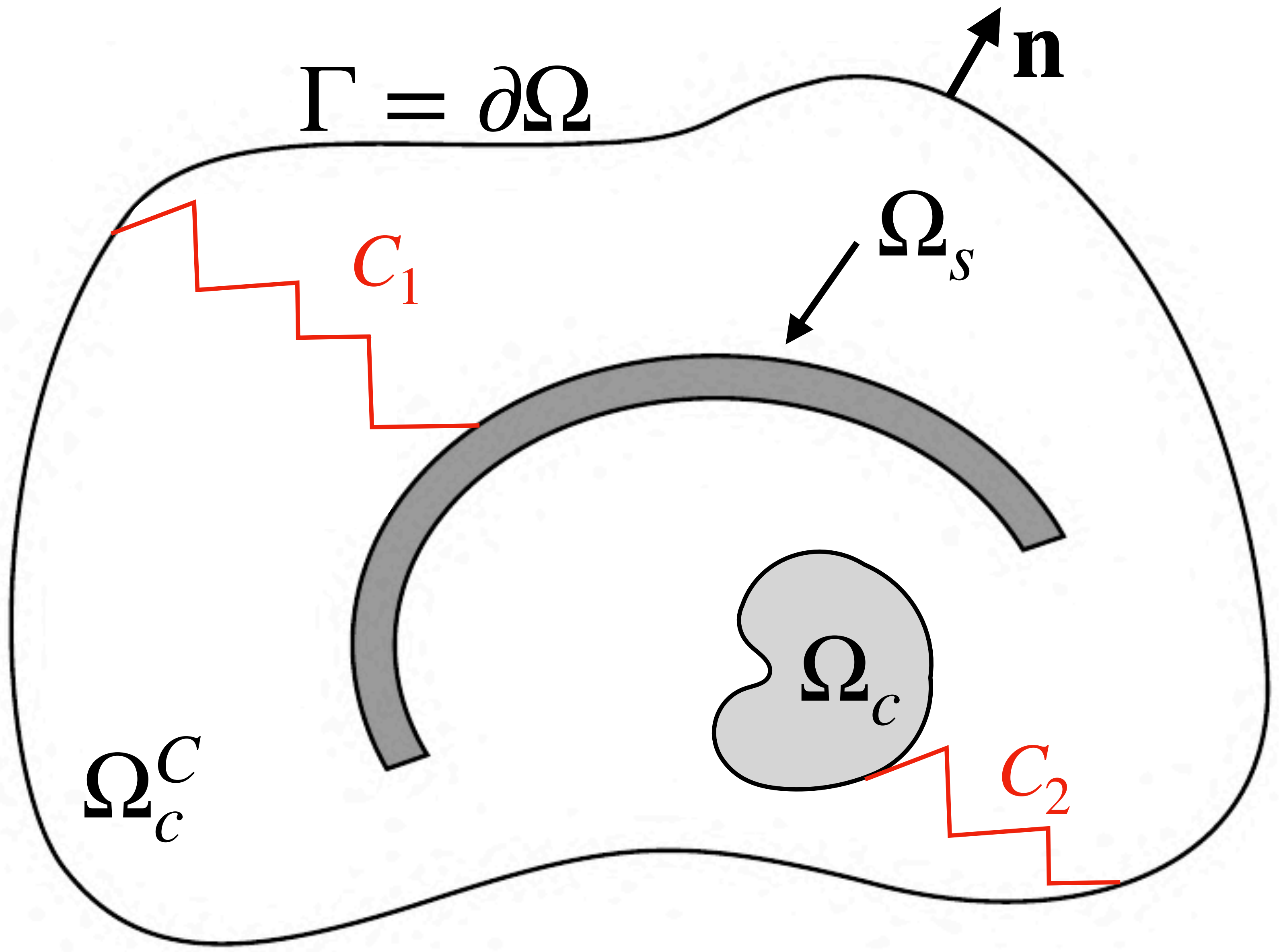
The magnetic field formulation

eddy current problems



$\Omega_s \Rightarrow$ HTS region ($\Omega_s \subset \Omega_c$)

$\Omega_c \Rightarrow$ conducting region

$C_i \Rightarrow$ i-th cohomology, i.e. thick cut

Weak form

$$\left(\rho \nabla \times \mathbf{h}, \nabla \times \mathbf{w}\right)_{\Omega_c} + \partial_t \left(\mu \mathbf{h}, \mathbf{w}\right)_{\Omega} + \langle \mathbf{n} \times \mathbf{e}, \mathbf{w} \rangle_{\Gamma_e} = \mathbf{0}$$

E-J power law

$$\rho(\mathbf{j}) = \frac{e_c}{j_c}\left(\frac{|\mathbf{j}|}{j_c}\right)^{n-1}$$

Discrete form for $\mathbf{h}$

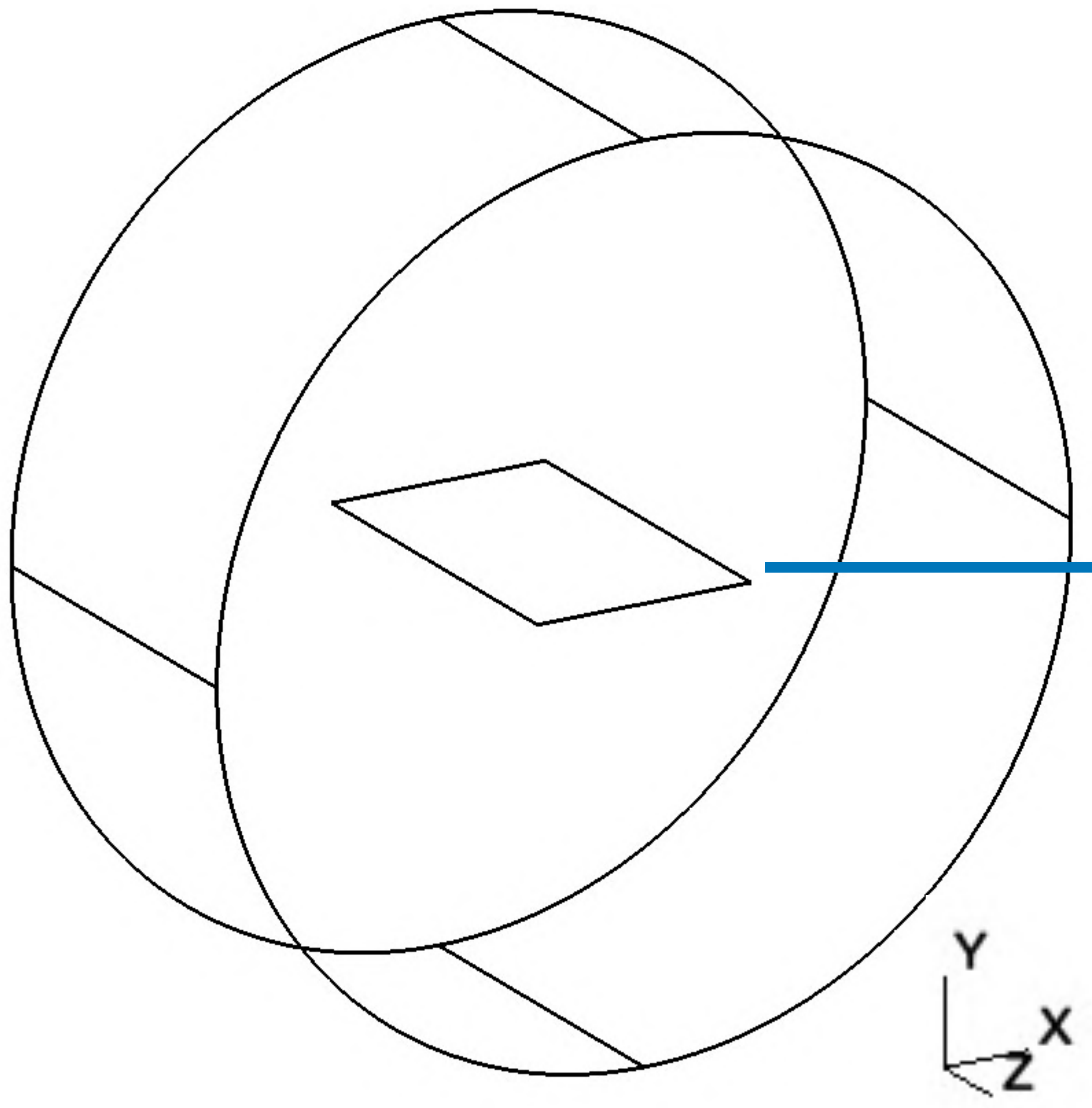
$$\mathbf{h} = \sum_{e \in \Omega_c} h_e \mathbf{w}_e + \sum_{n \in \Omega_c^C} -\phi_n \nabla w_n + \sum_{C_i} I_i \psi_i$$

where:

- $\mathbf{w}_e$ are the vector basis functions of each edge in Ω_c
- w_n are the nodal basis functions of each node in Ω_c^C
- ψ_i are discontinuous shape functions associated with a cut related to the current I_i to be imposed to each conducting subdomain i in Ω

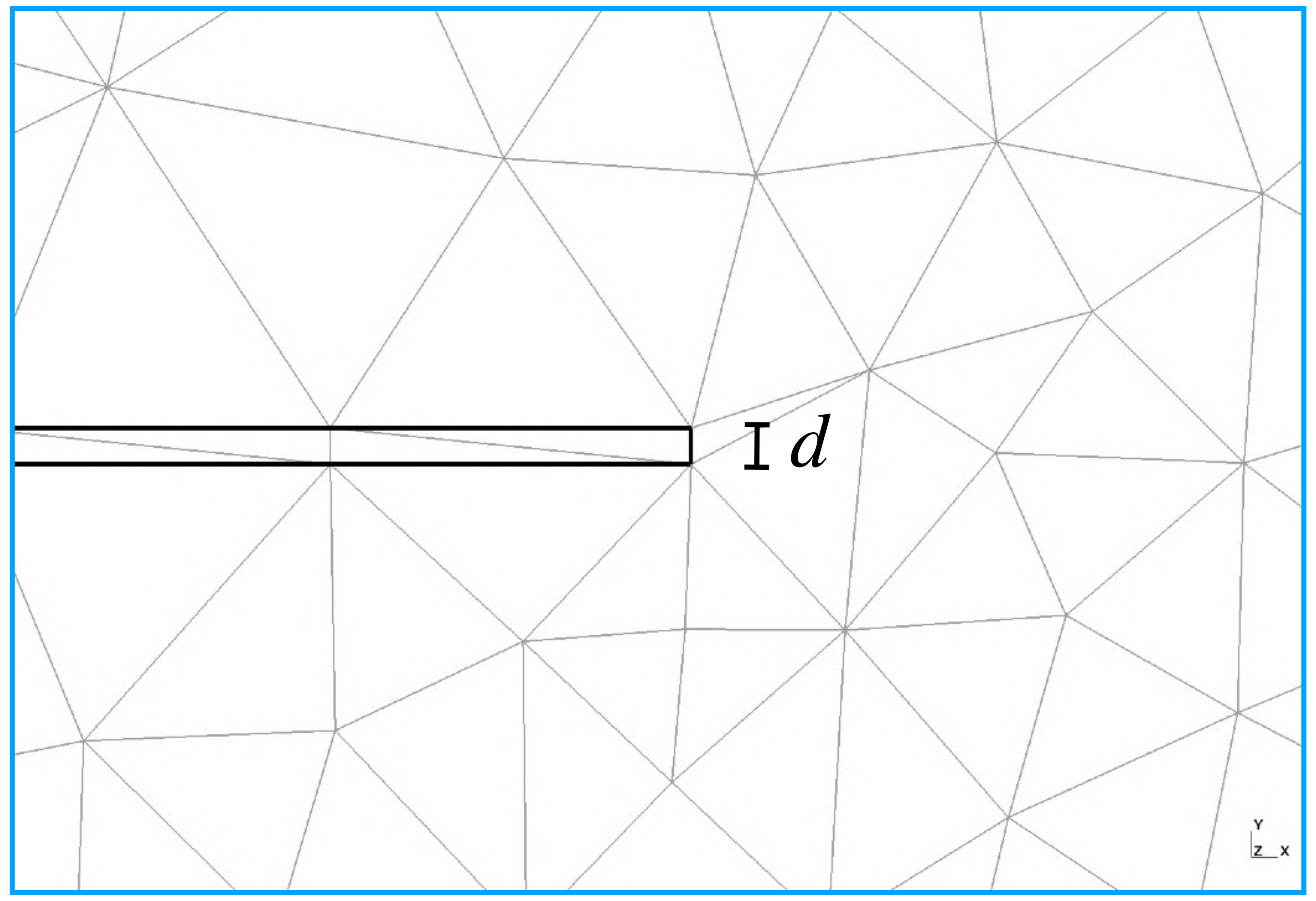
Modelling thin regions

discretization issues and the thin-shell (TS) model

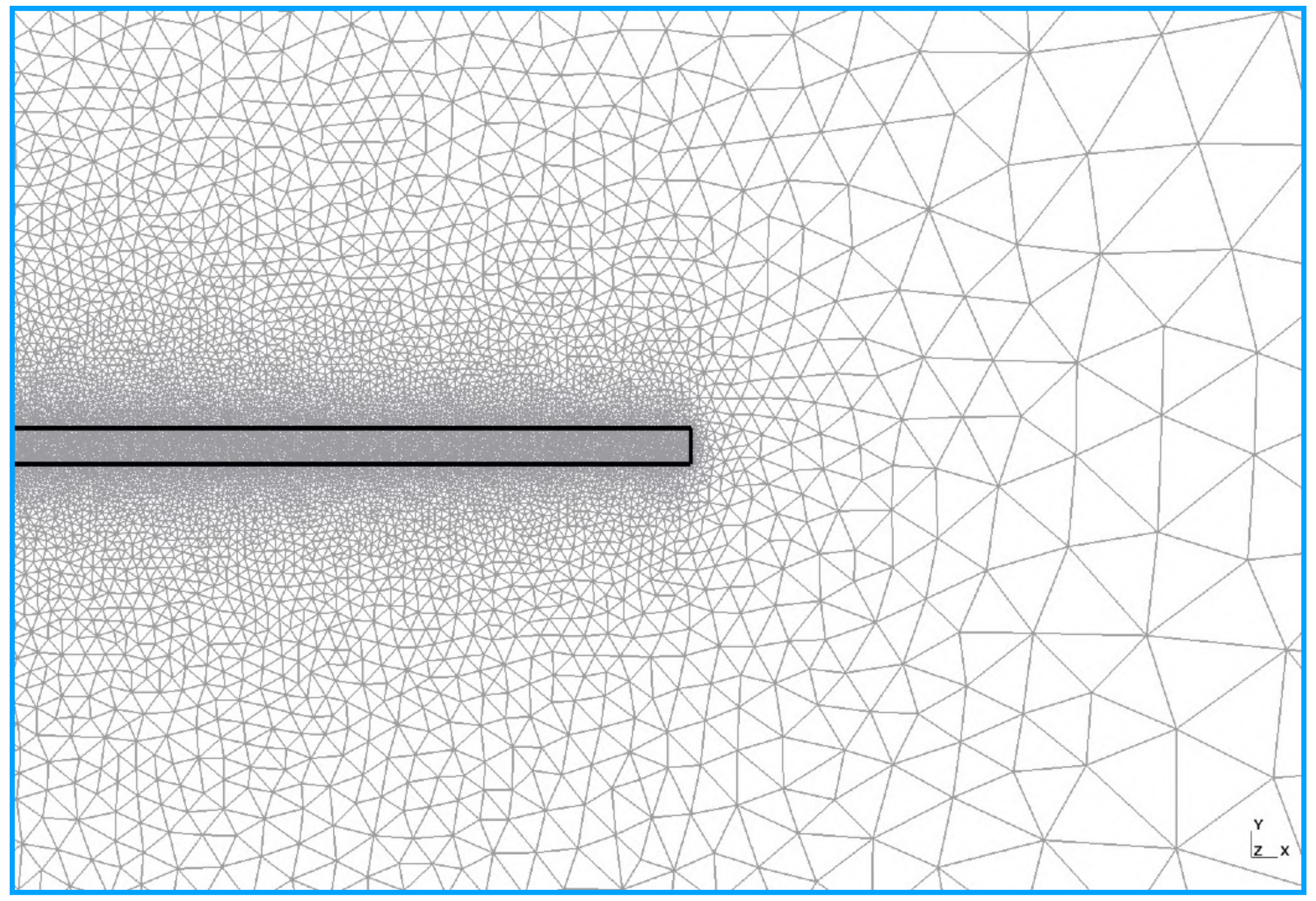


3-D example of a single thin region

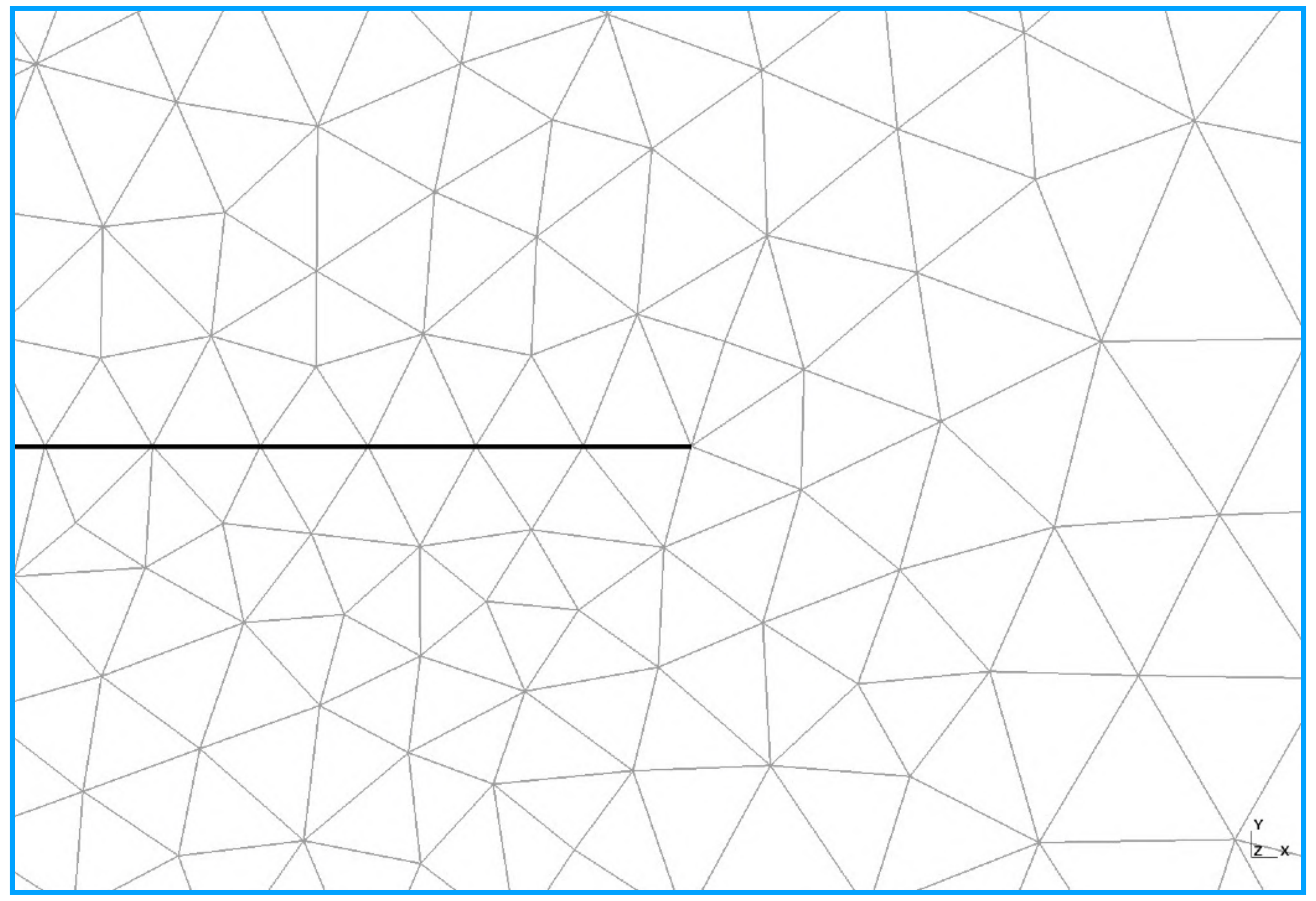
Coarser mesh



Fine mesh



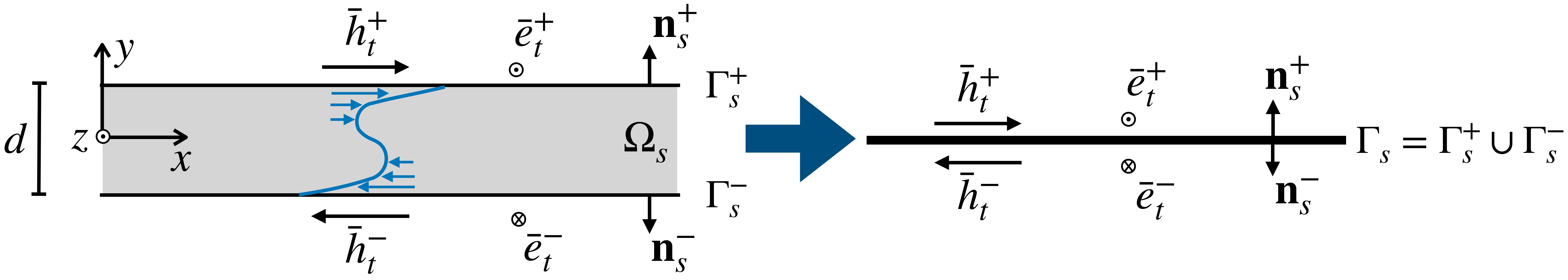
Thin-shell (TS) model



- Independent variables along top and bottom of the shell
- Discontinuity of normal and tangential fields
- Simpler mesh, with less elements

The classical thin-shell model for ohmic conductors

two impedance boundary conditions (IBCs) in harmonic regime [Mayergoyz,1995]



General case

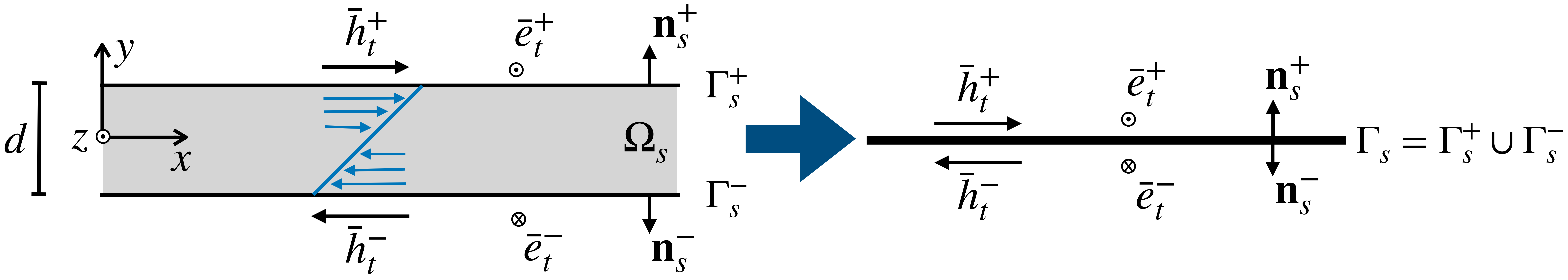
Ampère's law $$\bar{h}_t^+ - \bar{h}_t^- = \vec{\eta}_e(\bar{e}_t^+ + \bar{e}_t^-) \quad (1)$$

Faraday's law $$\bar{e}_t^+ - \bar{e}_t^- = \vec{\eta}_h(\bar{h}_t^+ + \bar{h}_t^-) \quad (2)$$

where $\vec{\eta}_h = \frac{i\omega\mu}{\vec{a}} \tanh\left(\frac{\vec{a}d}{2}\right)$ and $\vec{\eta}_e = \frac{\sigma}{\vec{a}} \tanh\left(\frac{\vec{a}d}{2}\right)$ with $\vec{a} = \frac{1+j}{\delta}, j = \sqrt{-1}, \delta = \sqrt{2/(\mu\sigma\omega)}, \omega = 2\pi f$

The classical thin-shell model for ohmic conductors

two impedance boundary conditions (IBCs) in harmonic regime [Mayergoyz,1995]



Case $\delta \gg d$

(1) $$\frac{\bar{h}_t^+ - \bar{h}_t^-}{d} = \sigma \frac{(\bar{e}_t^+ + \bar{e}_t^-)}{2}$$

(2) $$\frac{\bar{e}_t^+ - \bar{e}_t^-}{d} = -\,\partial_t \mu \frac{(\bar{h}_t^+ + \bar{h}_t^-)}{2}$$

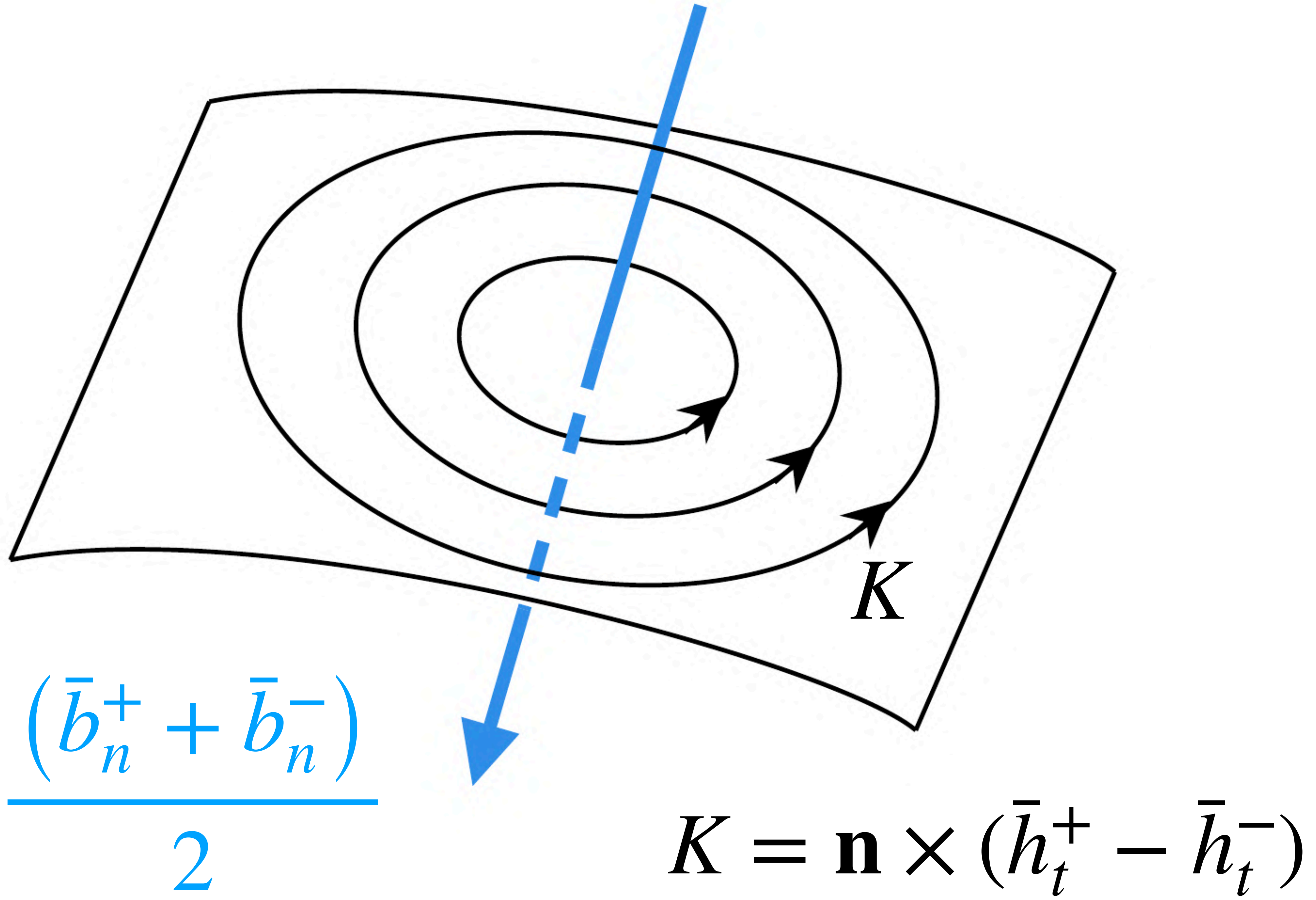
The classical thin-shell model for ohmic conductors

field's normal components [Krahenbuhl, 1993] : H-formulation example

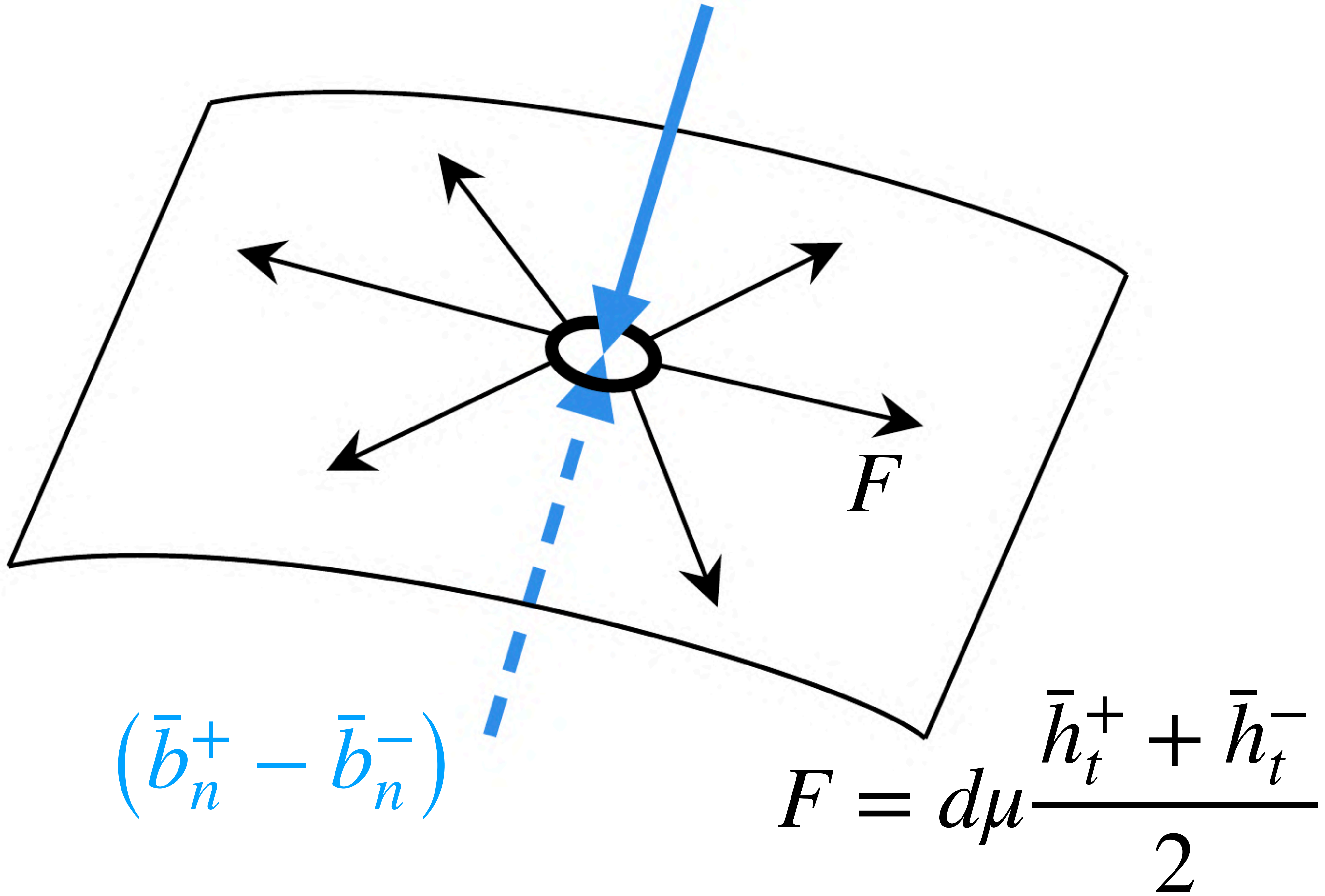
(1) $$\partial_t \sigma \frac{\left(\bar{b}_n^+ + \bar{b}_n^-\right)}{2} = \frac{\mathrm{div}_s(\bar{h}_t^+ - \bar{h}_t^-)}{d}$$

(2) $$\frac{\left(\bar{b}_n^+ - \bar{b}_n^-\right)}{d} = \mu \frac{\mathrm{div}_s(\bar{h}_t^+ + \bar{h}_t^-)}{2}$$

Faraday's law

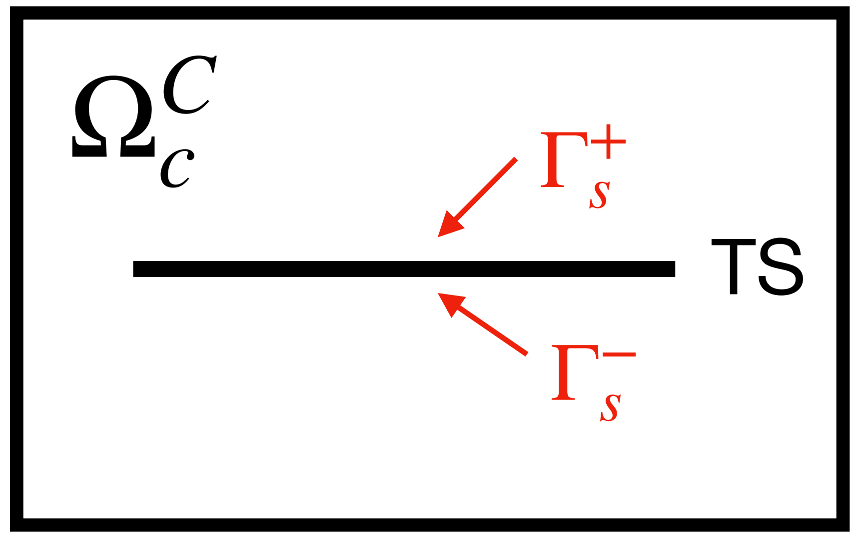


Magnetic flux continuity



The classical thin-shell model for ohmic conductors

connection to the global system of equations in dual formulations [Geuzaine, 2000]



Magnetic field formulation ($\mathbf{H}$):

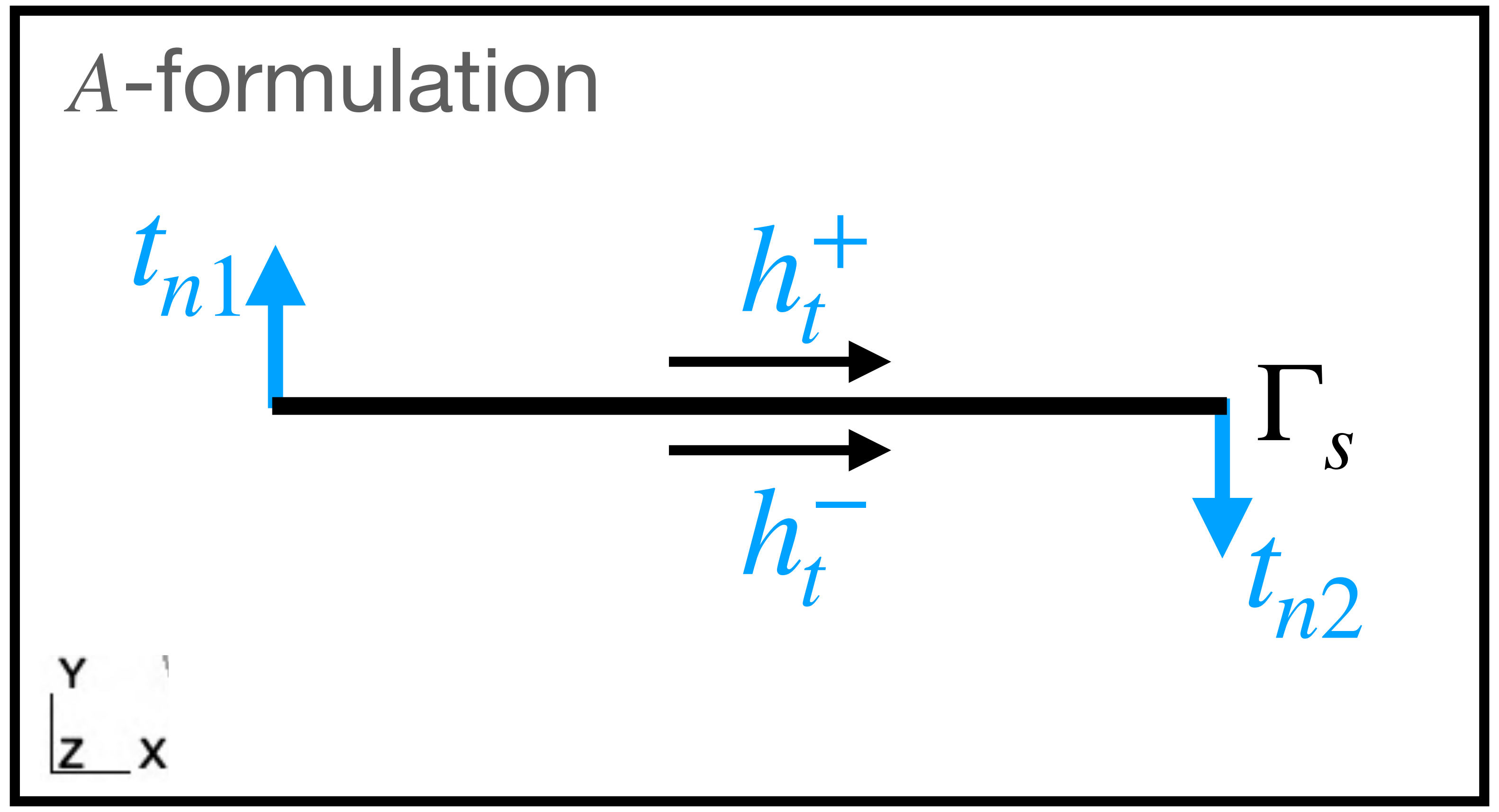
$$\left(\rho \nabla \times \mathbf{h}, \nabla \times \mathbf{w}\right)_{\Omega_c \backslash \Omega_s} + \left(\partial_t \mu \mathbf{h}, \mathbf{w}\right)_{\Omega \backslash \Omega_s} + \langle \mathbf{n}_s \times \mathbf{e}, \mathbf{w} \rangle_{\Gamma_s^+} - \langle \mathbf{n}_s \times \mathbf{e}, \mathbf{w} \rangle_{\Gamma_s^-} = 0$$

Magnetic vector potential formulation ($\mathbf{A}$):

$$(\nu \nabla \times \mathbf{a}, \nabla \times \mathbf{w})_{\Omega \backslash \Omega_s} + \left(\sigma \partial_t \mathbf{a}, \mathbf{w}\right)_{\Omega_c \backslash \Omega_s} + \langle \mathbf{n}_s \times \mathbf{h}, \mathbf{w} \rangle_{\Gamma_s^+} - \langle \mathbf{n}_s \times \mathbf{h}, \mathbf{w} \rangle_{\Gamma_s^-} = 0$$

T-A Formulation in terms of IBCs

approach proposed by [Zhang, 2017]



- T-formulation in 1-D $\partial_x(\rho \partial_x t_n) = \partial_t b_n$
- Current constraint of type $I = (t_{n1} - t_{n2})d$
- The current density $j_z = \partial_x t_n$

which is impressed in the A-formulation as $\dfrac{(h_t^+ - h_t^-)}{d} = j_z$

In addition:

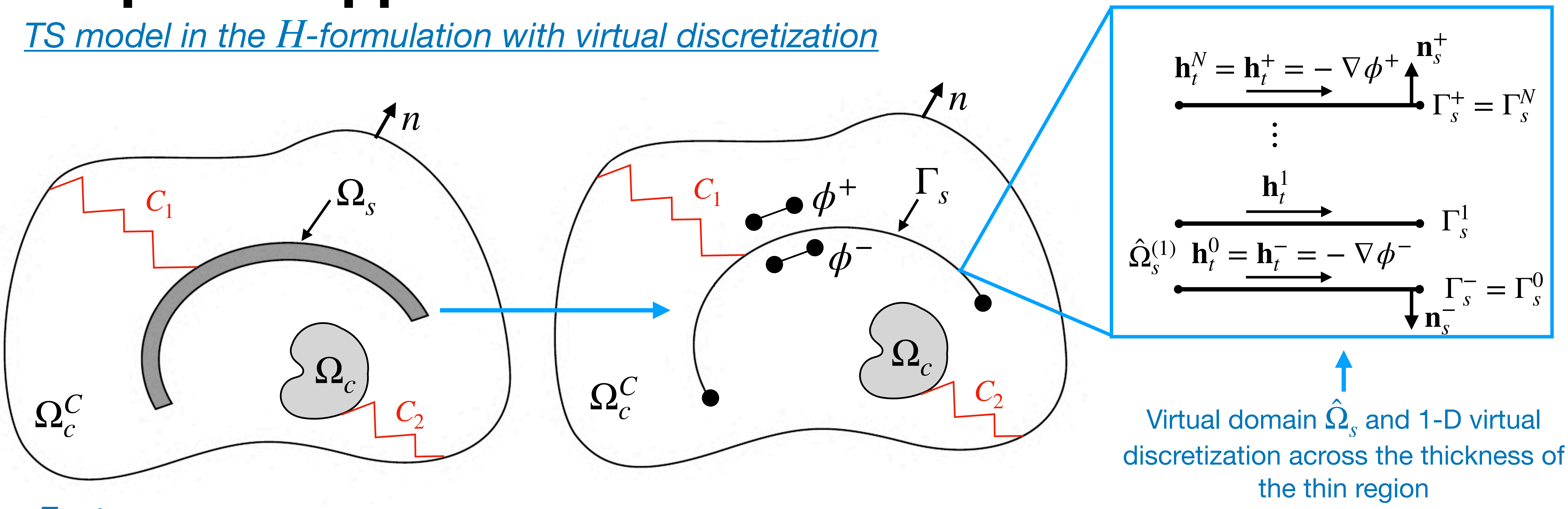
From COMSOL user guide $b_n = \mu \dfrac{h_n^+ + h_n^-}{2}$ using Faraday's law in the form $\partial_x e_t^\pm = -\mu \partial_t h_n^\pm$

we have $\boxed{\dfrac{h_t^+ - h_t^-}{d} = \sigma \dfrac{(e_t^+ + e_t^-)}{2}}$ → Impedance condition (1)

and $\boxed{e_t^+ - e_t^- = 0}$ → Impedance condition (2) $\dfrac{\bar{e}_t^+ - \bar{e}_t^-}{d} = -\partial_t \mu \dfrac{(\bar{h}_t^+ + \bar{h}_t^-)}{2}$ is not respected!

Proposed approach

TS model in the H-formulation with virtual discretization



Features:

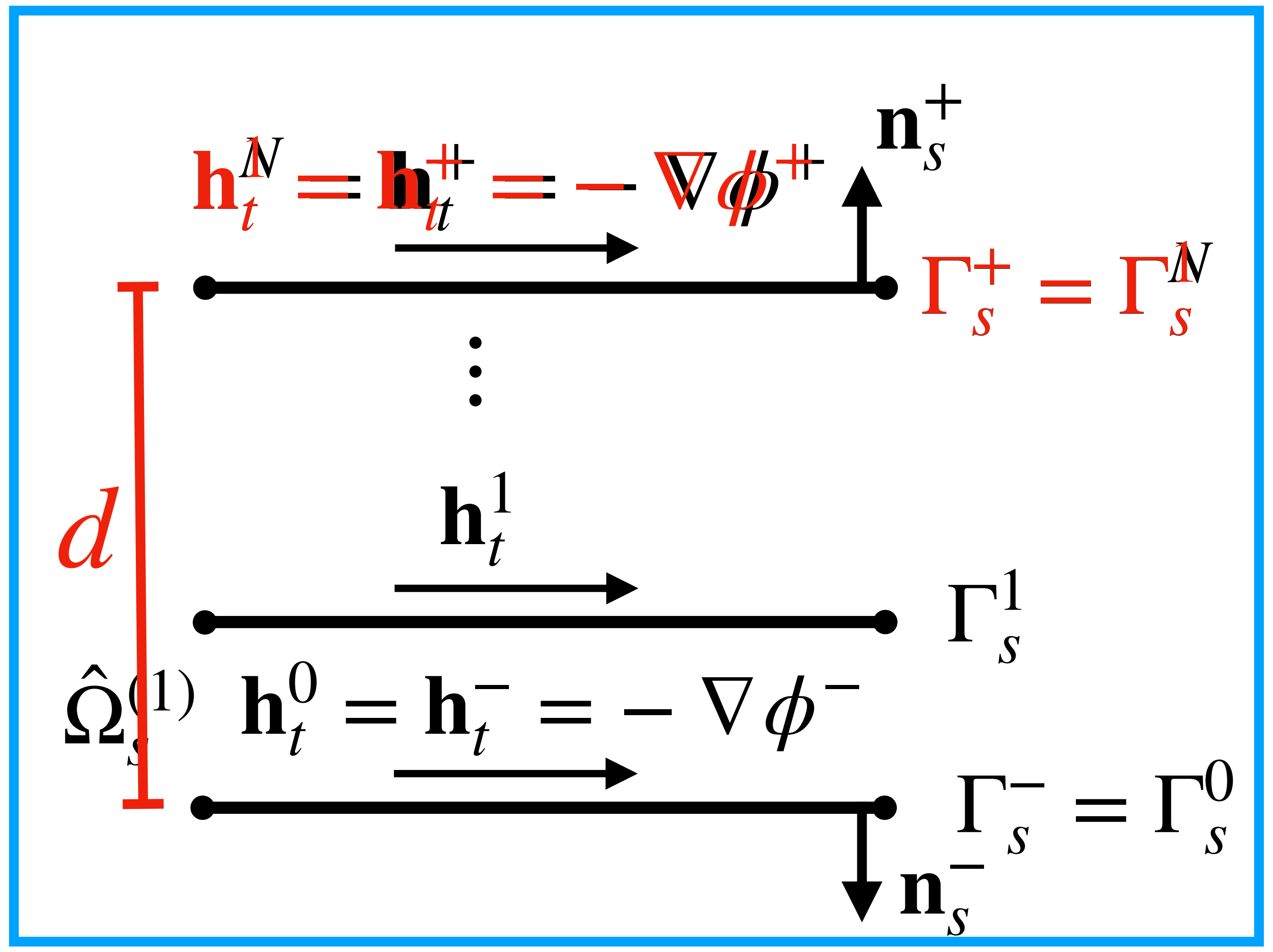
- Thin regions are represented by **lower dimensional geometries**
- Nodes, edges and surfaces are duplicated $\Rightarrow \Omega_c^C$ become multiply connected and $\mathbf{h}_t$ discontinuous over Γ_s
- Boundaries of the lower dimensional geometry are either a single point (2-D) or curve (3-D), i.e. no independent variables top/bottom
- Thick cuts (C_i) associated to each conductor Ω_i are determined purely from the mesh [Pellikka, 2013]

Proposed approach

IBCs derivation

The boundary conditions in the weak form are given by

$$\langle \mathbf{n}_s \times \mathbf{e}, \mathbf{w} \rangle_{\Gamma_s^+} - \langle \mathbf{n}_s \times \mathbf{e}, \mathbf{w} \rangle_{\Gamma_s^-} = \sum_{k=1}^{N} \sum_{j=1}^{2} \langle \rho^{(k)} \mathbf{h}_t^m, \mathbf{w} \rangle_{\Gamma_s^m} \cdot \mathcal{S}_{ij}^{(k)} + \sum_{k=1}^{N} \sum_{j=1}^{2} \partial_t \langle \mu^{(k)} \mathbf{h}_t^m, \mathbf{w} \rangle_{\Gamma_s^m} \cdot \mathcal{M}_{ij}^{(k)} \quad \forall\, i = 1,2$$



Virtual domain $\hat{\Omega}_s$ and 1-D virtual discretization across the thickness of the thin region

Remark

With $N = 1$, and evaluating $\mathcal{S}^{(1)}$ and $\mathcal{M}^{(1)}$ analytically we find the system

$$\begin{bmatrix} \mathbf{e}_t^+ \\ -\mathbf{e}_t^- \end{bmatrix} = \frac{\rho^{(1)}}{d} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{h}_t^+ \\ \mathbf{h}_t^- \end{bmatrix} + \frac{\partial_t \mu^{(1)} d}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \mathbf{h}_t^+ \\ \mathbf{h}_t^- \end{bmatrix}$$

which is equivalent to the IBCs in the classical TS model when $\delta \gg d$ and dual to the T-A-formulation

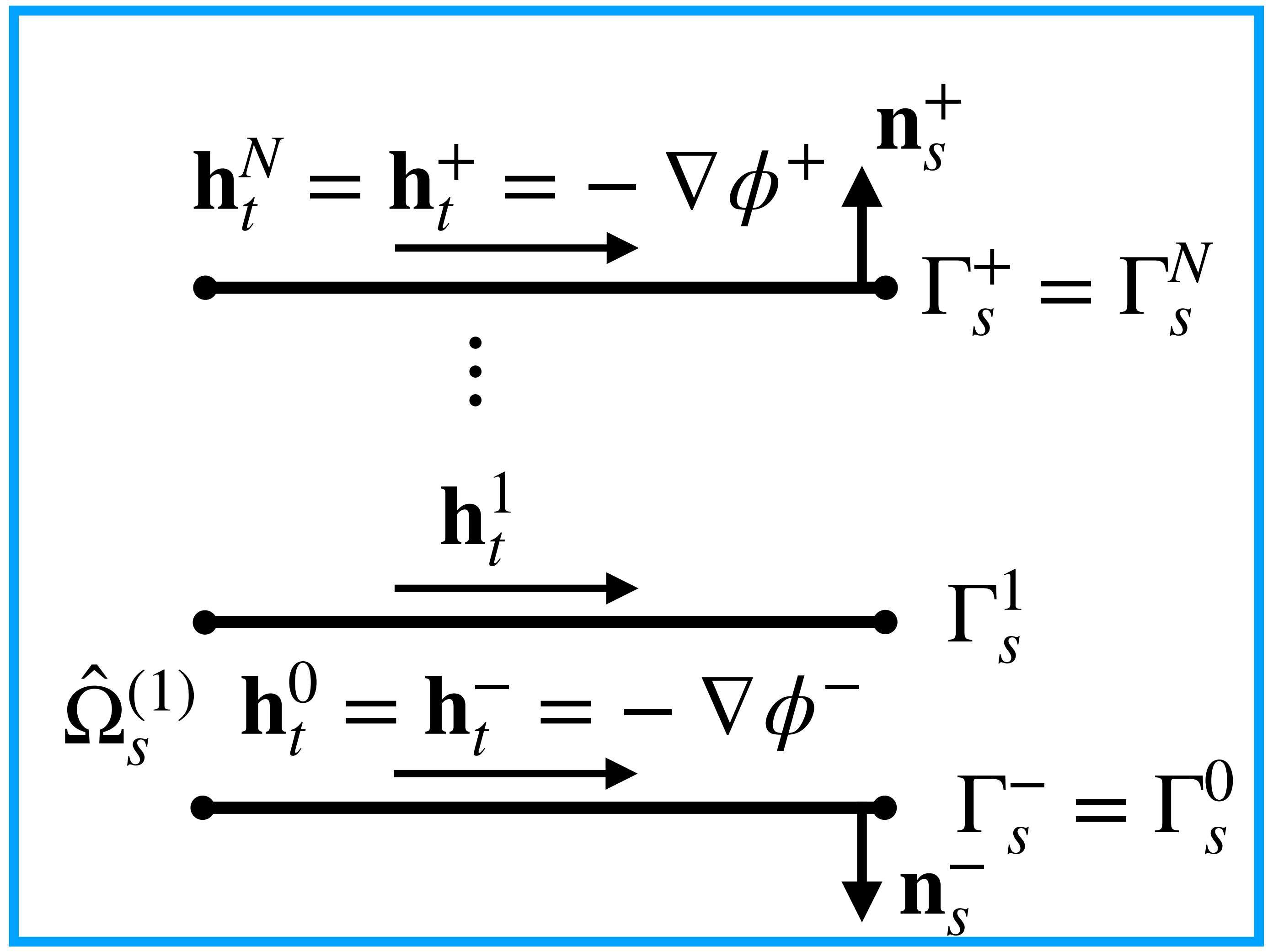
Proposed approach

current density and power-law treatment

The profile of $\mathbf{h}^{(k)}$ is linear across $\Delta y^{(k)} = d/N$ and

$$\mathbf{j}_z^{(k)} = \mathbf{n}_s \times \partial_y \mathbf{h}^{(k)} = \mathbf{n}_s \times \left(\frac{\mathbf{h}_t^k - \mathbf{h}_t^{k-1}}{\Delta y^{(k)}} \right)$$

is constant in $\Omega_s^{(k)}$



Virtual domain $\hat{\Omega}_s$ and 1-D virtual discretization across the thickness of the thin region

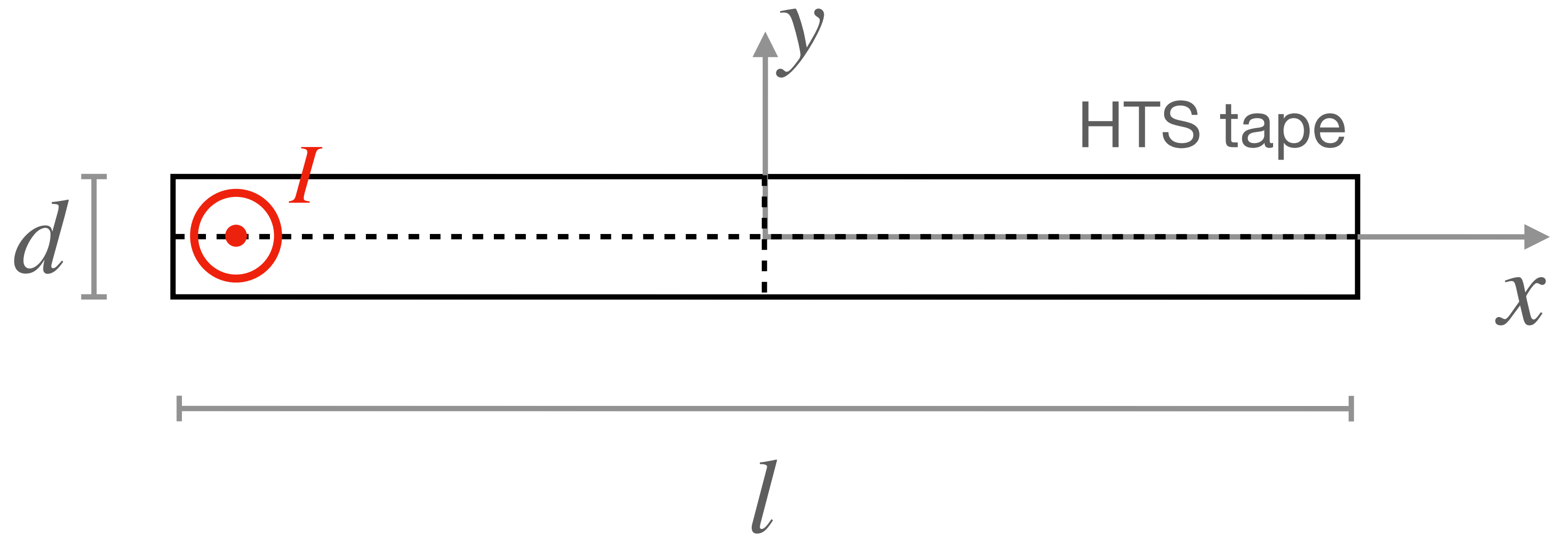
The local 1-D E-J power-law is

$$\rho^{(k)} = \frac{e_c}{j_c} \left(\frac{|\mathbf{n}_s \times (\mathbf{h}_t^k - \mathbf{h}_t^{k-1})|}{j_c \Delta y^{(k)}} \right)^{n-1}$$

The proposed TS model was implemented in **Gmsh** [Geuzaine, 2009] and solved using **GetDP** [Dular].

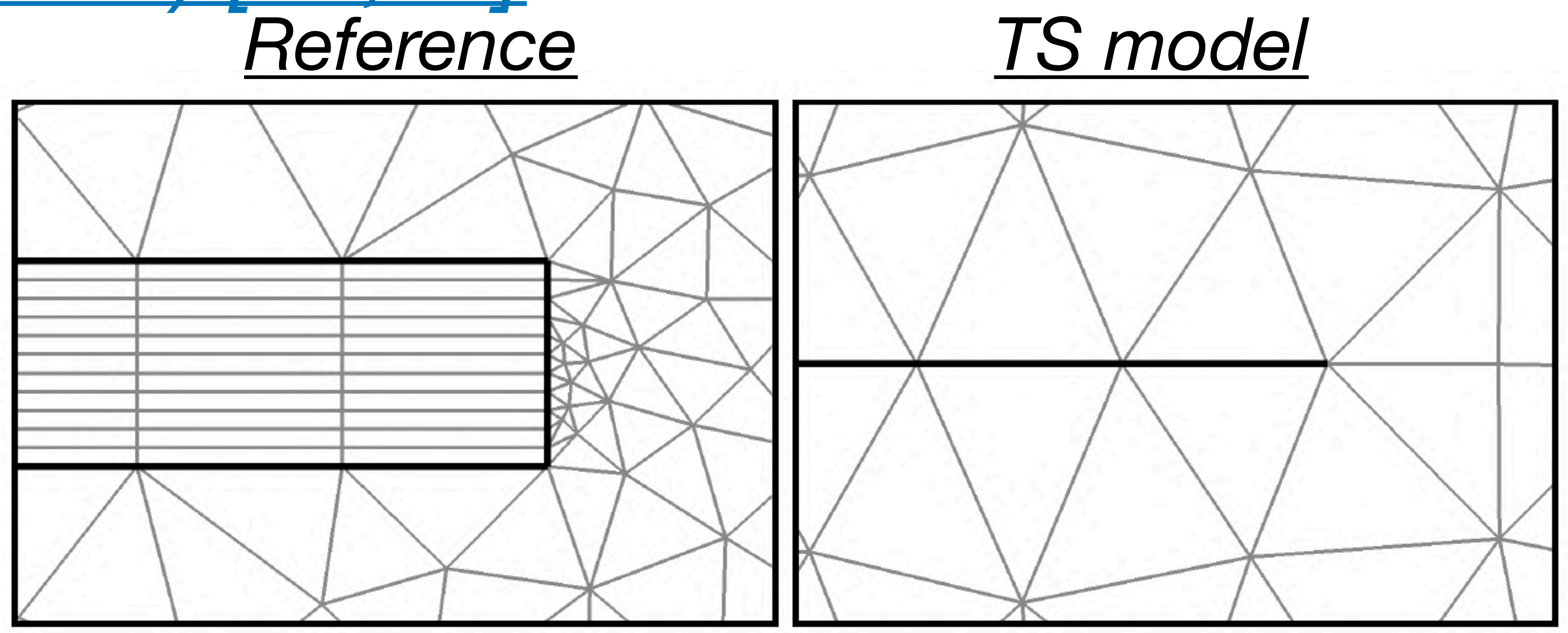
2-D Validation

single HTS tape (only the HTS layer is modelled using the TS model) [13,14]



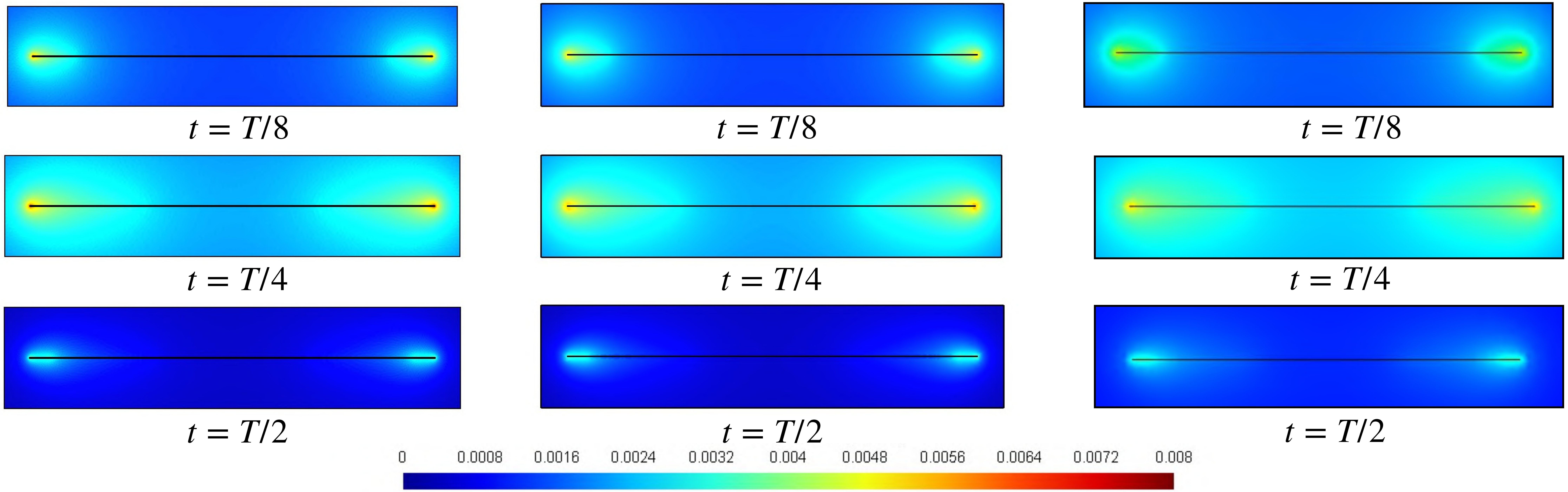
Simulations parameters:

- $e_c = 10^{-4}$ V/m
- $j_c = 5 \times 10^8$ A/m^2
- $n = 21$
- Imposed current: $0.9 I_c$
- $l = 4$ mm
- $d = 10\ \mu$m



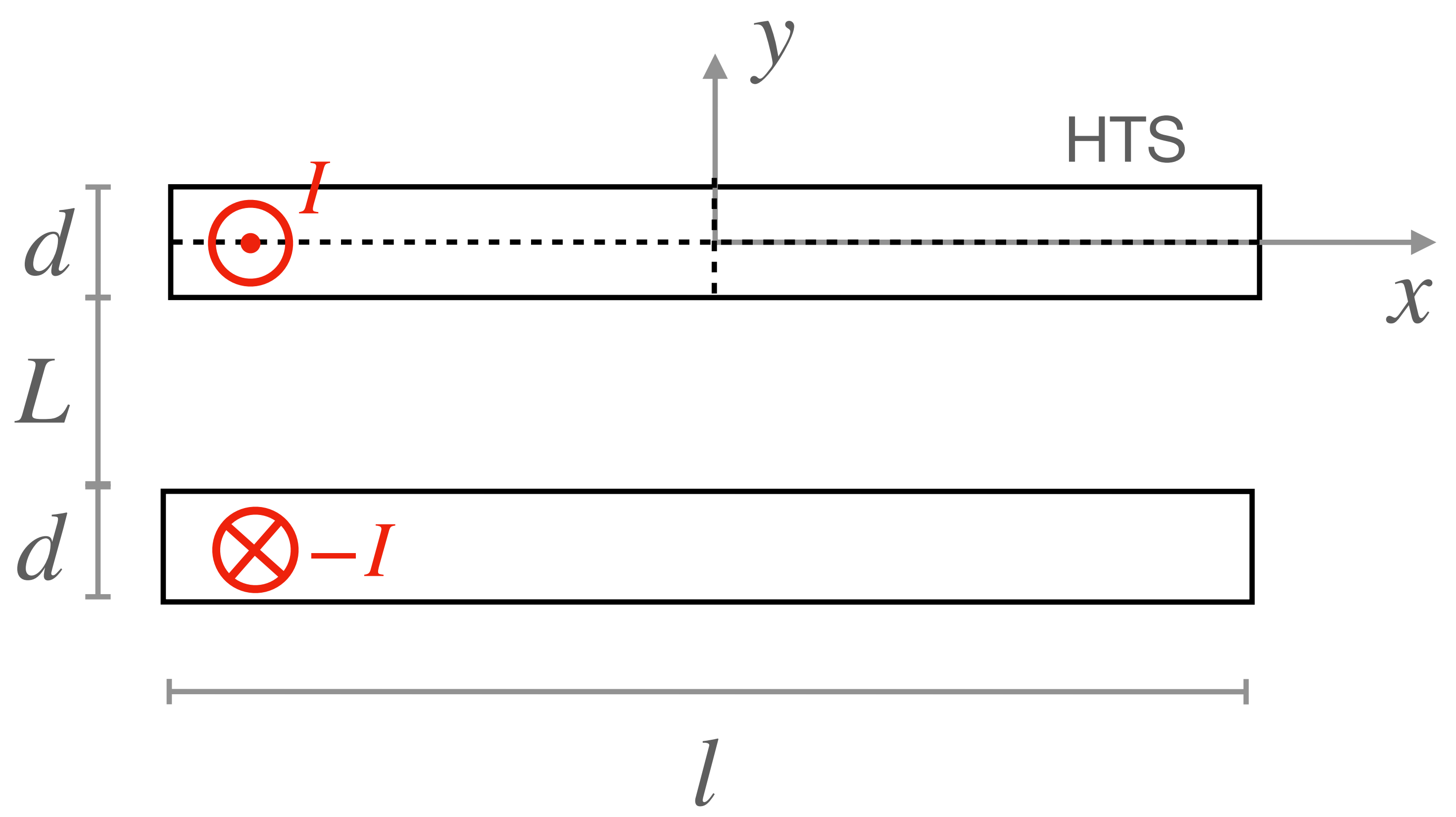
Magnetic flux density:

H-ϕ-formulation (reference) | *H-ϕ TS model ($N = 1$)* | *T-A-formulation*



2-D Validation

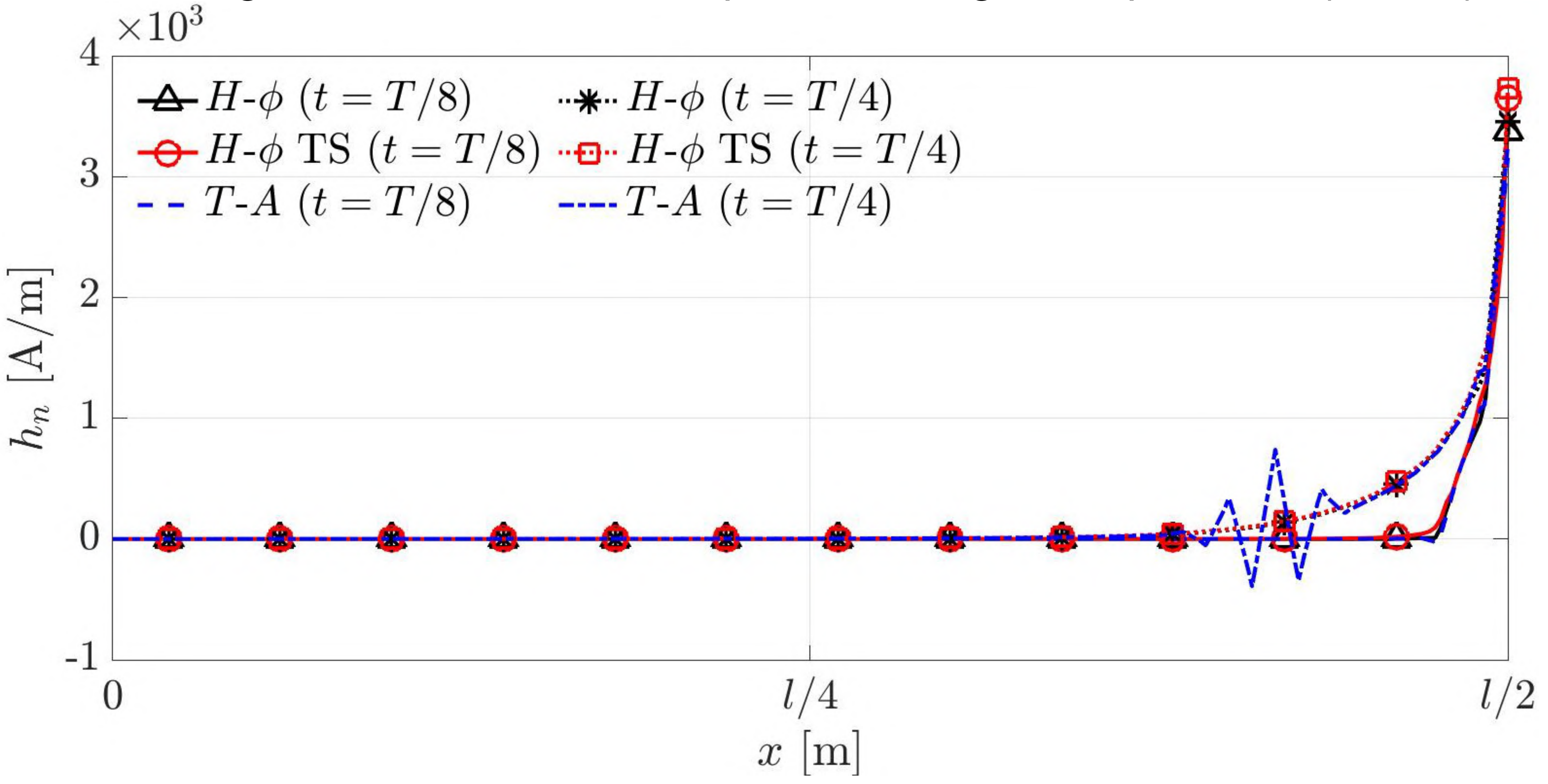
two closely packed tapes carrying anti-parallel currents [Grilli, 2010]



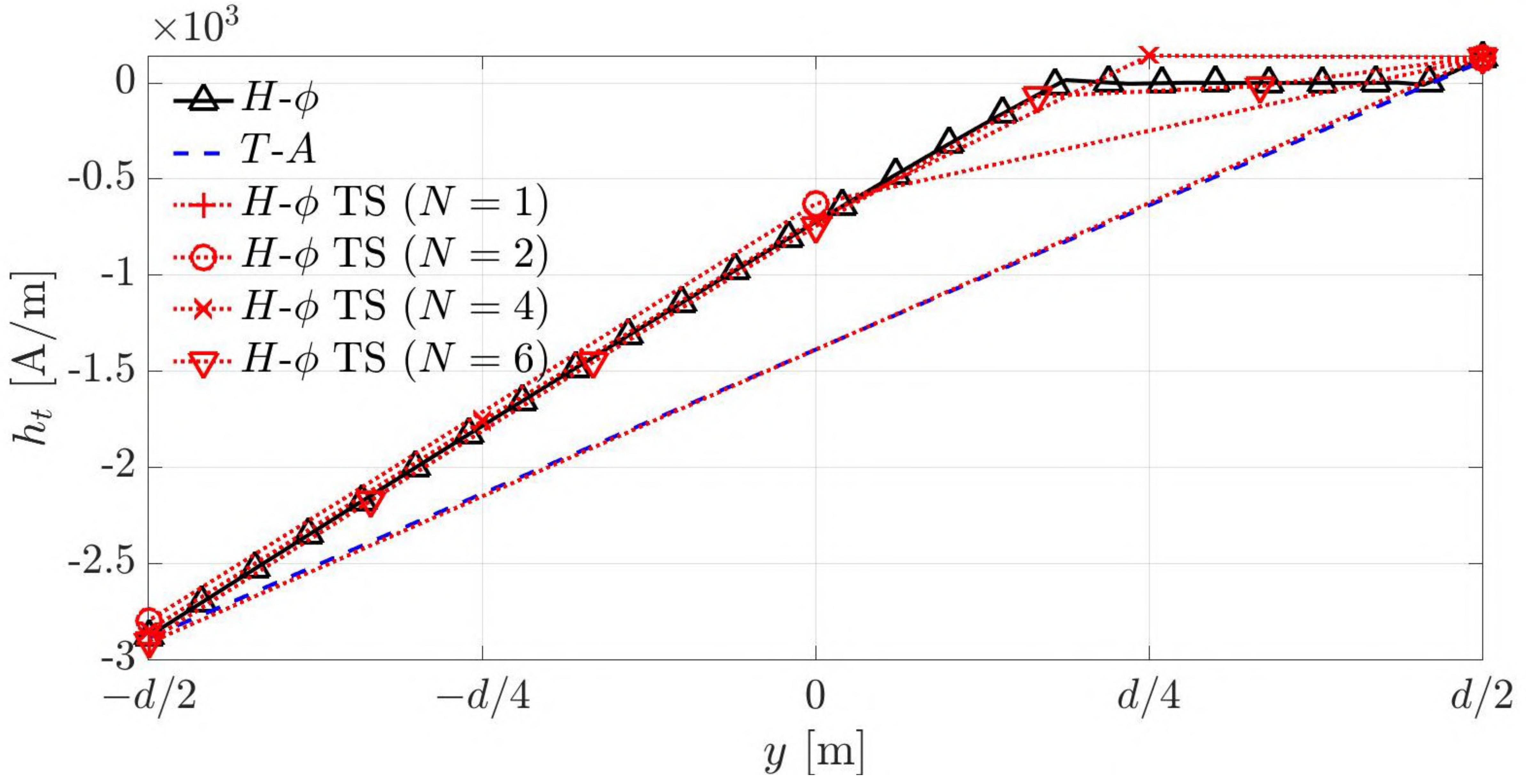
Simulations parameters:

- $e_c = 10^{-4}$ V/m
- $j_c = 5 \times 10^8$ A/m^2
- $n = 21$
- Imposed current: $0.9 I_c$
- $l = 4$ mm
- $d = 10\ \mu$m
- $L = 250\ \mu$m

Magnetic field normal component along the tape width ($N = 1$)



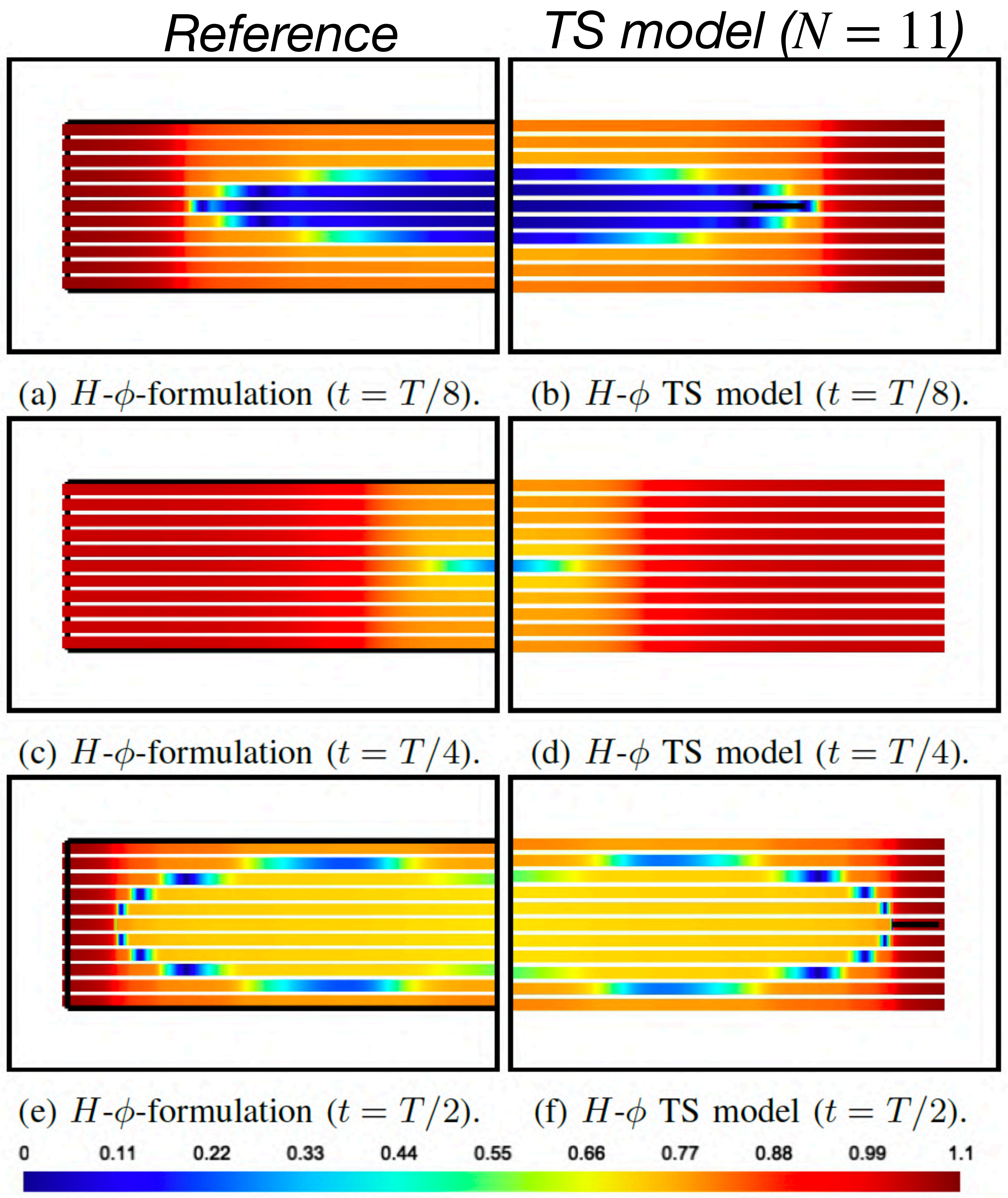
Magnetic field tangential component across the tape thickness



2-D Validation

current density distribution in the tape

Single tape

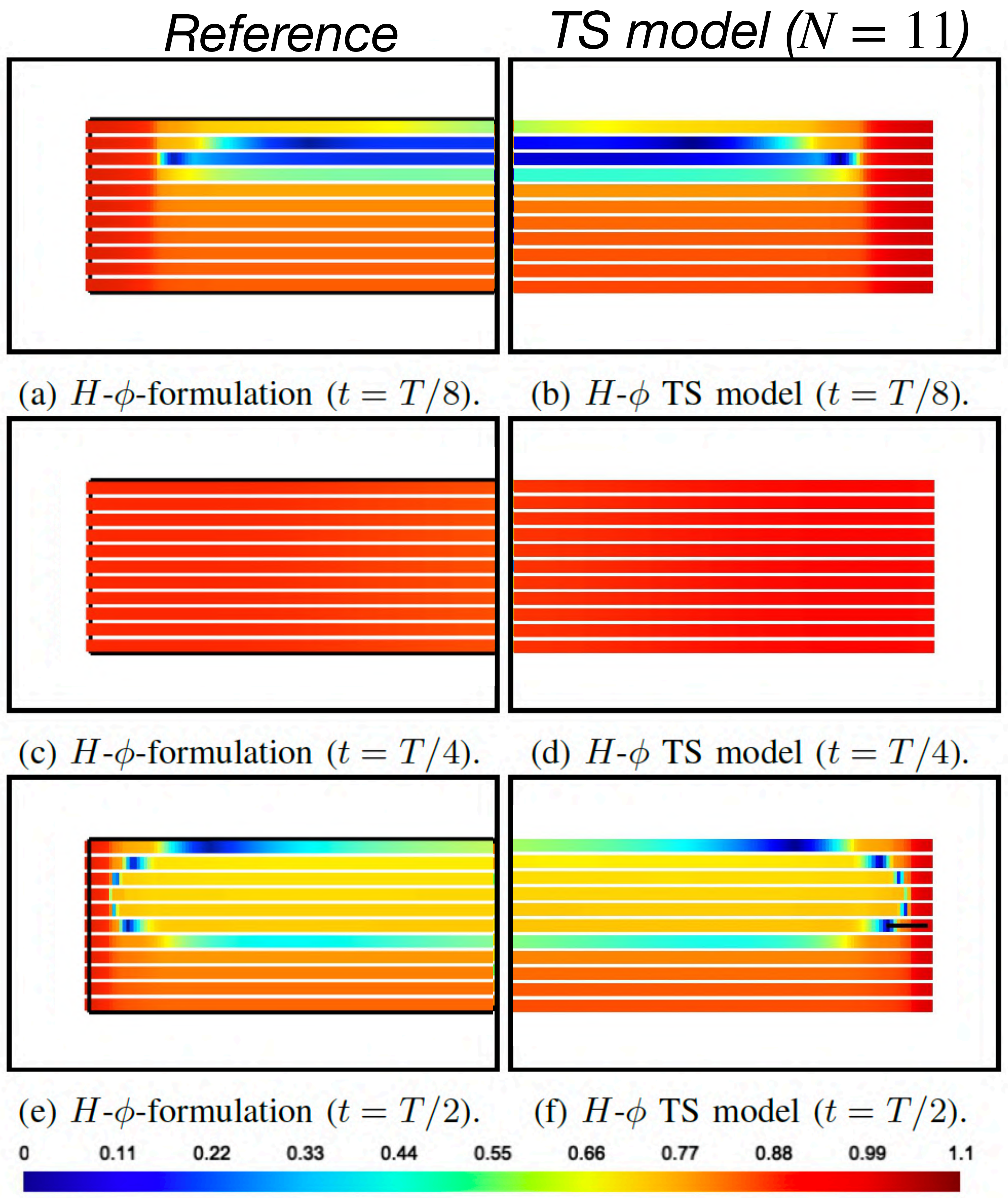


(a) H-ϕ-formulation ($t = T/8$). (b) H-ϕ TS model ($t = T/8$).

(c) H-ϕ-formulation ($t = T/4$). (d) H-ϕ TS model ($t = T/4$).

(e) H-ϕ-formulation ($t = T/2$). (f) H-ϕ TS model ($t = T/2$).

Closely packed tapes



(a) H-ϕ-formulation ($t = T/8$). (b) H-ϕ TS model ($t = T/8$).

(c) H-ϕ-formulation ($t = T/4$). (d) H-ϕ TS model ($t = T/4$).

(e) H-ϕ-formulation ($t = T/2$). (f) H-ϕ TS model ($t = T/2$).

2-D Validation

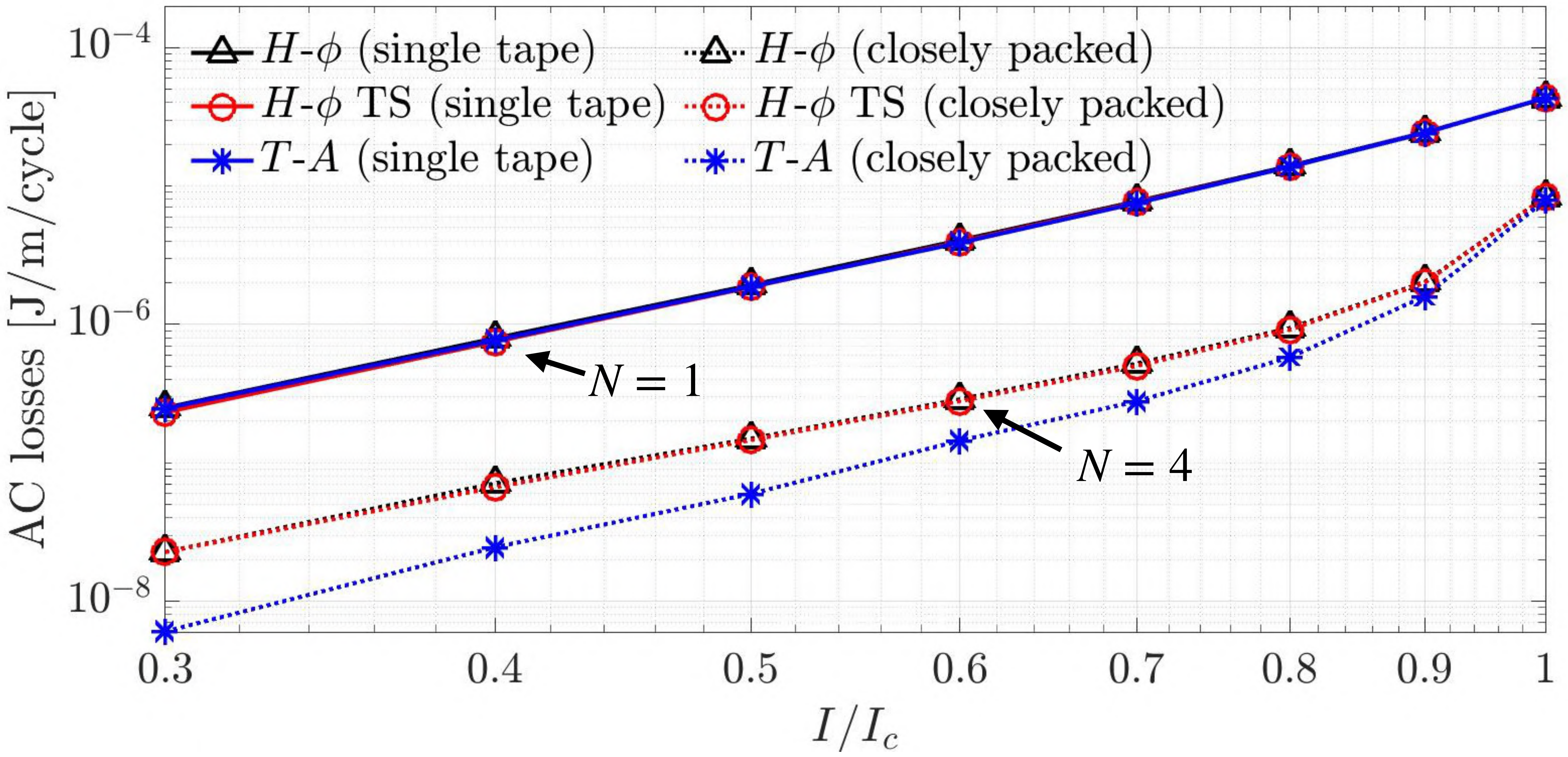
AC losses computation

Instantaneous AC loss:

$$\mathscr{L}(t) = \sum_{k=1}^{N} \int_{\Gamma_s^k} \rho^{(k)} H^{(k)T} \mathcal{S}^{(k)} H^{(k)} d\Gamma$$

where $H^{(k)} = \begin{bmatrix} h_t^k \\ h_t^{k-1} \end{bmatrix}$

Total AC losses per cycle as a function of the transport current



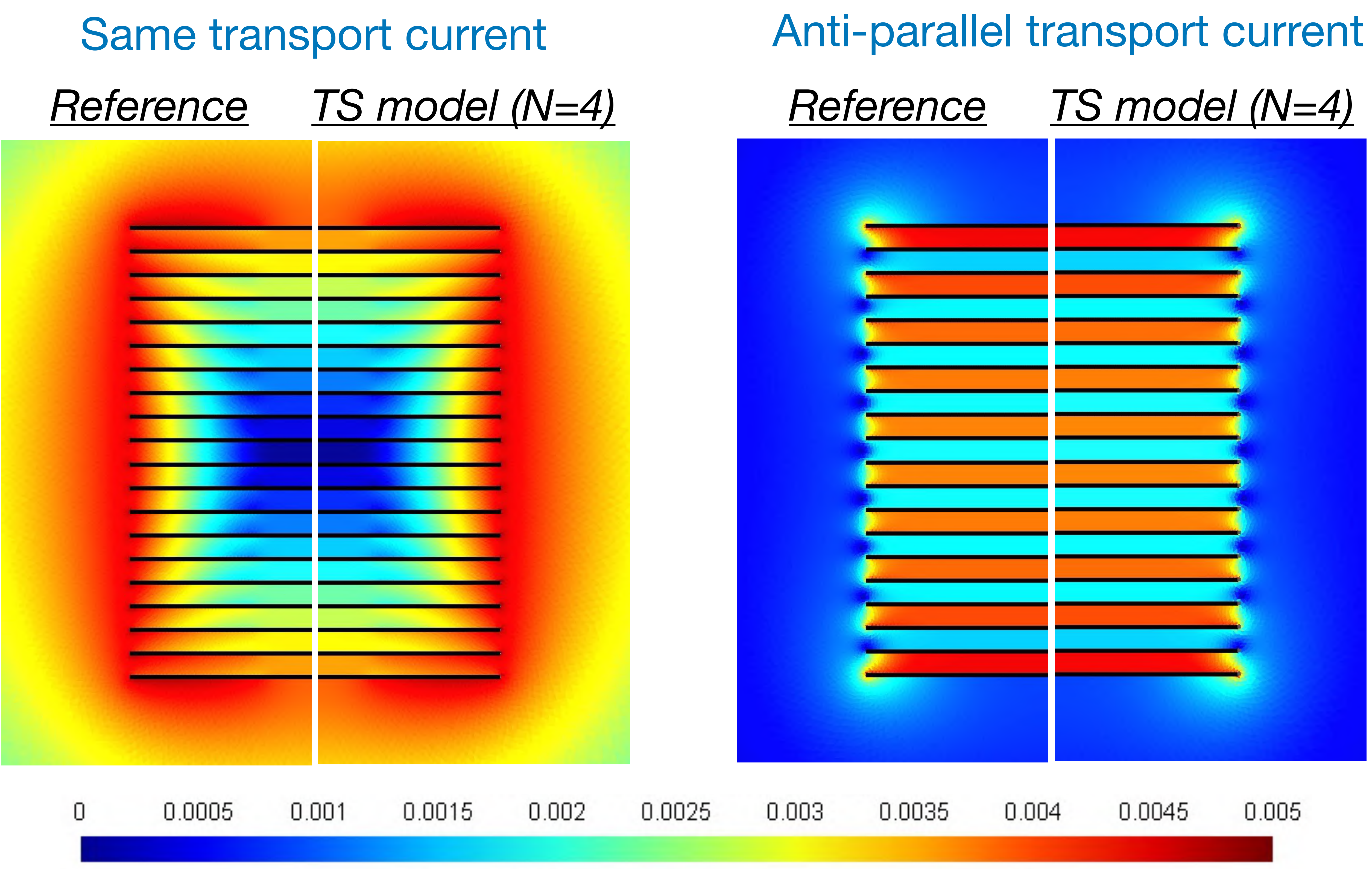
Number of DoFs and CPU time with different N values

	H-ϕ	TS ($N = 1$)	TS ($N = 2$)	TS ($N = 4$)	TS ($N = 6$)	TS ($N = 11$)
DoFs	33188	18727	19127	19927	20727	22727
Time	4015.65s	553.57s	563.34s	806.30s	1252.21s	3079.08s

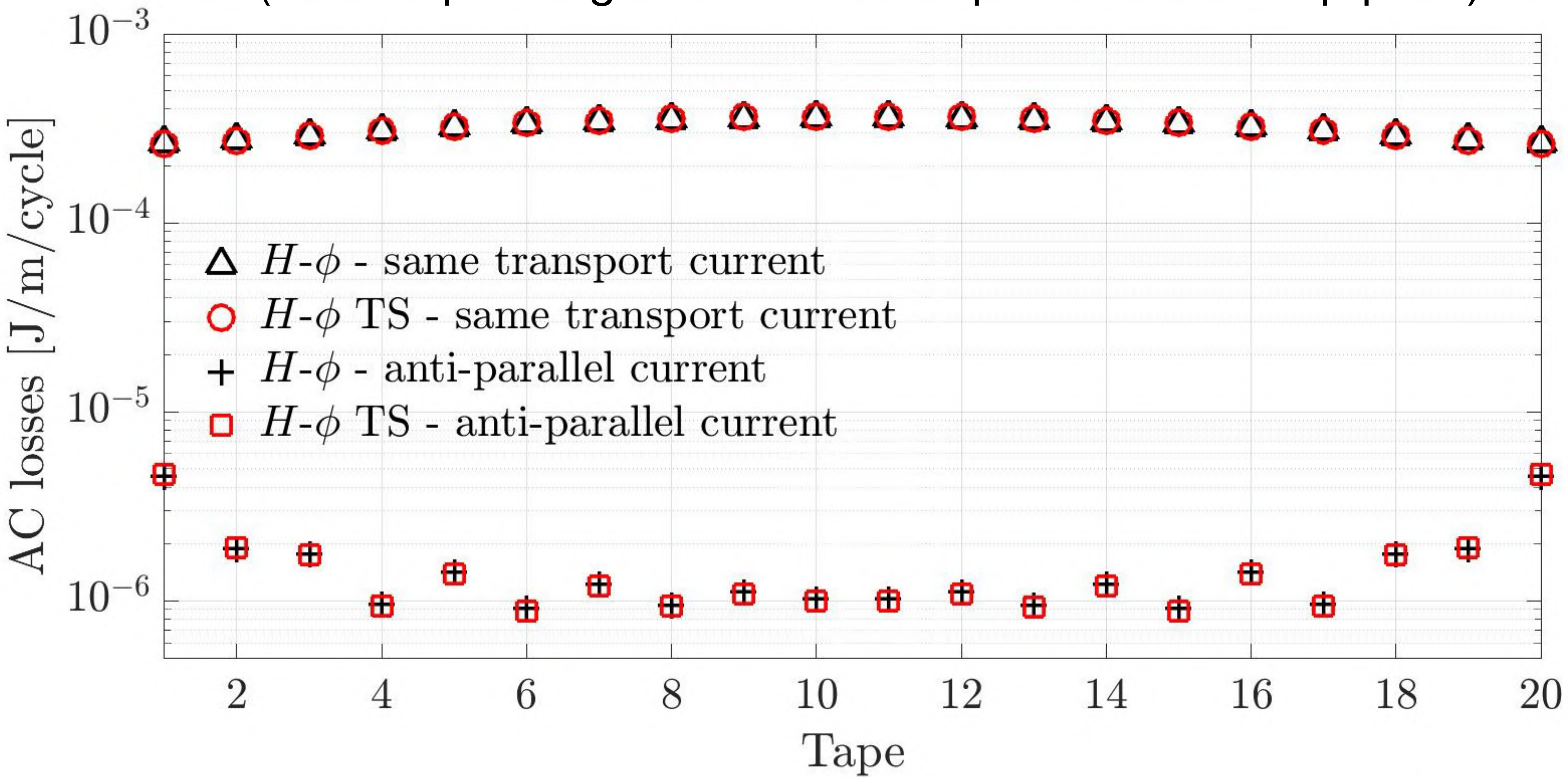
2-D Application #1

infinitely long representation of a racetrack coil

Magnetic flux density

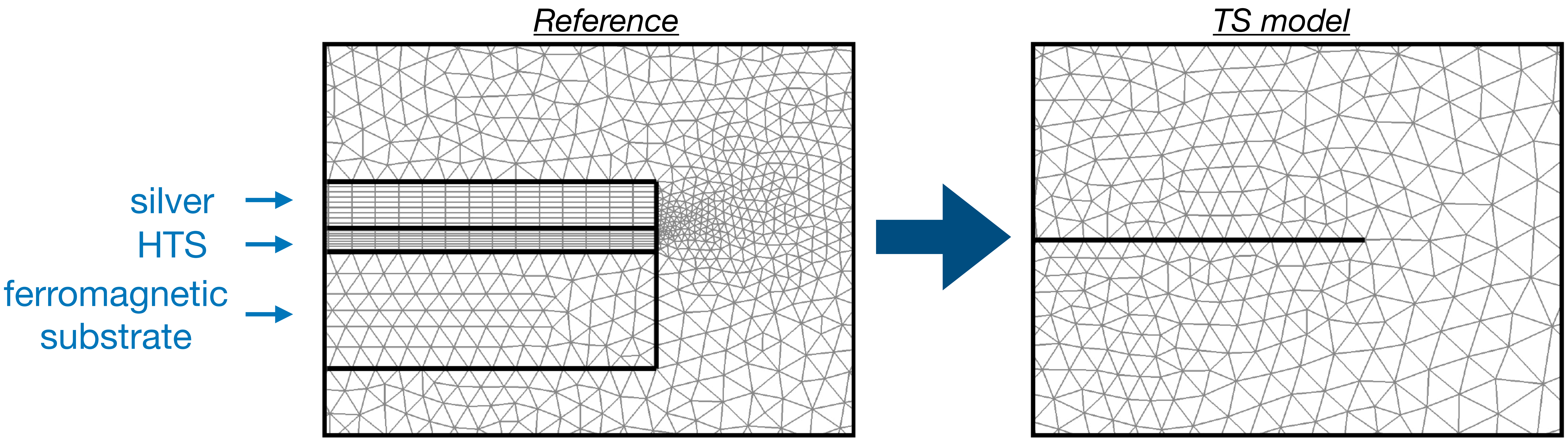


Total AC losses per cycle in each tape with $I = 0.9I_c$
(1 corresponding to the bottom tape and 20 the top pate)

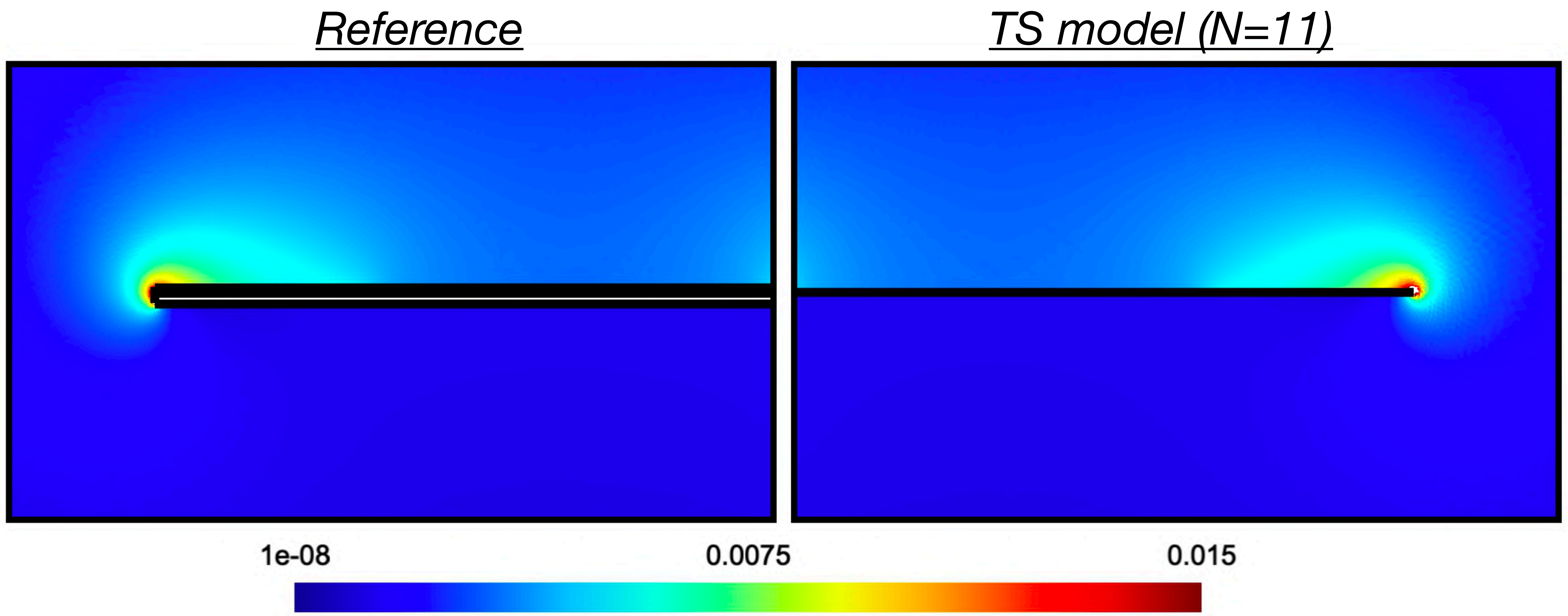


2-D Application #2

full HTS tape: substrate + HTS + silver stabilizer



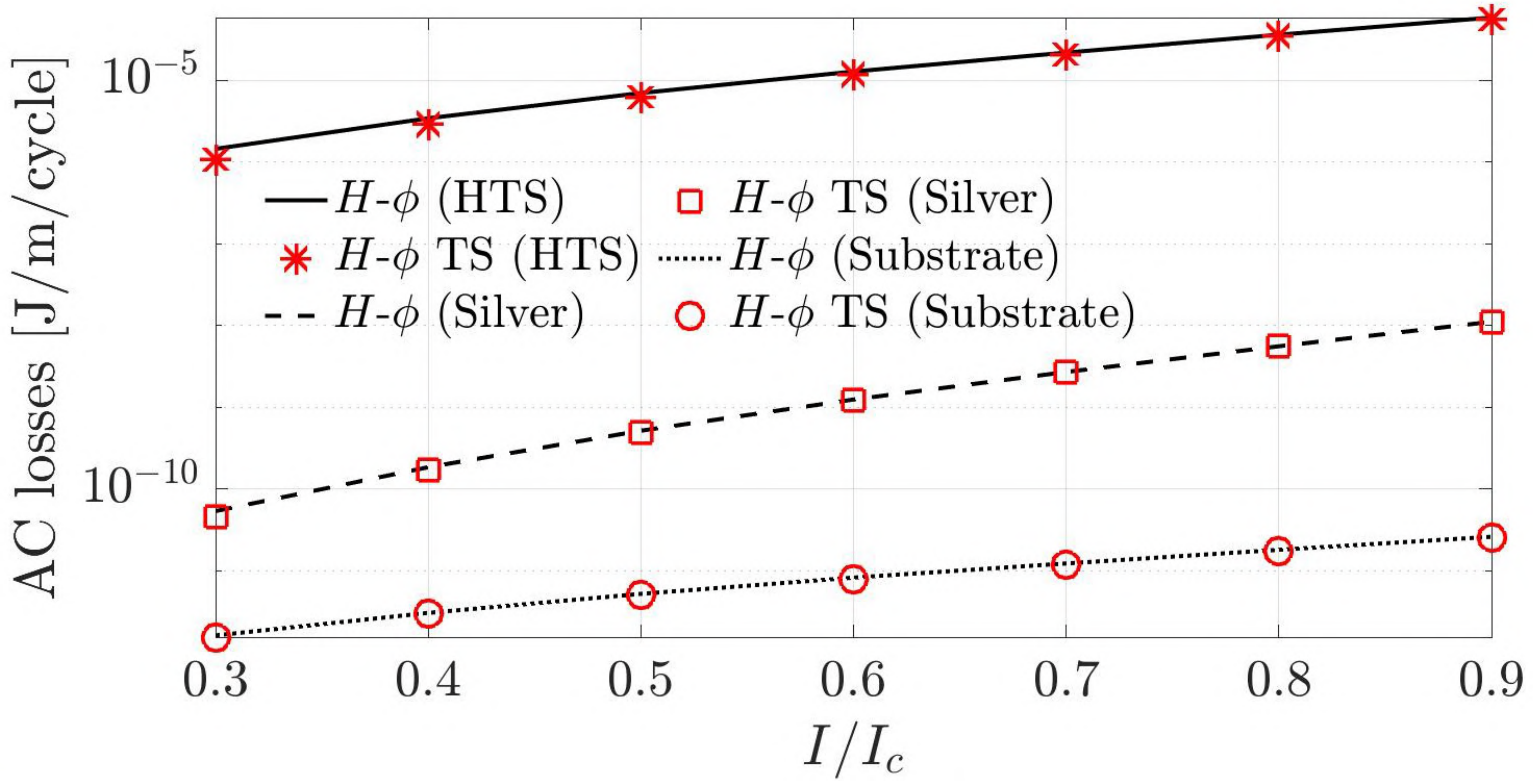
Magnetic flux density:



2-D Application #2

full HTS tape: substrate + HTS + silver stabilizer

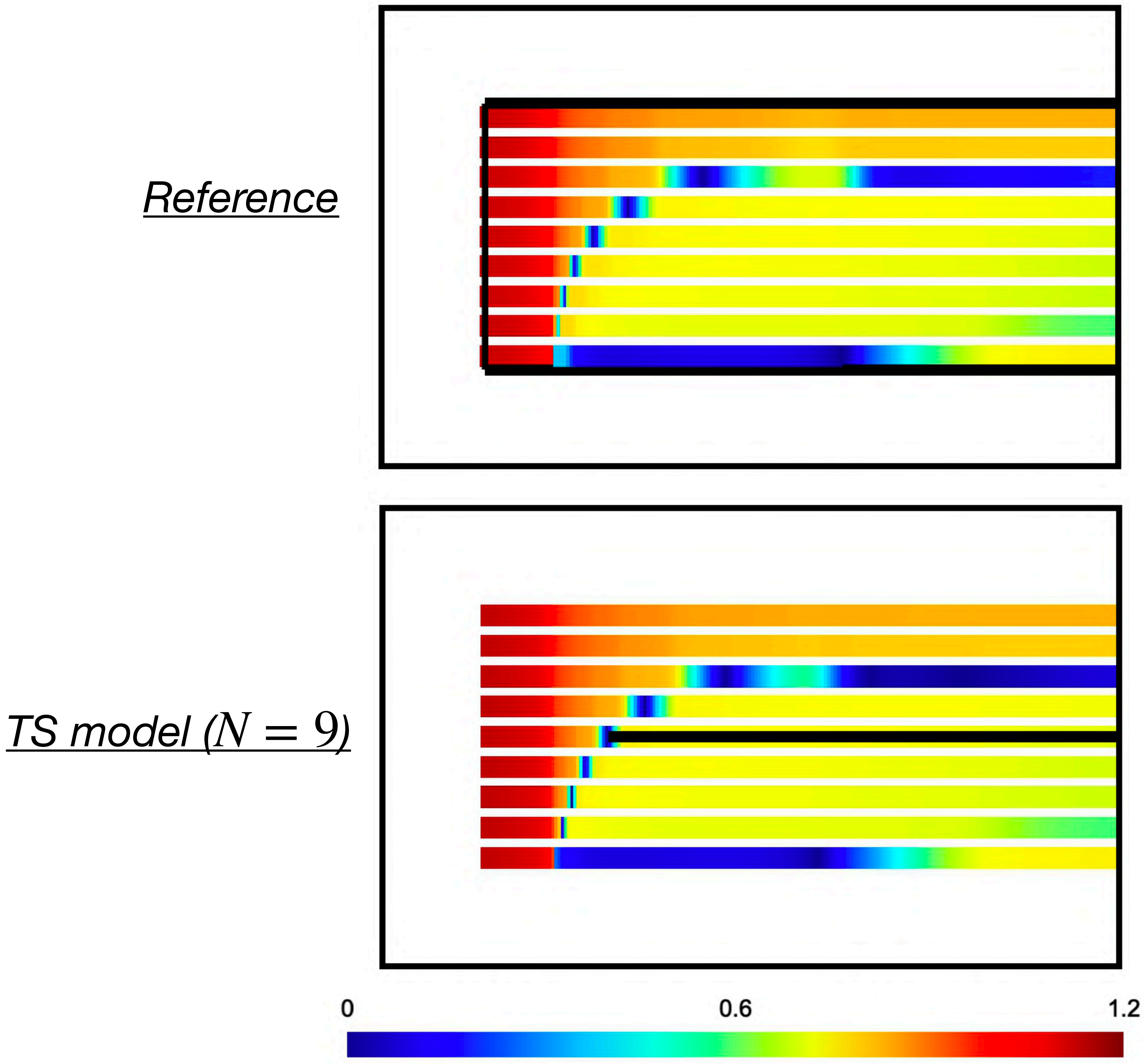
Total AC losses per cycle as a function of the transport current



Number of DoFs and CPU time

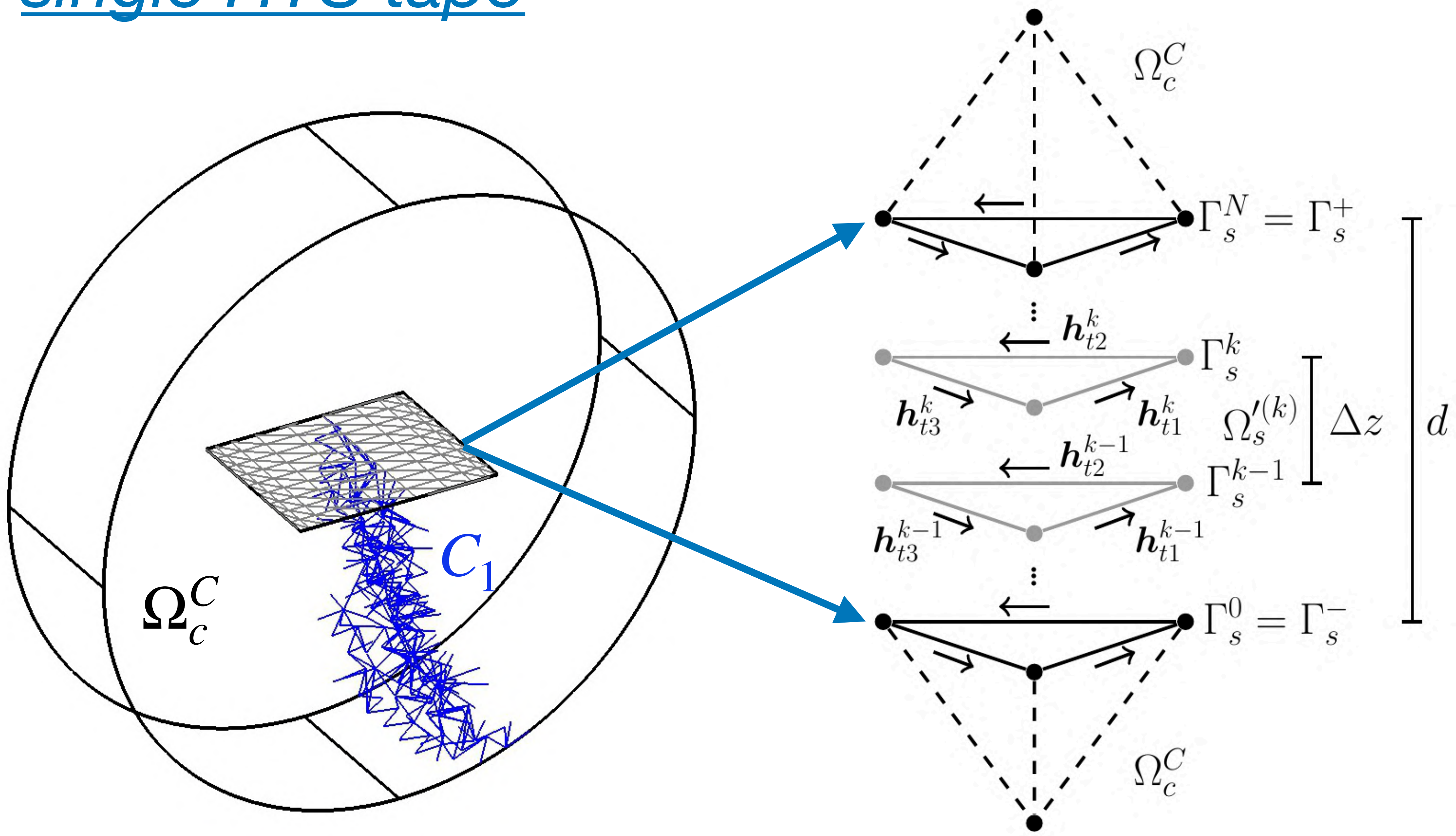
	DoFs	CPU time [s]
H-ϕ	58,411	9152.39
H-ϕ TS	41,025	4681.31

Current density in the HTS layer at $t = T/2$

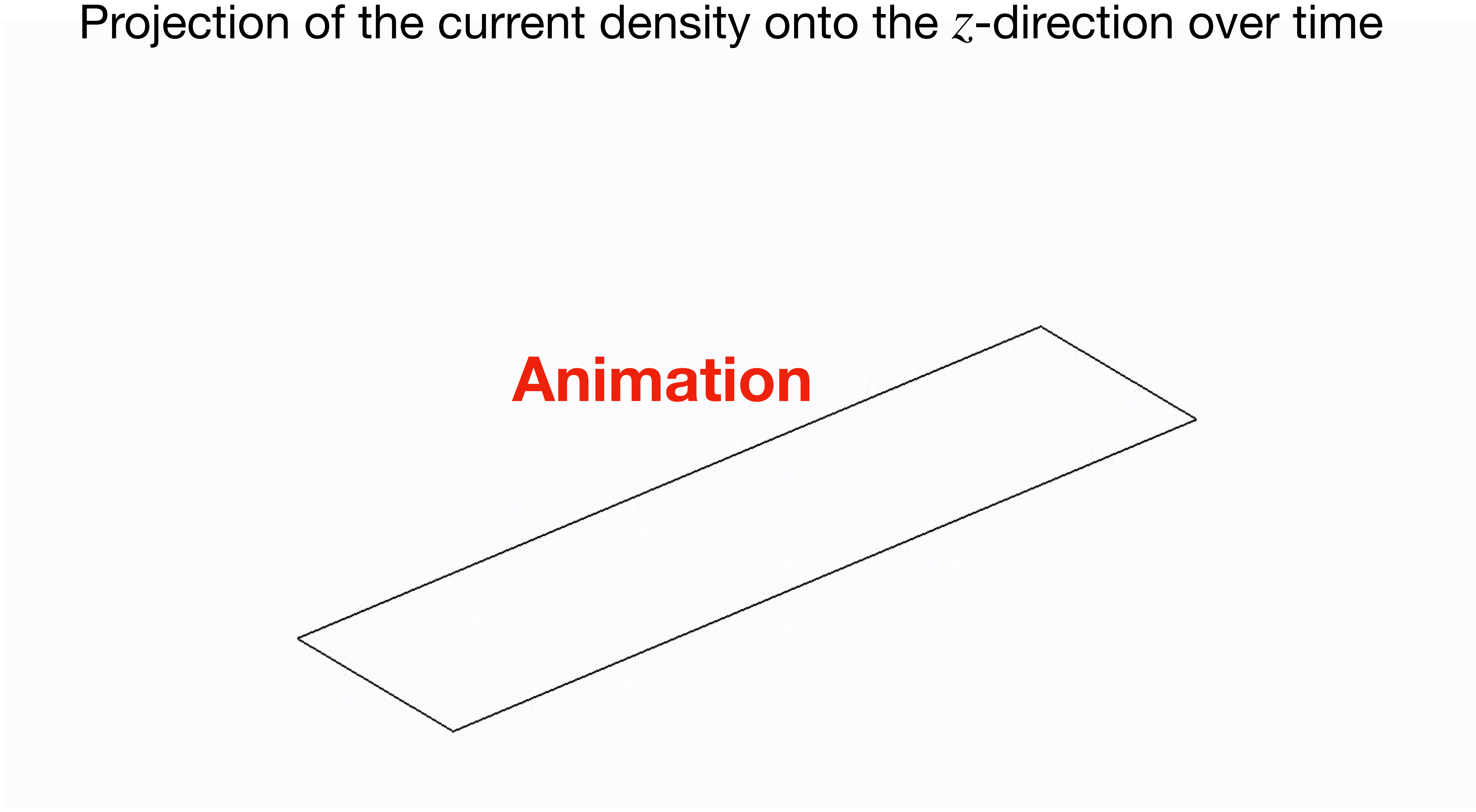


3-D Validation

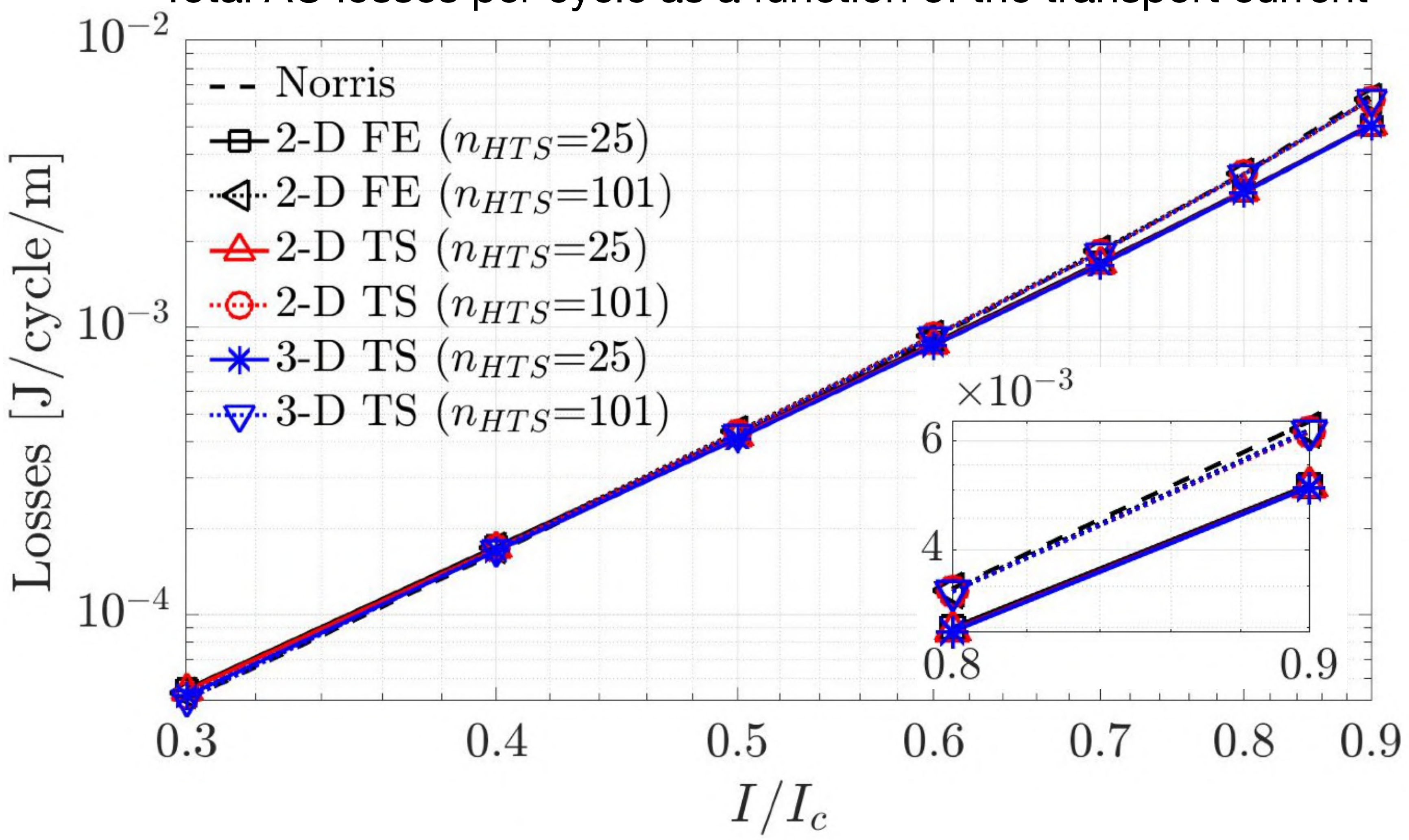
single HTS tape



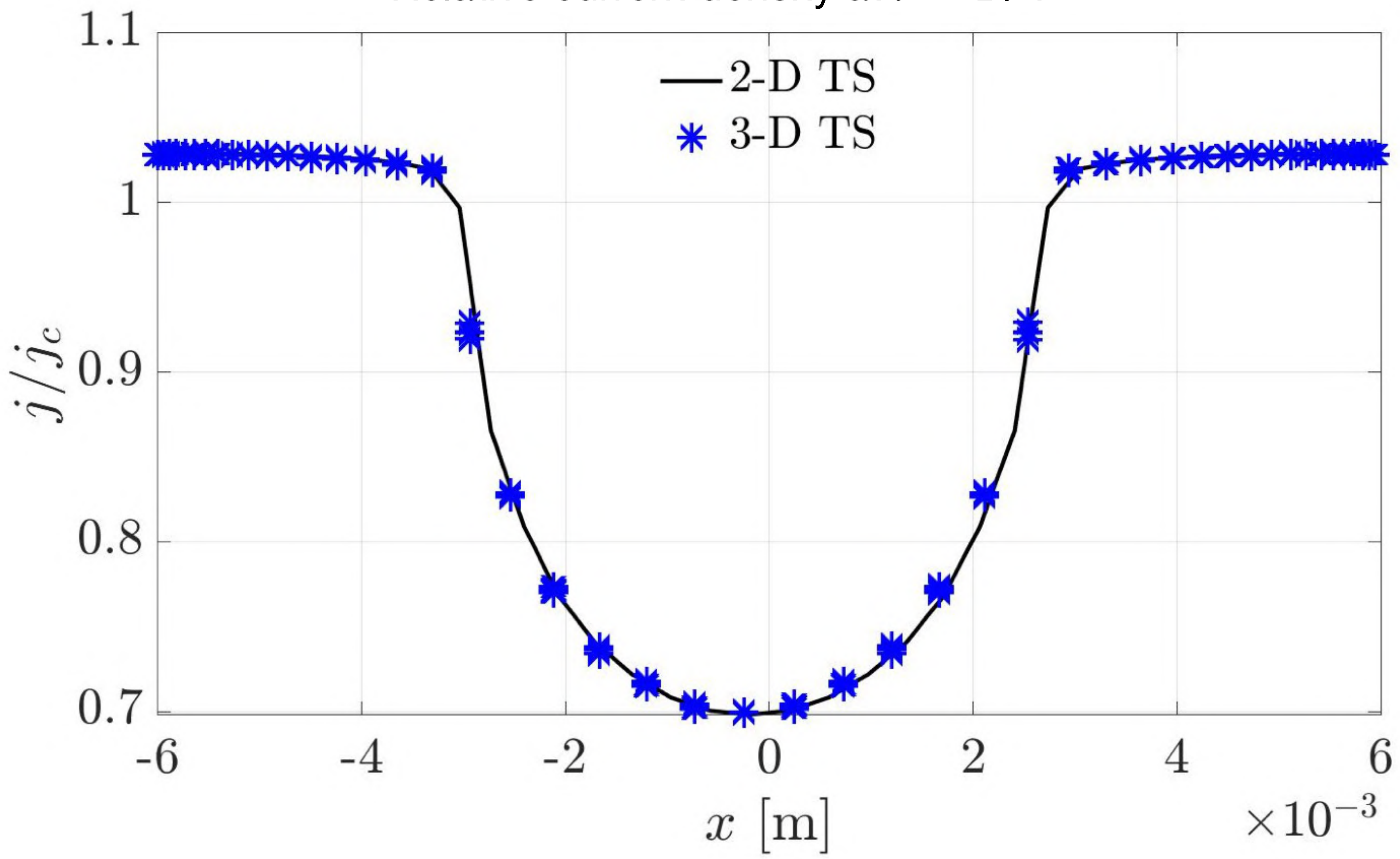
Projection of the current density onto the z-direction over time



Total AC losses per cycle as a function of the transport current

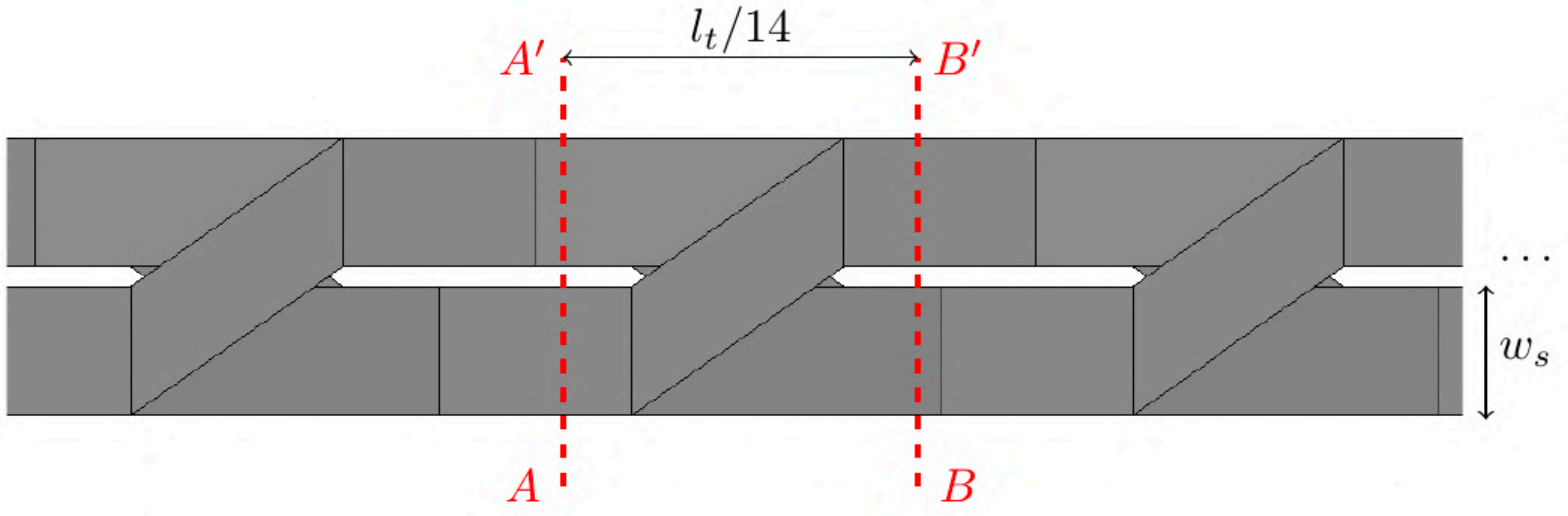


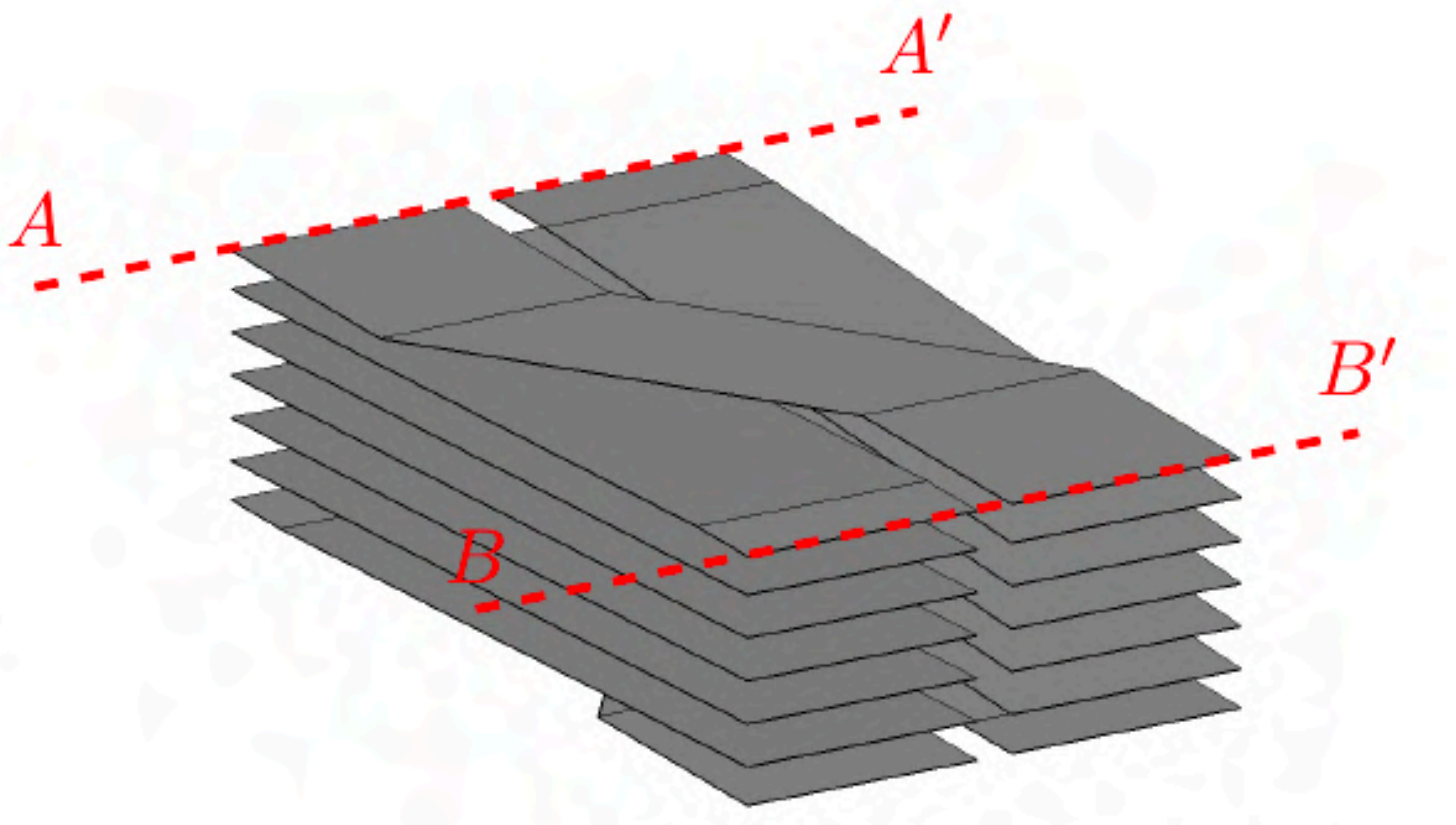
Relative current density at $t = T/4$



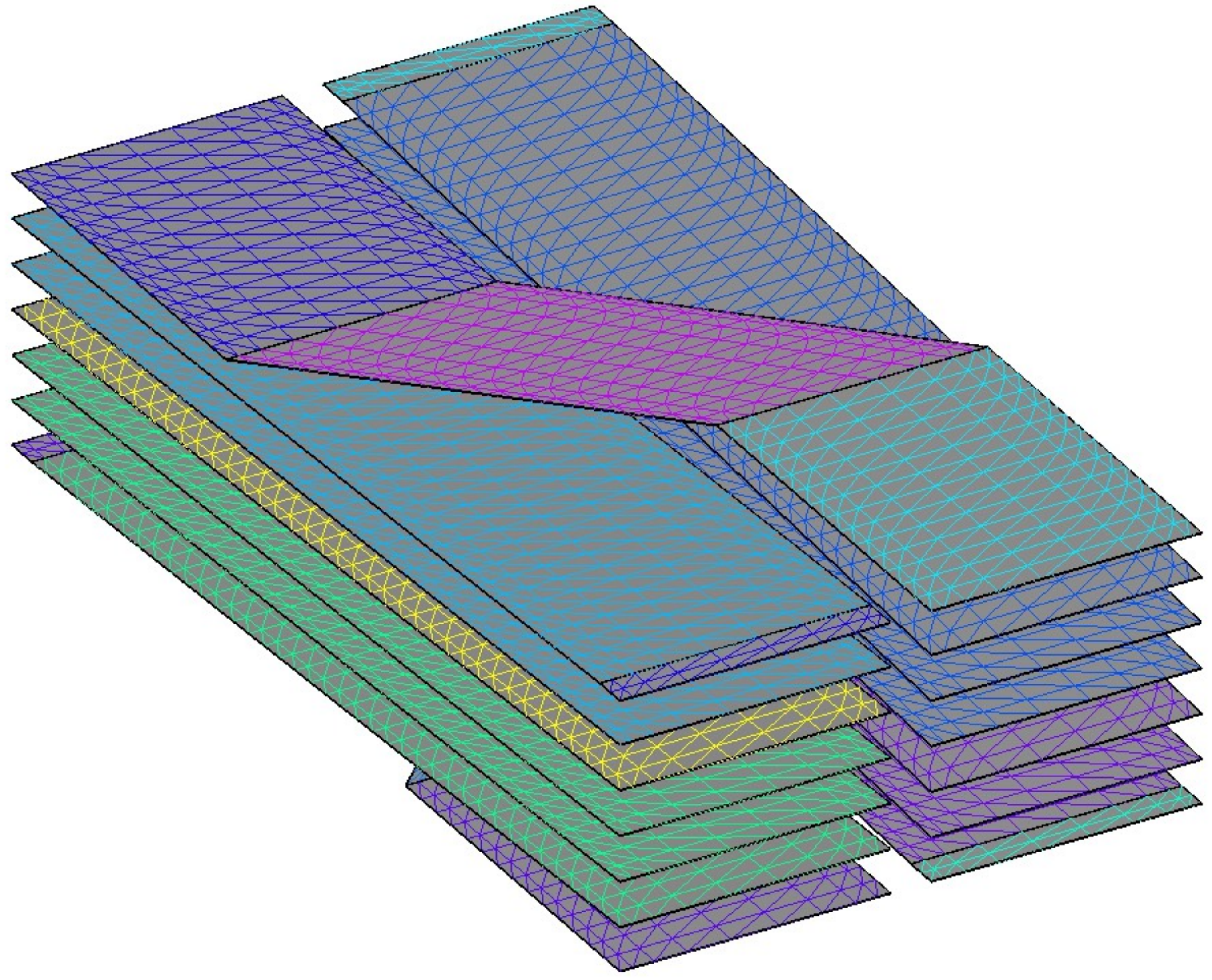
3-D Application

Roebel cable [Zermeno,2013]



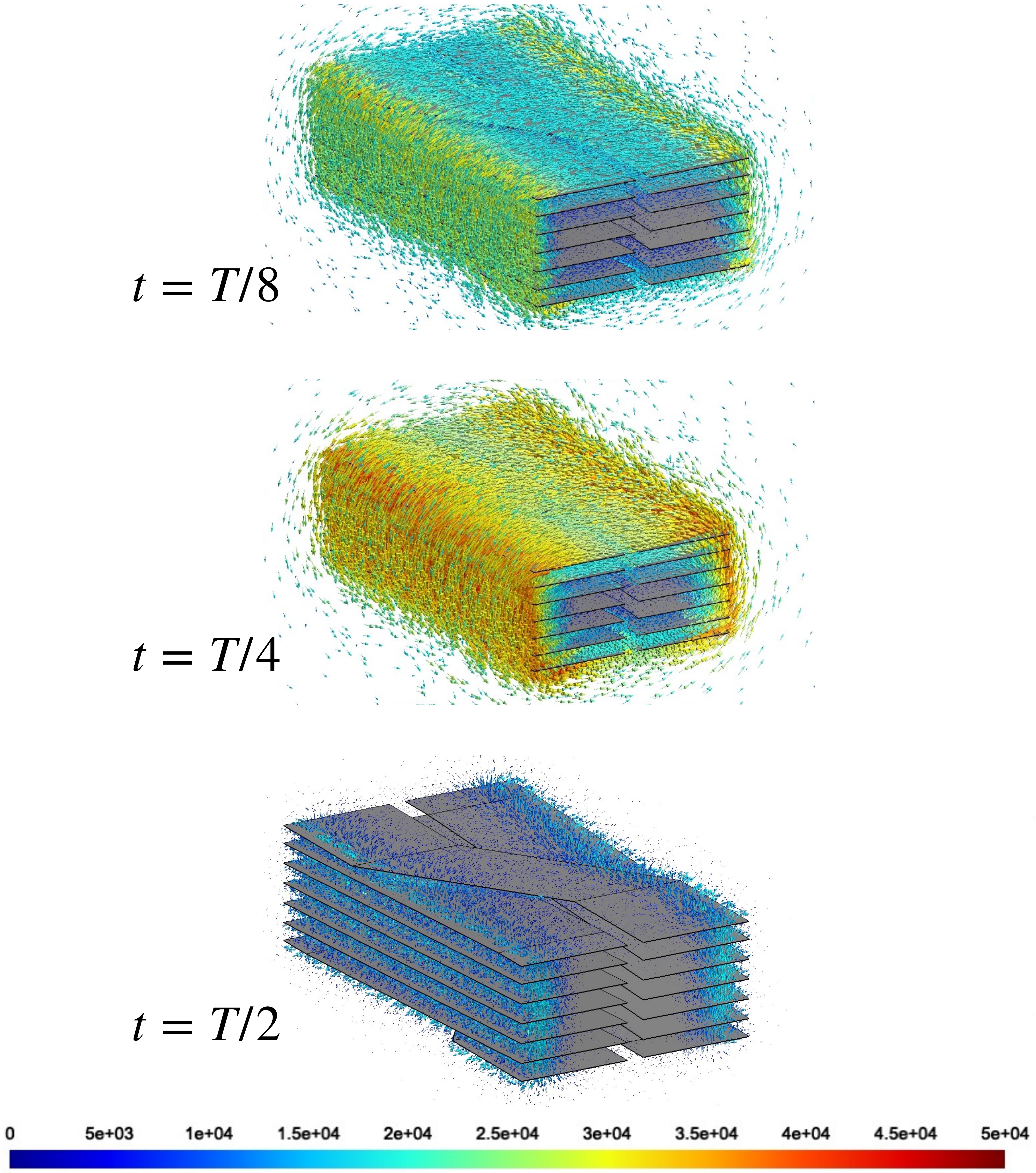


Parameter	Value
Number of strands (n_s)	14
Transposition length (l_t)	109 [mm]
Cable width (w_c)	4.3 [mm]
Gap between strands (g)	0.3 [mm]
Strands width (w_s)	2 [mm]
Tape's thickness (d)	10 [μm]
Operating frequency (f)	50 [Hz]
Critical current (I_c)	465 [A]
Critical electric field (E_c)	10^{-4} [V/m]
Power-law exponent (n)	21

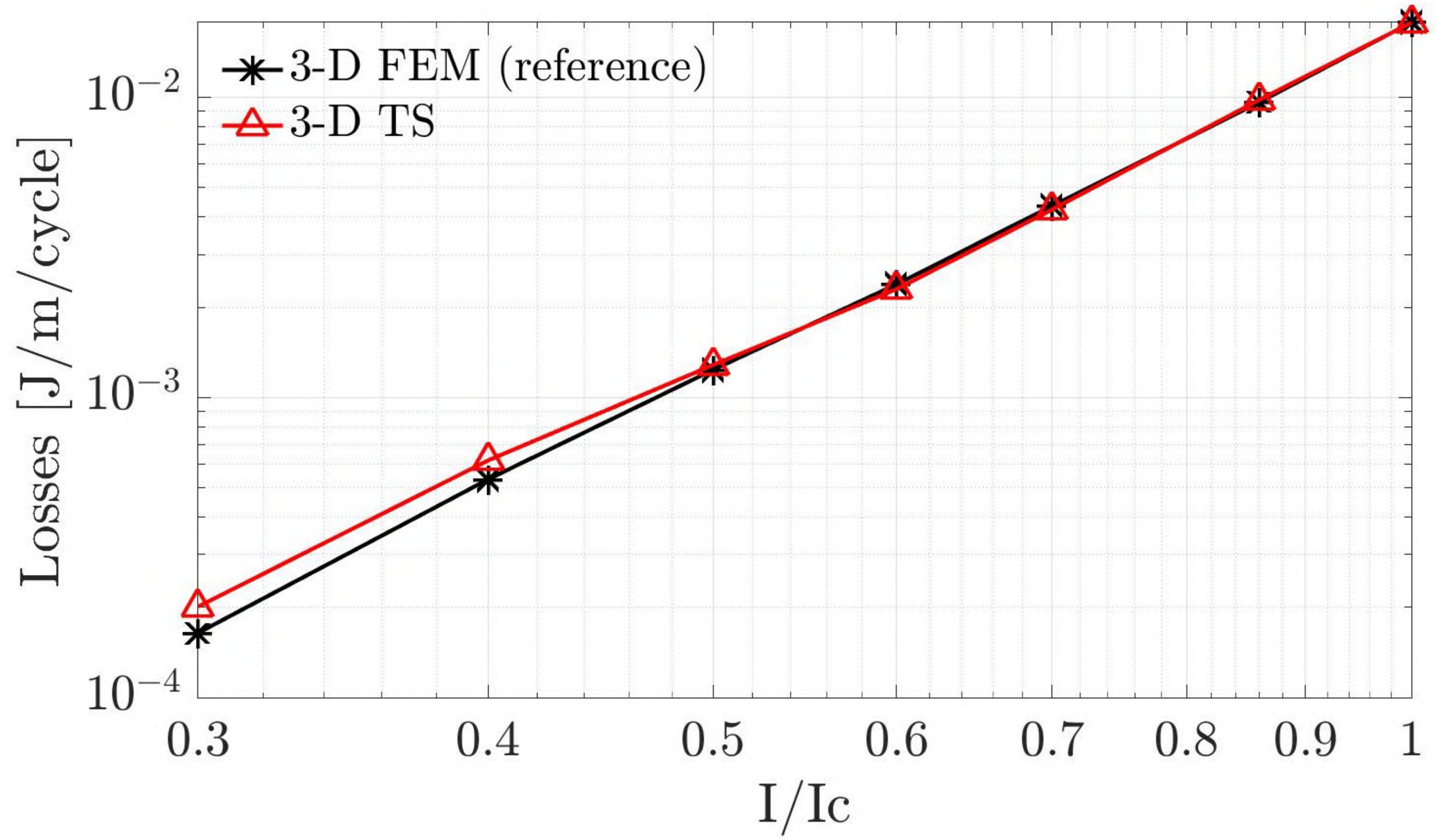


3-D Application

[Roebel cable [Zermeno,2013]](#)



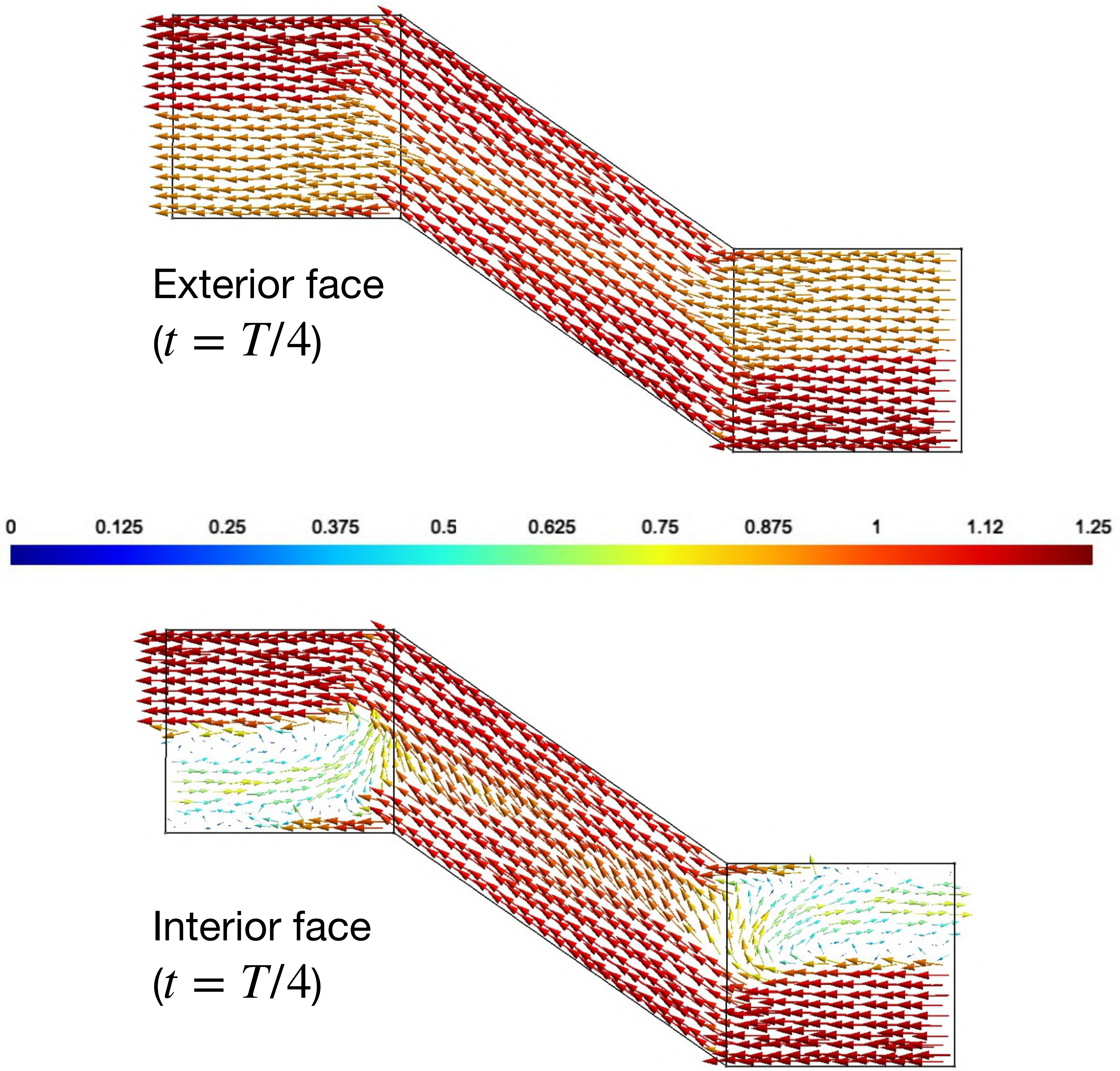
Total AC losses per cycle as a function of the transport current



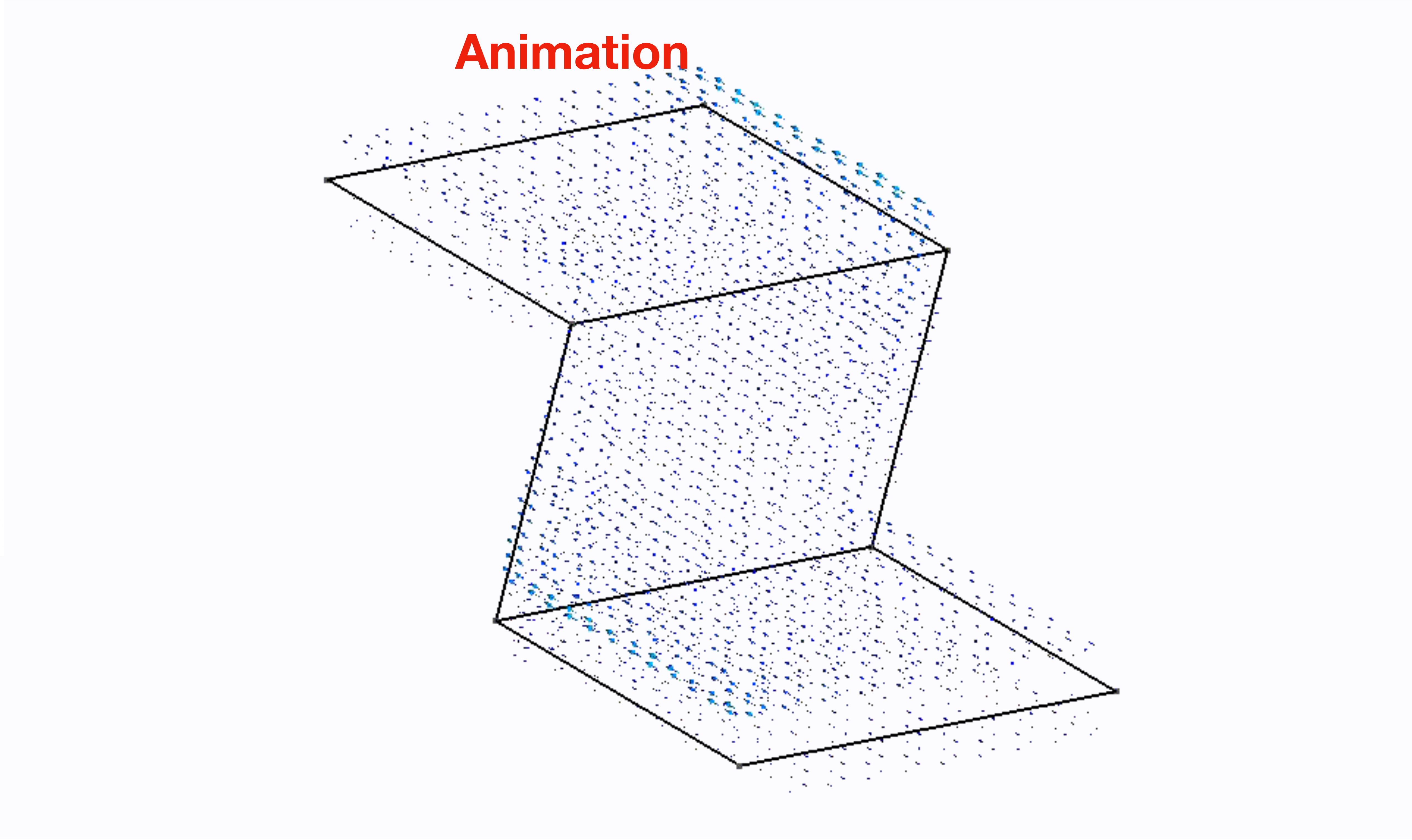
Model	AC Losses (mJ/cycle/m)	Relative difference	Number of DoFs	DoFs Reduction
3-D FEM (reference)	9.64	-	483,520	-
3-D TS-FEM ($N = 4$)	9.81	1.76%	120,097	75%

3-D Application

[Roebel cable [Zermeno,2013]](#)



Projection of the current density across the strands thickness



Conclusions

A new finite element thin-shell (TS) model was proposed and demonstrated to compute local and global quantities in HTS simulations efficiently

Characteristics of proposed TS model:

- Greatly **simplifies meshing** and **reduces the number of DoFs**
- Couples naturally with either **the H, H-ϕ or A formulations** (A-formulation not shown here)
  - **No spurious oscillation** such as in the T-A formulation
- Complete electromagnetic formulation in terms for **magnetic and electric fields**
  - Case $N = 1$ **is equivalent to the T-A-formulation**, but with better **numerical stability**
- Allows considering **any type of problem involving HTS tapes**, including:
  - Closely packed tapes $\Rightarrow$ both **edge and top/bottom losses are taken into account**
  - Tapes with **multiple layers** with different properties, e.g. **ferromagnetic substrates**, etc

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Thank you!

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