# Reduction in energy dissipation rate with increased effective applied field and variation in energy stored and dissipated with relative phase of two sinusoidal components of field.

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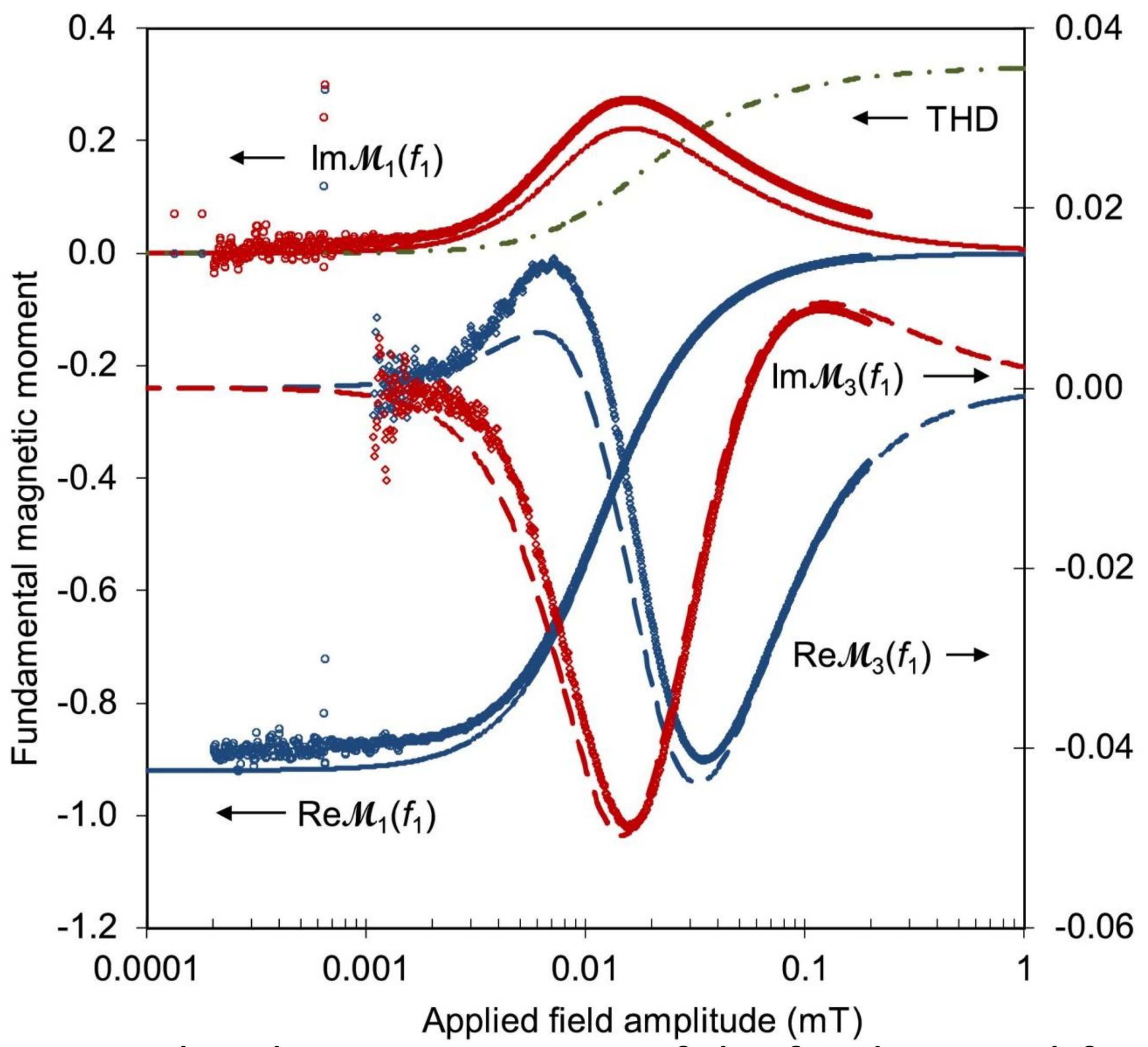
## Introduction

In a superconductor in the critical state, the distribution of induced shielding currents is quasi-static and depends on the value of the applied field and its history. This hysteresis leads to the fact that in a time-periodically variable field the current distribution and its magnetic moment depend on the peak value of the field but not on the rate of change of the field. The magnetization loop and its area, i.e. the losses, are given by the ratio of the peak value of the field and the critical current density. This suggests that by applying a field waveform with the same effective (rms) value but a larger crest factor $|B_p|/B_{rms}$, it is possible to reduce losses. This is diametrically opposed to the flux-flow regime or normal state, where the losses are proportional to the rms value of the field.

If, instead of a sinusoidal field $B(t) = B_1 \sin 2\pi f_1 t$, we apply a square wave field whose Fourier series expansion is $B(t) = B_1 \sum_{n=1,3,\ldots}^{N} \frac{\sin(n2\pi f_1 t)}{n}$, its peak and rms values are $B_p = B_{rms} = \frac{\pi}{4} B_1$, where the rms value is greater than the rms value of the sinusoidal field $B_{rms} = \frac{1}{\sqrt{2}} B_1$. It is obvious that by adding suitable components of the field we can increase its effective value and at the same time reduce its peak value. Both the amplitudes and the relative phases of the components affect the resulting value.

## Experiment and model

Prove of the critical state in the sample: 2G HTS wire #SCS4050 [1].




The applied field is $B(t) = B_1 \cos(2\pi f t)$

The magnetic moment of a sample is given by Fourier coefficients $M_n$, $m(t) = \sum_{n=0}^{\infty} M_n(f) \exp(i2\pi f t)$. The coefficient are calculated from experimental data [2].

The energy stored (real part of energy) and dissipated per applied field cycle (imaginary part of energy) is

$$\mathrm{Re}\, E(f) + i\, \mathrm{Im}\, E(f) = -\frac{B_1 M_1(f)}{2}.$$

The power dissipated per field cycle is $P = f\pi B_1 \mathrm{Im}\, M_1(f)$.

Fig. 1. Real and imaginary parts of the fundamental-frequency and third-harmonic frequency complex amplitudes of the magnetic moment, normalized to the magnetic moment of the sample with perfect screening, as a function of the applied field amplitude [3]. Symbols are for experimental data. Curves represent the amplitudes calculated on the basis of the model.

When the applied field is synthesized from harmonic components:

a) in the critical state, the nonlinear response due to hysteresis leads to mixing of harmonic components of the field and of the magnetic moment

b) in flux-flow or normal states, the response is linear for which the principle of superposition applies.

Normal state $P \propto \sum_{n=1,3,5,\ldots}^{N} (nf)^2 \sigma \left(\frac{B_1}{n}\right)^2 \propto \frac{N+1}{2}$, where $\sigma$ is the conductivity

Critical state $P = \sum_{n=1,3,5,\ldots}^{N} nf\pi B_1 \mathrm{Im}\, M_1(nf)$

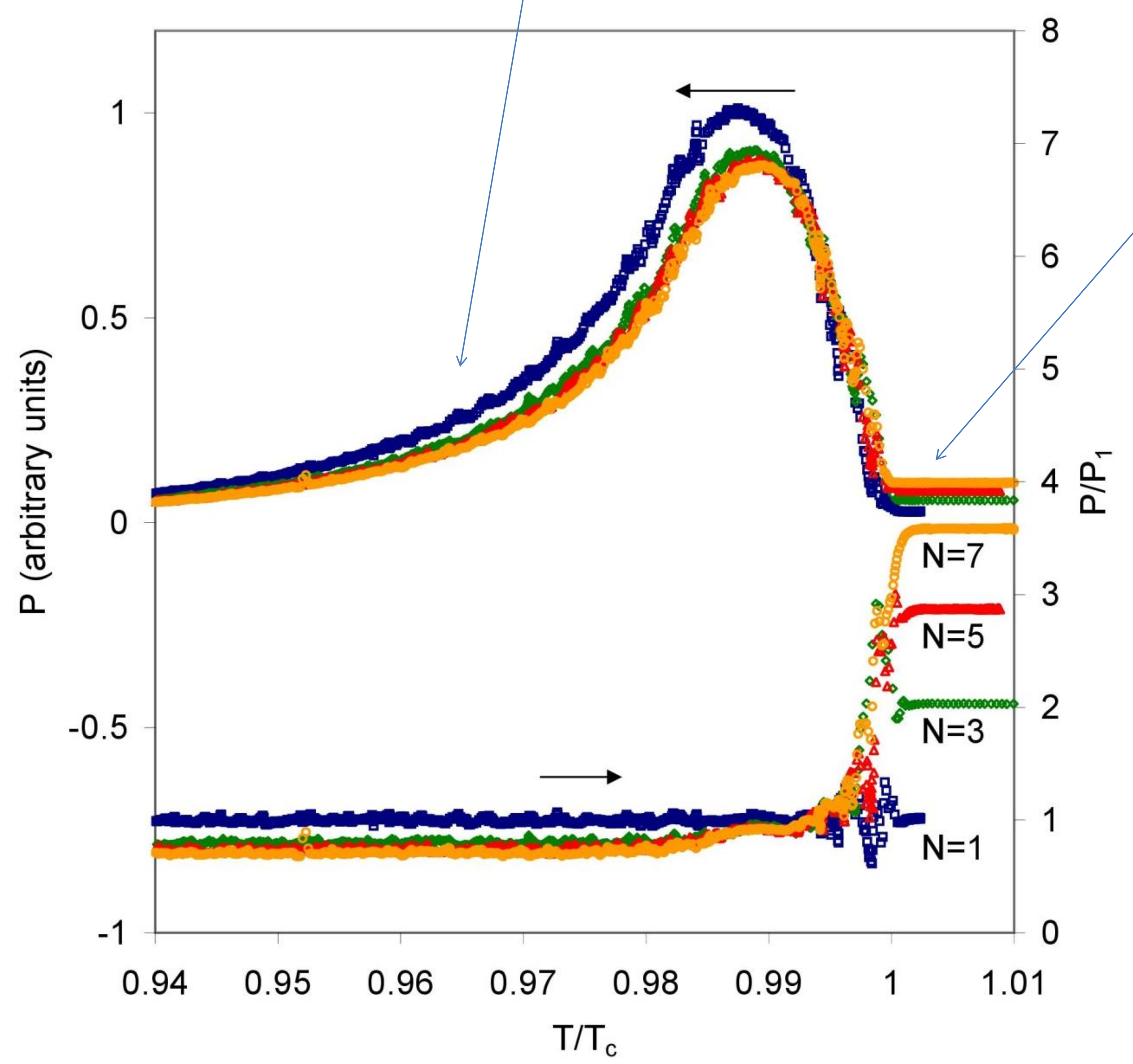



Fig 2. Top: temperature dependence of the total dissipated power in Nb film for the square wave field synthesized from the first one, two, three, and four terms in series expansion with $B_1$ = 100 μT [4]. Bottom: the total energy dissipation rate normalized to the energy dissipation rate in the pure sinusoidal field.

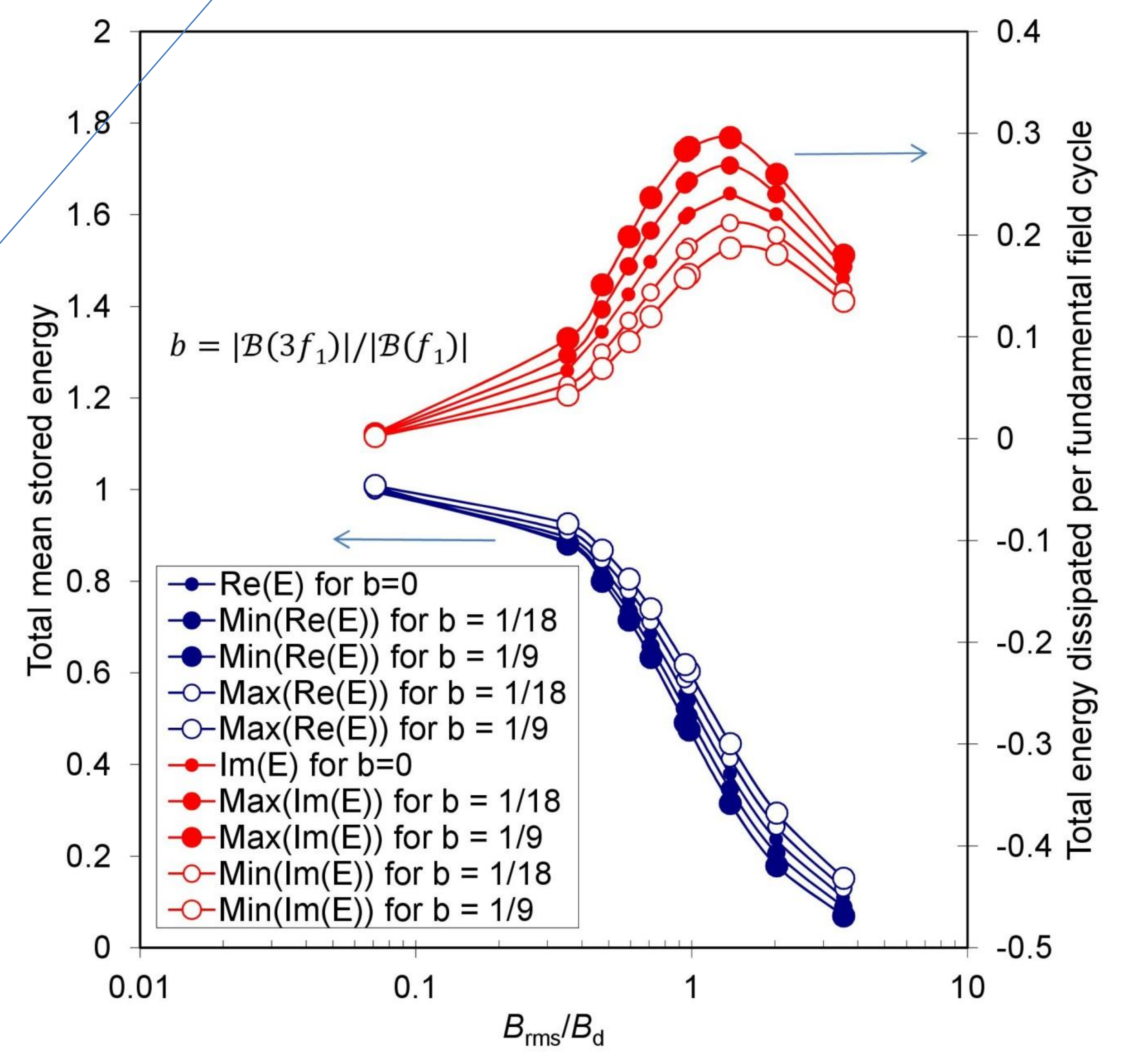



Fig. 3. Normalized theoretical extreme values of total mean stored energy and total energy dissipated per the fundamental field cycle, for different values of the relative phase of the fundamental and third harmonic components of the field, as a function of $B_{rms}/B_d$, where $B_d = \mu_0 j_c d/\pi$ is the characteristic field [3].

## Conclusion

In an applied periodic field, the energy of a system with hysteresis, both the mean accumulated energy and the energy dissipated per the cycle of the fundamental component of the applied field, varies with the amplitudes and relative phases of the sinusoidal field components at harmonics of the fundamental frequency. Appropriate selection of components can lead to reduced losses, even if the new field has a larger effective value than a purely sinusoidal field. This can be used in practice. Complete analytical expressions for the initial magnetization curve and thus for the full hysteresis loop offer an effective way to study such a system.

In the flux-flow regime and in the normal state, the response of the system is linear and the principle of superposition applies here - each added component of the field increases the dissipation of energy.

References:

[1] 2G HTS wire #SCS4050 produced by SuperPower Inc., ⟨http://www.superpowerinc.com⟩.

[2] Z. Janů and F. Soukup, Review of Scientific Instruments **88** (2017) 065104; doi: 10.1063/1.4984943.

[3] Z. Janů and T. Chagovets, Physica B **504** (2017) 9–12, doi: 10.1016/j.physb.2016.10.003

[4] Z. Janů and F. Soukup, Supercond. Sci. Technol. **28** (2015) 085016, doi:10.1088/0953-2048/28/8/085016