Numerical investigation of critical states in superposed superconducting films

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Magneto-optical images of critical states

Critical states in assemblies of films of uneven cross-sections are more complex than expected [1]. In a previous work [2], it was shown how the magnetic coupling between the films, which induces a breaking of the square symmetry, and a magnetic-field dependence of J_c can lead to such critical states.

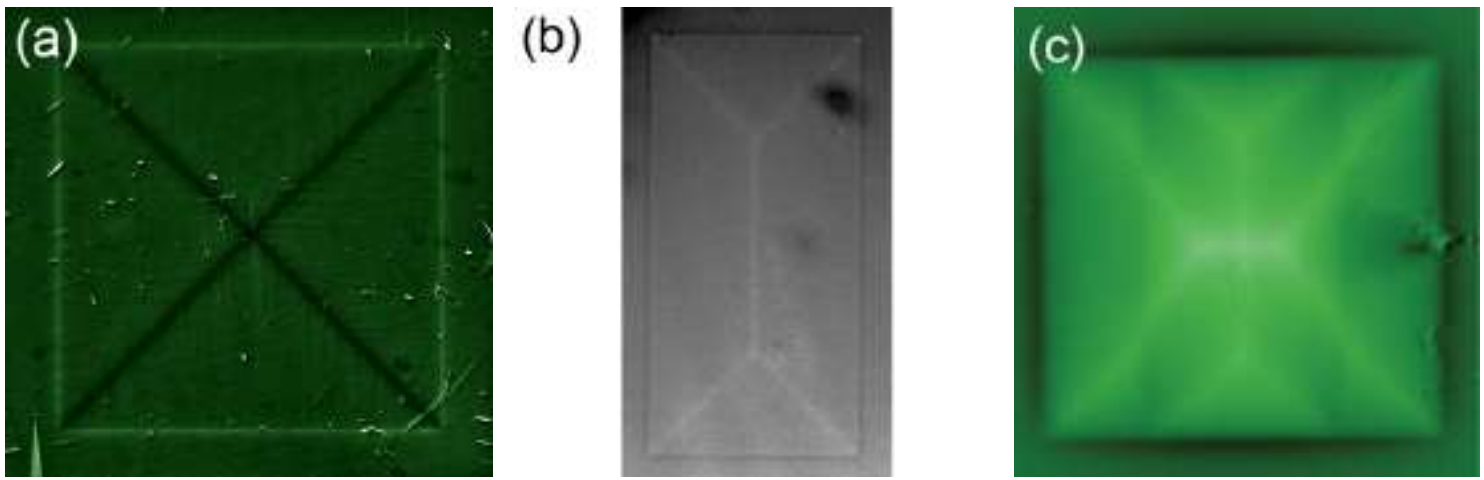

Figure: Magneto-imaging of the critical state in (a) a square film, (b) a rectangular strip and (c) their centred superposition.

[1] T. Tamegai *et al.*, Physica C **533**, 74-79 (2017).
[2] L. Burger *et al.*, SUST **32**, 125010 (2019).

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What do we investigate?

We proceed to a **parametric investigation of the geometrical parameters of the assemblies**, which includes the off-centring of the strip with respect to the square film, the thickness of the insulating layer that lies between the strip and the square film, or the width of the rectangular strip. Three-layers assemblies are also briefly investigated.

In order to do so, we use **numerical simulations in the framework of the finite-element method**.

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Geometry of a two-layers assembly

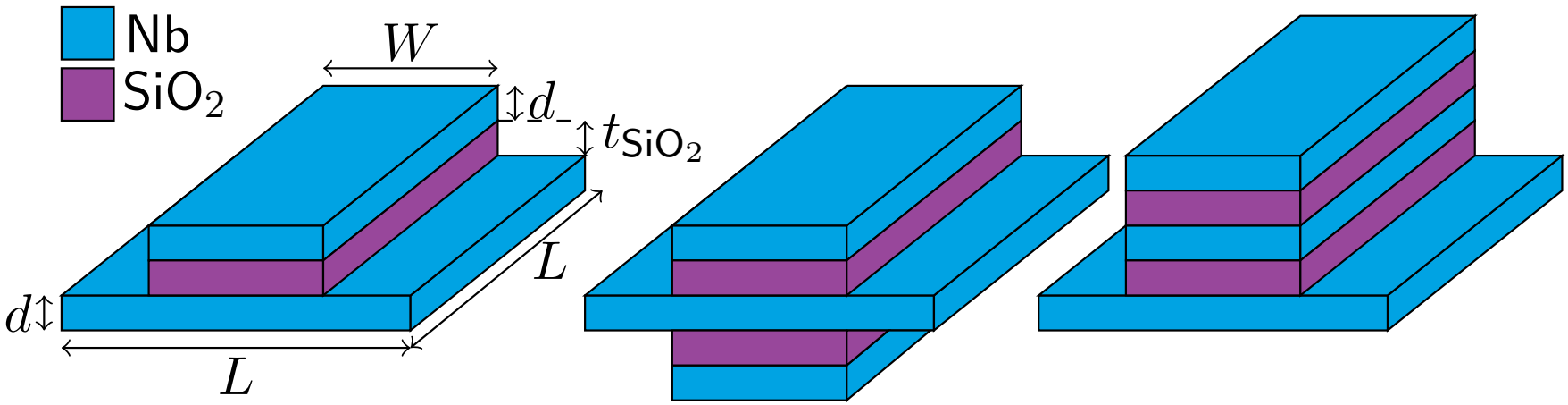


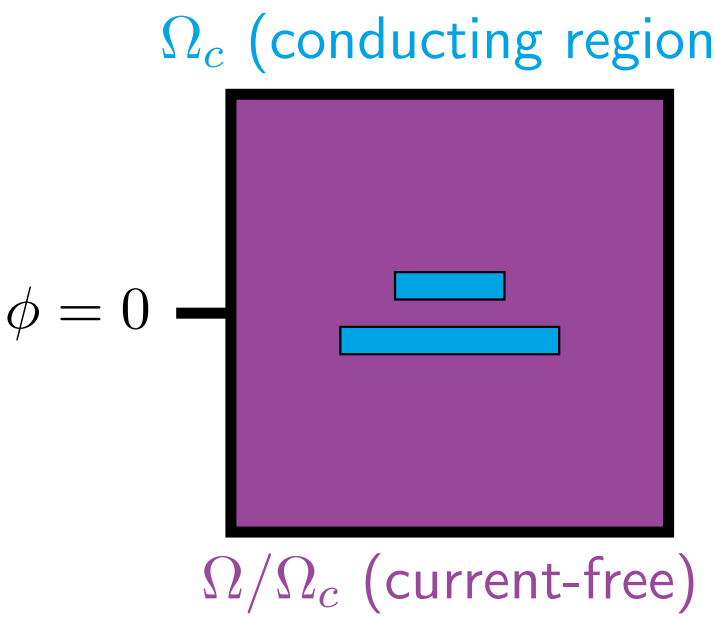
Figure: Geometry of the investigated two- and three-layers assemblies made of thin strips and square films. The applied field is perpendicular to the cross section of the films.

If not stated otherwise, it is assumed that $L = 200$ μm, $W = 100$ μm, $d = 300$ nm, $t_{SiO_2} = 300$ nm, and that the films are centred.

H-ϕ formulation and parameters

Finite-element $H - \phi$ formulation (solved in GetDP [3]) :

$$\int_\Omega \mu_0 \, \dot{\mathbf{h}} \cdot \psi \, \mathrm{d}\Omega + \int_\Omega \mu_0 \, \dot{\mathbf{H}}_a \cdot \psi \, d\Omega + \int_{\Omega_c} \rho(|\nabla \times \mathbf{h}|) \nabla \times \mathbf{h} \cdot \nabla \times \psi \, \mathrm{d}\Omega_c = 0$$



$\psi \equiv$ linear edge test functions
$\psi = -\nabla\phi$ in Ω/Ω_c
$\phi \equiv$ linear nodal test functions

[3] C. Geuzaine *et al.*, International Journal for Numerical Methods in Engineering **79**, 1309-1331 (2009)

Constitutive laws

The constitutive laws are

$$\rho = \frac{E_c}{J_c(|\mathbf{B}|)} \left(\frac{|\mathbf{J}|}{J_c(|\mathbf{B}|)} \right)^{n-1},$$

$$J_c(|\mathbf{B}|) = \frac{J_{c0}}{(1 + |\mathbf{B}|/B_0)^{\alpha}},$$

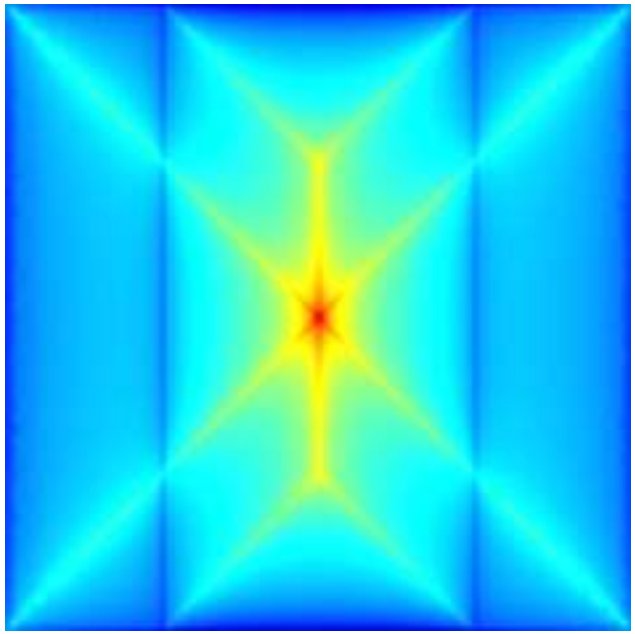
$$\mathbf{B} = \mu_0 \mathbf{H}$$

	J_{c0} [MA/cm^2]	B_0 [mT]	α [-]	n [-]
Rectangular strip	5.4	4.9	0.51	19
Square film	3.4	1.25	0.42	19

Table: This set of parameters is used, if not stated otherwise. $E_c = 1$ μV/cm and $\mu_0 = 4\pi \times 10^{-7}$ H/m in all simulations.

The critical role of $J_c(|\mathbf{B}|)$: B_z with infinite B_0

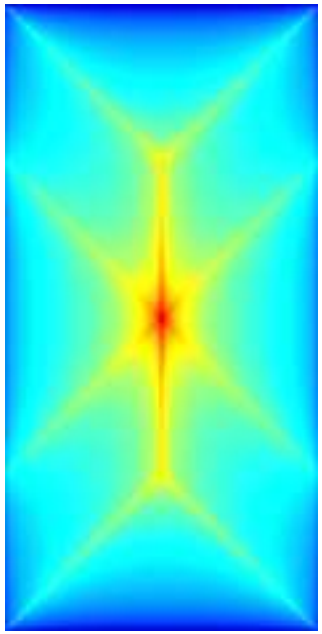
Square film



B_z (G)

-88.9 68.8 226

Rectangular strip



B_z (G)

-88.3 66.5 221

Figure: In both film: $J_{c0} = 2$ MA/cm^2, $B_0 \to \infty$, $\alpha = 1$.

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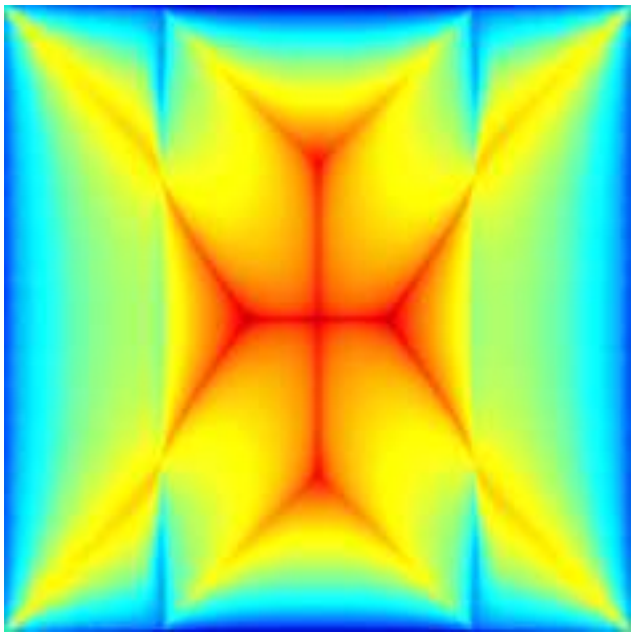
The critical role of $J_c(|\mathbf{B}|)$: B_z with finite B_0

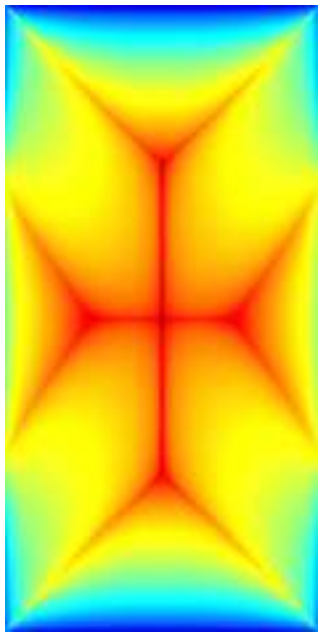
Square film



B_z (G)

-164 25.8 216

Rectangular strip



B_z (G)

-163 28.6 220

Figure: In both films, $J_{c0} = 12$ MA/cm^2, $B_0 = 5$ mT, $\alpha = 1$.

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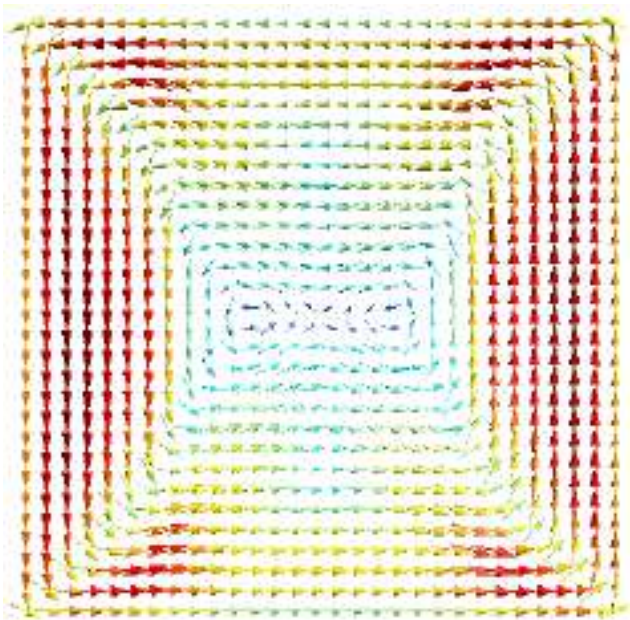
The critical role of $J_c(|\mathbf{B}|)$: $\mathbf{J}$ with finite B_0

Square film



Rectangular strip

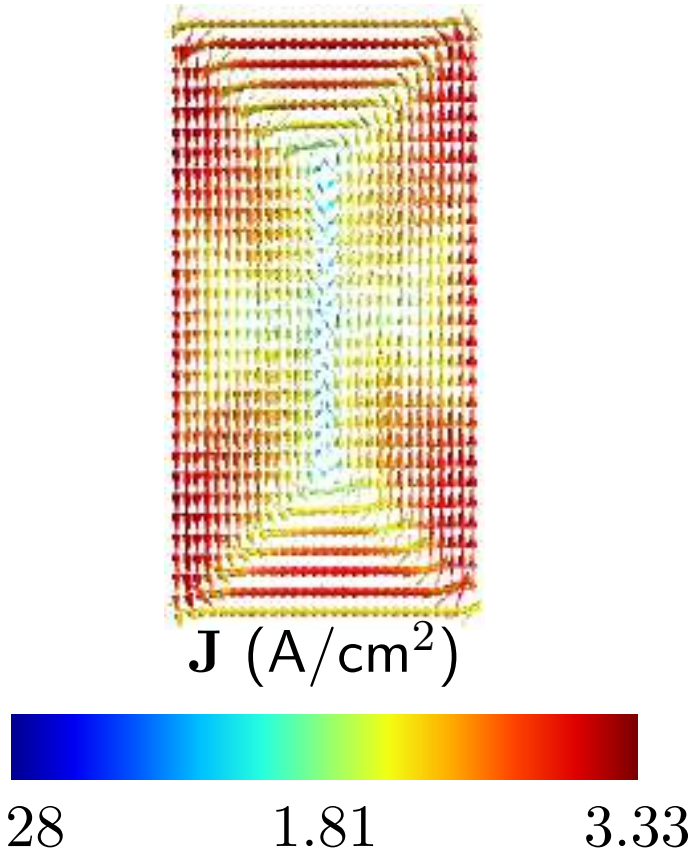


Figure: In both films, $J_{c0} = 12$ MA/cm^2, $B_0 = 5$ mT, $\alpha = 1$.

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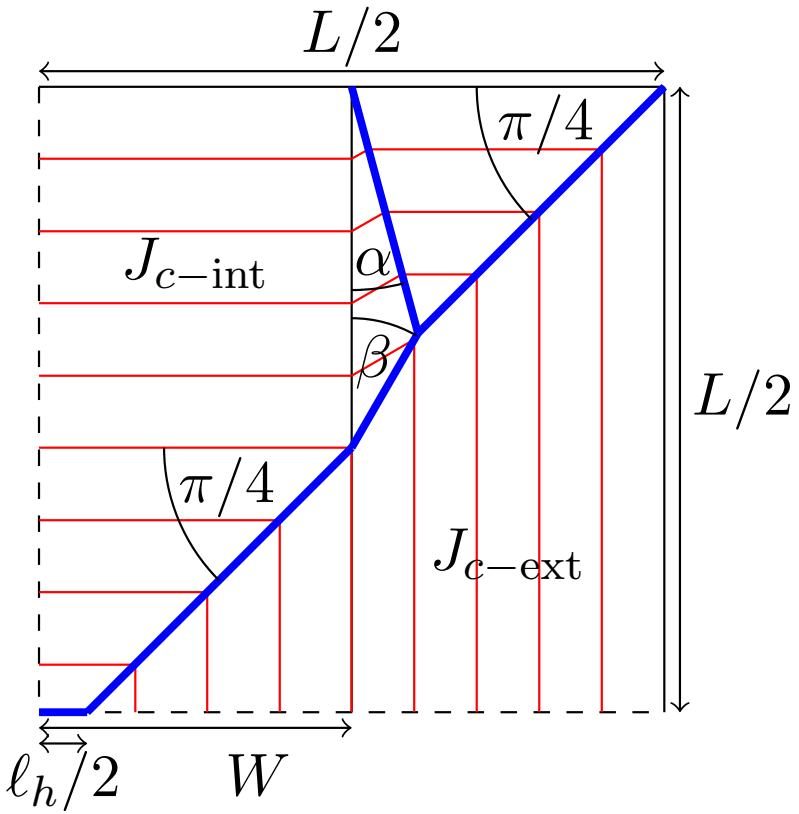
A simplified depiction of the d-lines based on the critical state model



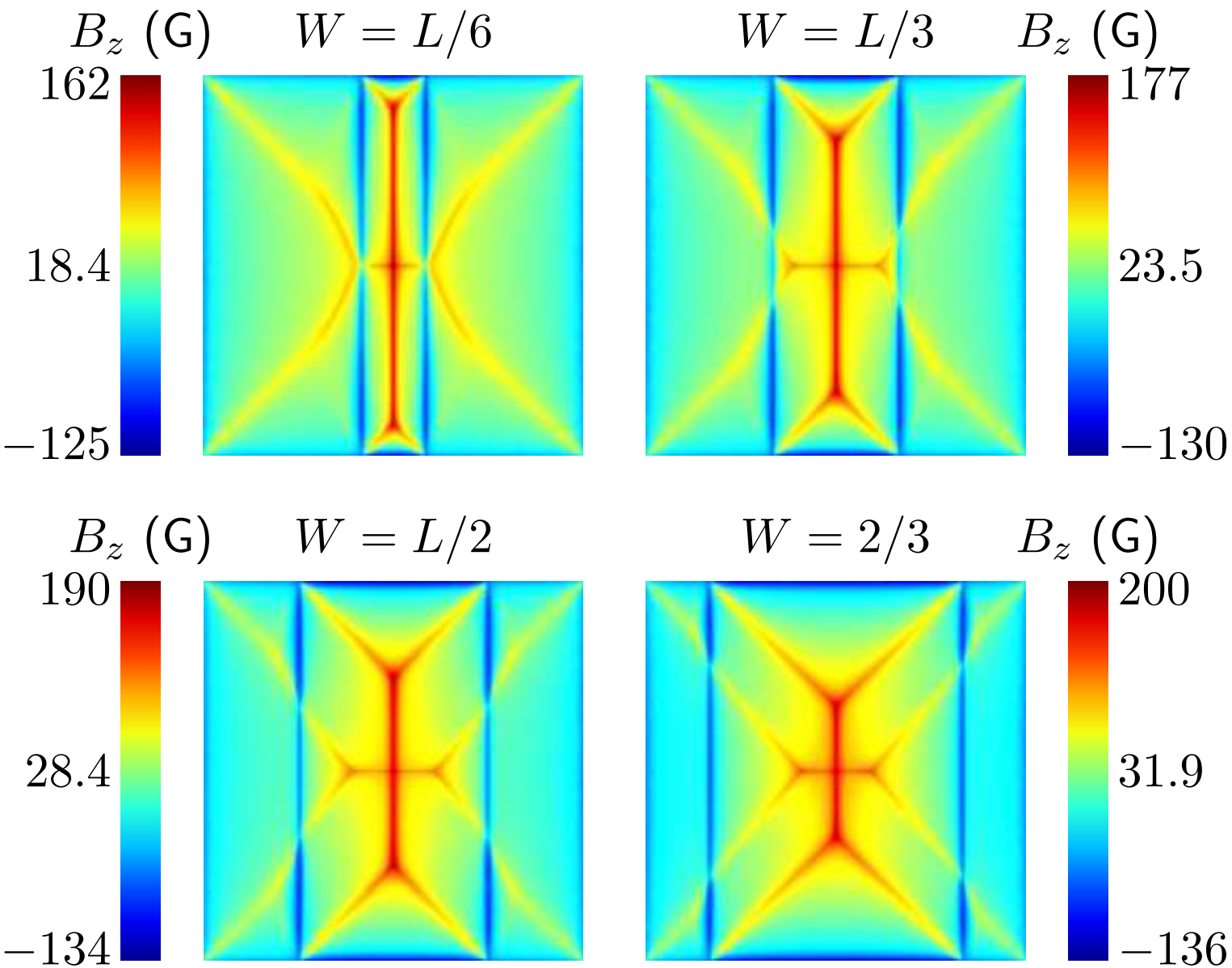
$$\beta = \frac{\pi}{4} - \alpha$$

$$\frac{J_{c-\mathrm{ext}}}{J_{c-\mathrm{int}}} = \frac{1}{\cos 2\alpha}$$

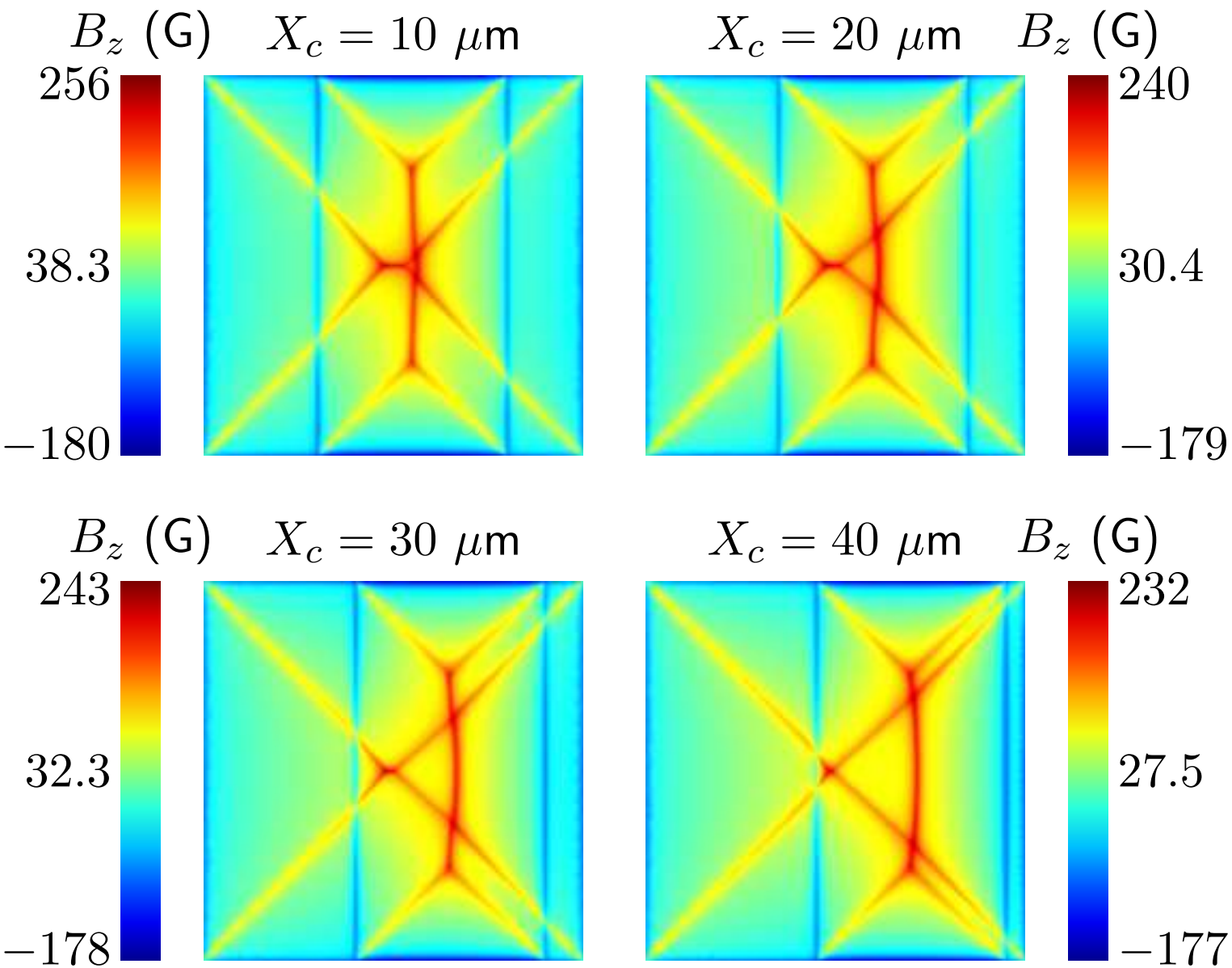
$$\frac{\ell_h}{L} = \frac{1}{2}\left(\frac{J_{c-\mathrm{ext}}}{J_{c-\mathrm{int}}} - 1\right)$$

Figure: Dashed lines indicate mirror symmetry lines. Current lines are in red and d-lines are in blue, both of which are depicted in the particular case $J_{c-\mathrm{int}}/J_{c-\mathrm{ext}} = \sqrt{3}/2$.

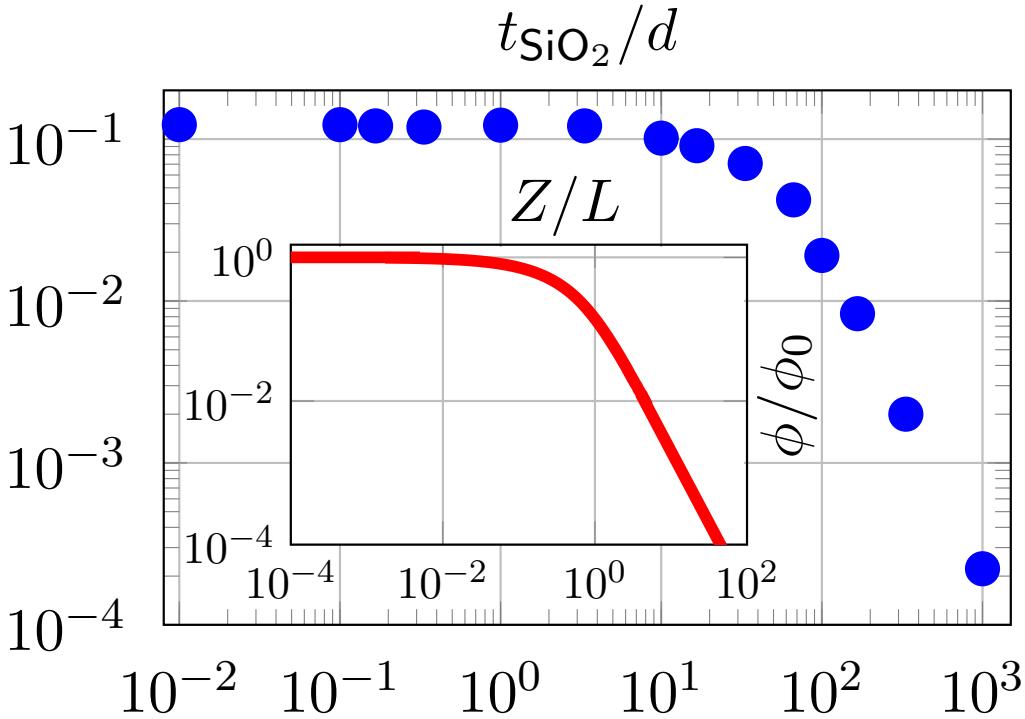
Influence of the width of the strip, W



Influence of off-centring the strip by a distance X_c



Influence of off-centring the strip by a distance X_c



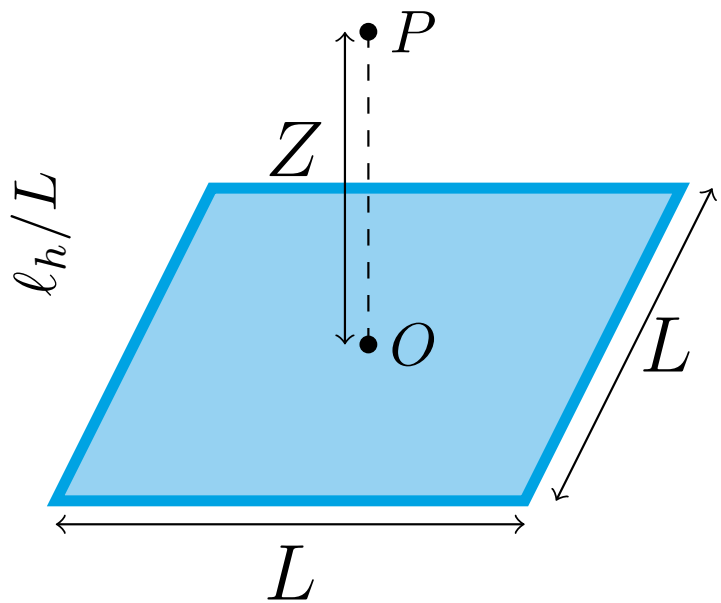


Figure: In both films: $B_0 =$ mT, $J_{c0} = 12$ MA/cm^2 and $\alpha = 1$.

Two-layers systems vs. three-layers systems

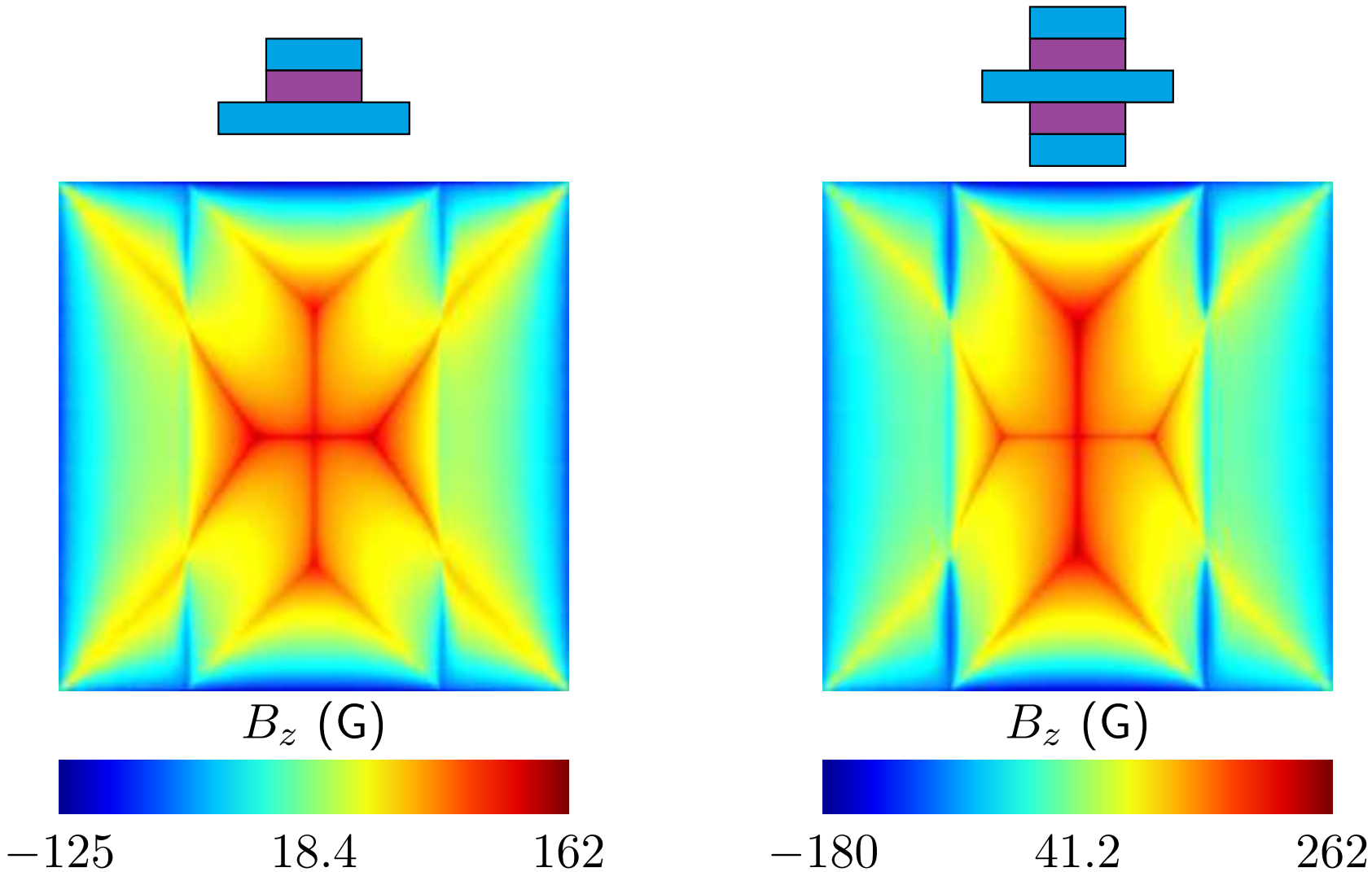


Figure: In all films: $B_0 = 5$ mT, $J_{c0} = 12$ MA/cm^2, $\alpha = 1$.

Three-layers systems: influence of the layers' positioning

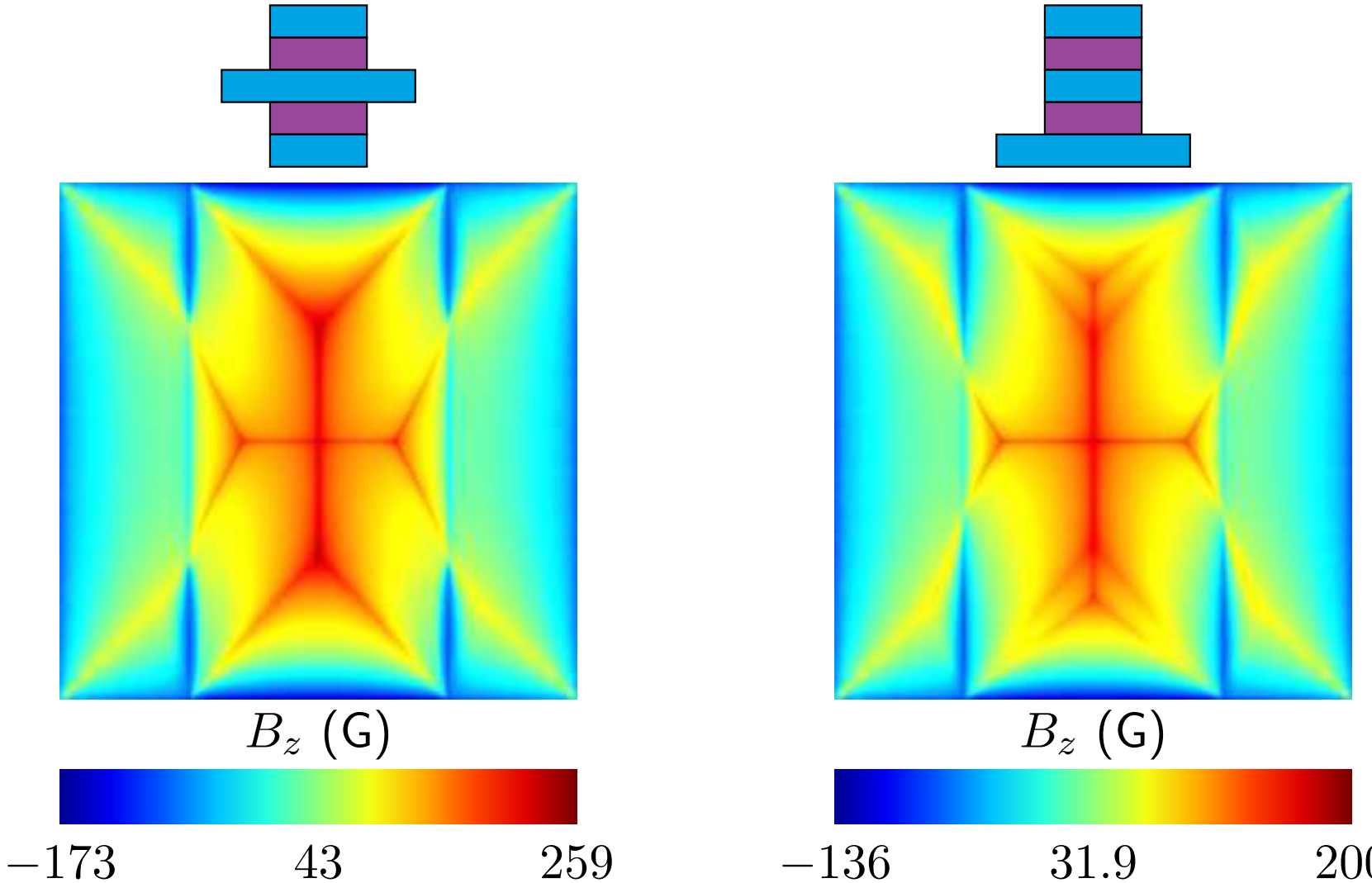
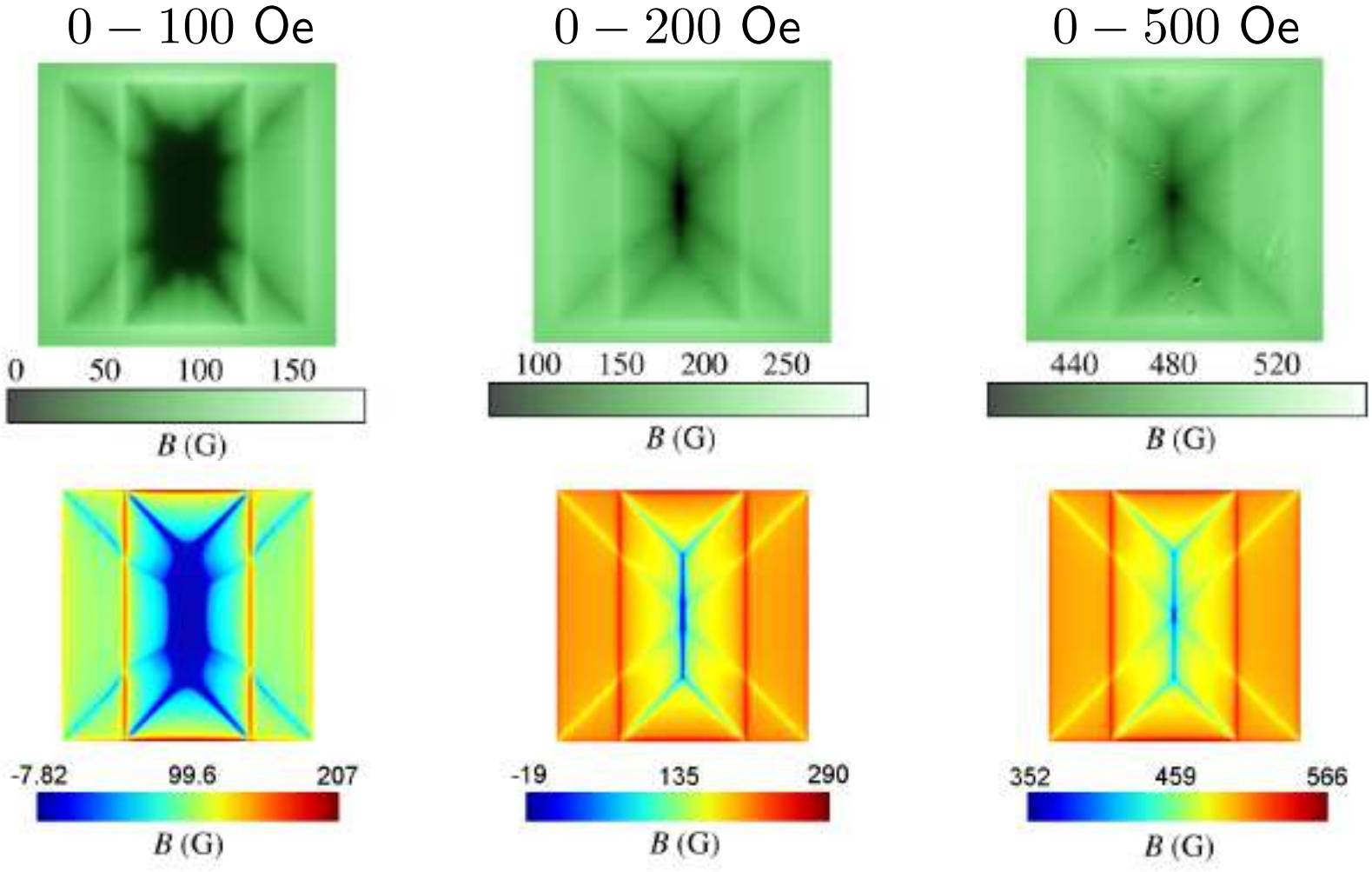
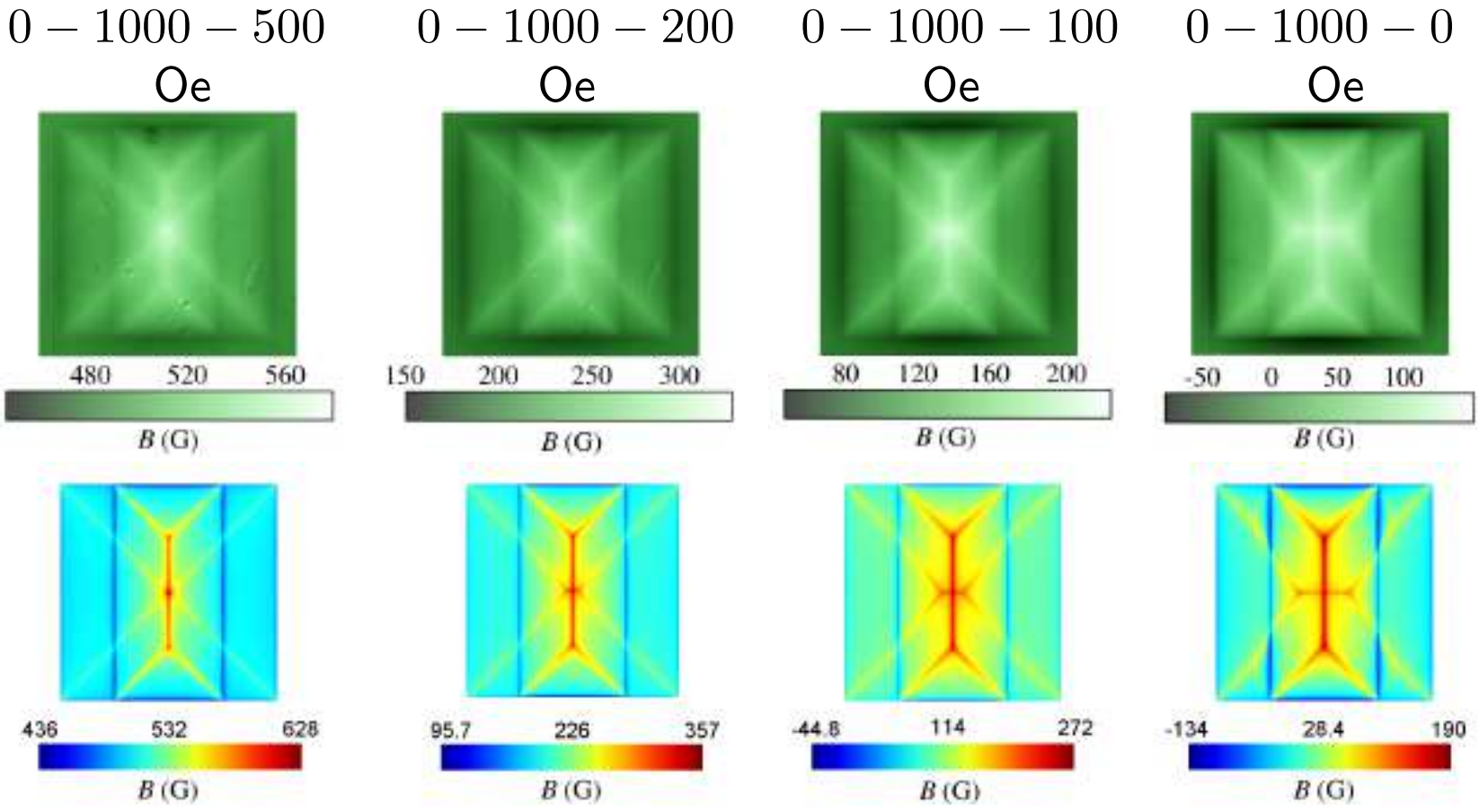


Figure: In all films: $B_0 = 5$ mT, $J_{c0} = 12$ MA/cm^2, $\alpha = 1$

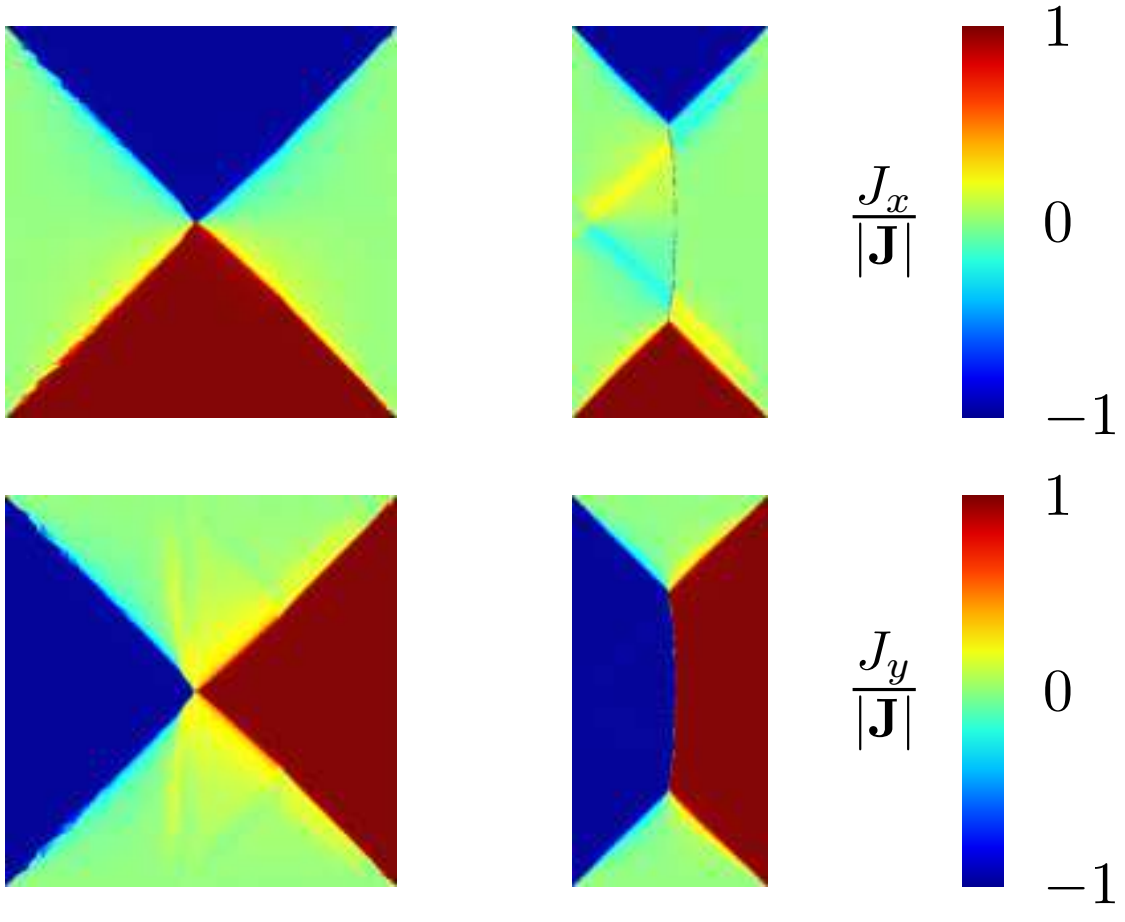
Applied-field-dependent critical states in two-layers assemblies: increasing applied field



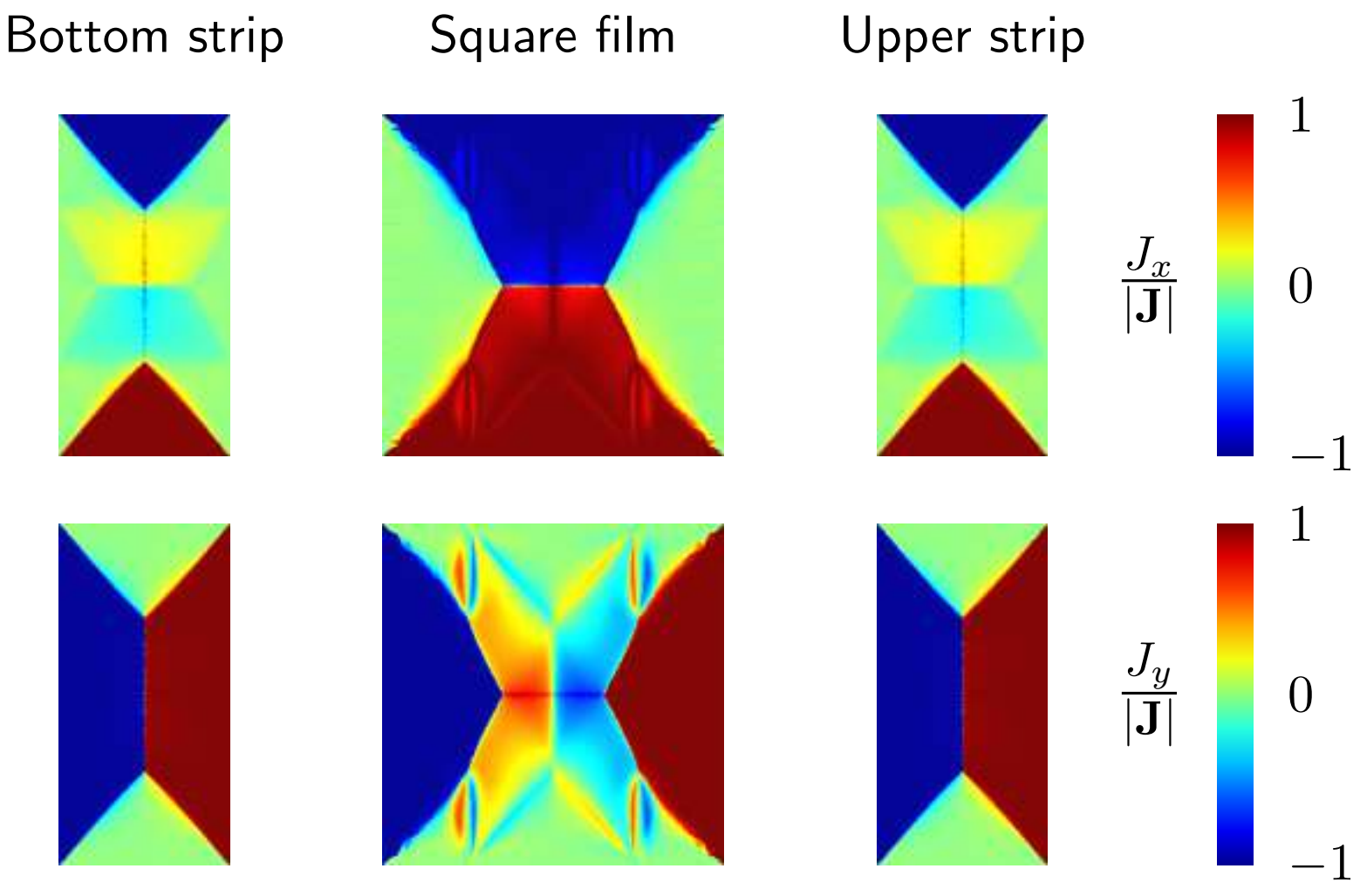
Applied-field-dependent critical states in two layers assemblies: decreasing applied field



Current density direction in an off-centred two-layers assembly ($X_c = 40\ \mu$m)



Current density direction within a three-layers assembly (the square film in between the strips)



Current density direction within a three-layers assembly (two strips above the square film)