SIMPLE MODEL OF PULSED FIELD MAGNETIZATION IN SUPERCONDUCTORS

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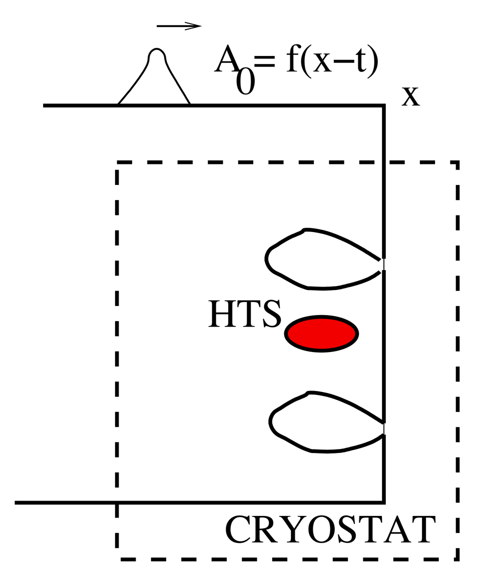
SUMMARY

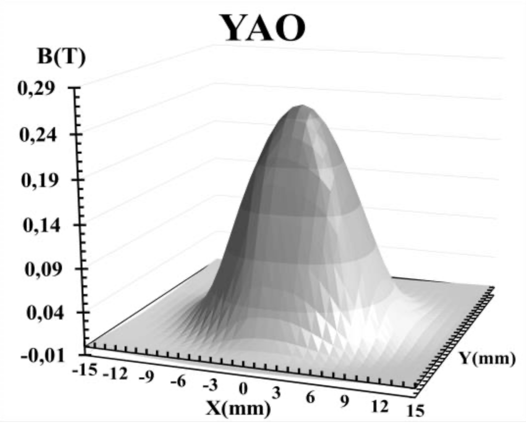
Pulsed field magnetisation leads to trapped field for long times. We model this phenomenon using 1D Maxwell-Ginzburg-Landau (Abelian Higgs) relativistic, out of equilibrium theory.

EXPERIMENTS AND ANALYSIS

(a)



(b)



(a) Schematics of experiment, [1].
(b) Trapped magnetic field in a GdBCO sample (*CRISMAT - Caen*).

1. Threshold in magnetic field above which the sample switches to permanent magnetization, [2].
2. Non trivial fixed point in the equations of motion.

MODEL : ABELIAN-HIGGS

- The Lagrangian [4] : $x \in \Omega \subset \mathbb{R}, t \in [0,T]$:

$$\mathcal{L} = A_t^2 - A_x^2 + \left[\frac{1}{\kappa^2}|\psi_t|^2 - \frac{1}{\kappa^2}|\psi_x|^2 - \frac{i}{\kappa}A(\psi^*\psi_x - \psi\psi_x^*) + |\psi|^2(1-A^2) - \frac{1}{2}|\psi|^4\right]\mathbb{1}_{\Omega_d},$$

where A is the vector potential, ψ the order parameter, $\Omega_d = (-L, L) \Subset \Omega$, $\kappa = \frac{\lambda}{\xi}$ the Ginzburg-Landau parameter.

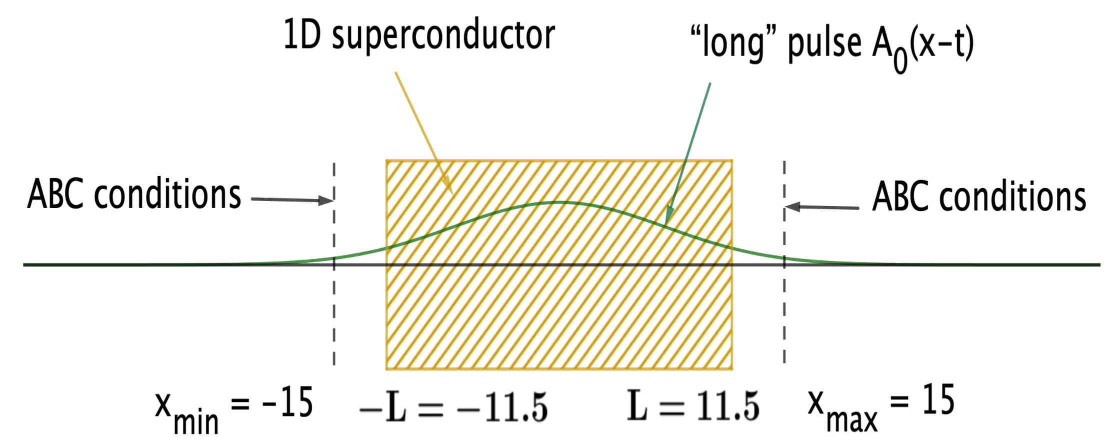
- Equations of motion:

$$A_{tt} = A_{xx} + 1_{\Omega_d}\left\{-\frac{i}{2\kappa}\left(\psi^*\psi_x - \psi\psi_x^*\right) - A|\psi|^2\right\},$$

$$\frac{1}{\kappa^2}\psi_{tt} = -\frac{1}{\kappa^2}\psi_{xx} - \frac{i}{\kappa}\left(A_x\psi + 2A\psi_x\right) + \psi\left(s - |\psi|^2 - A^2\right),$$

$$-\frac{i}{\kappa}\psi_x - A\psi = 0 \text{ at } x = \pm L.$$

NUMERICAL METHODS



- ODE solver for the time advance, finite differences for the space distribution.
- *Absorbing Boundary Conditions* : $\frac{\partial A}{\partial x} = \pm \frac{\partial A}{\partial t}$.
- $A = A' + A_0(x-t)$, (pulse of arbitrary support).
- Defect function s, [3]

$$\begin{cases} s(x) = -1, & \text{if defect,} \\ s(x) = 1, & \text{otherwise.} \end{cases}$$

- x_d : space between defect,
  a_0 : amplitude of the pulse,
  $w_d \in (0,1)$: density of defects.

REFERENCES

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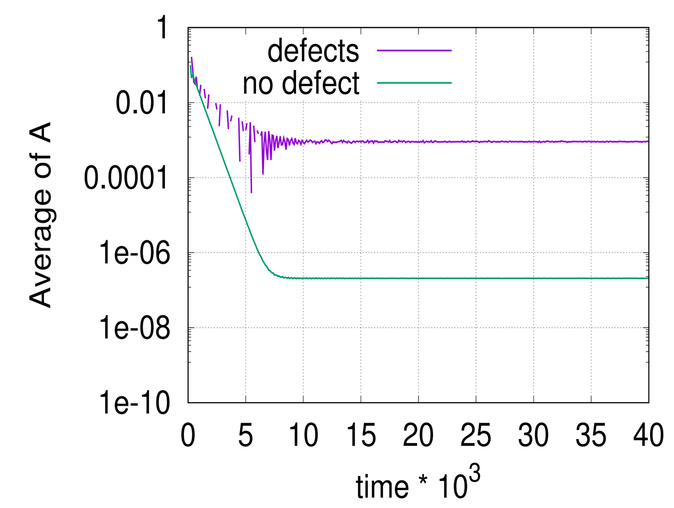
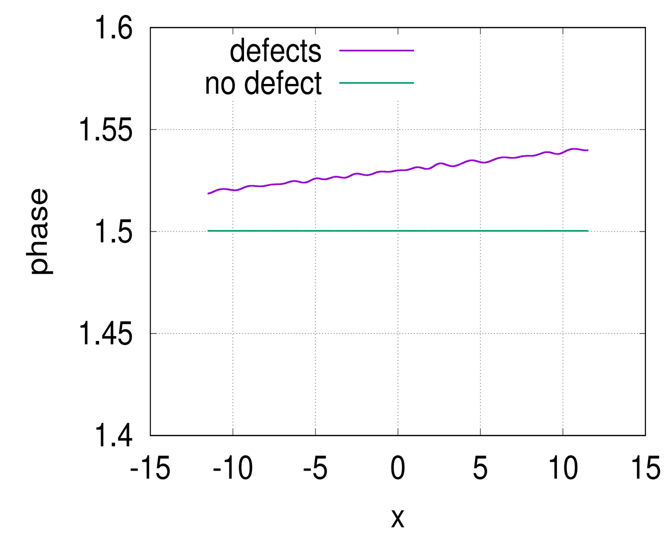
[3] T.S. Alstrom, M.P. Sorensen, N.F. Pedersen, S. Madsen. Magnetic Flux Lines in Complex Geometry Type-II Superconductors Studied by the Time Dependent Ginzburg-Landau Equation. In *Acta Applicandae Mathematicae, 115(1), 63-74*, 2010.

[4] H. B. Nielsen, P. Olesen. vortex-line models for dual strings. In *Nuclear Physics B61 ,45-61. North-Holland Publishing Company*. 1973.

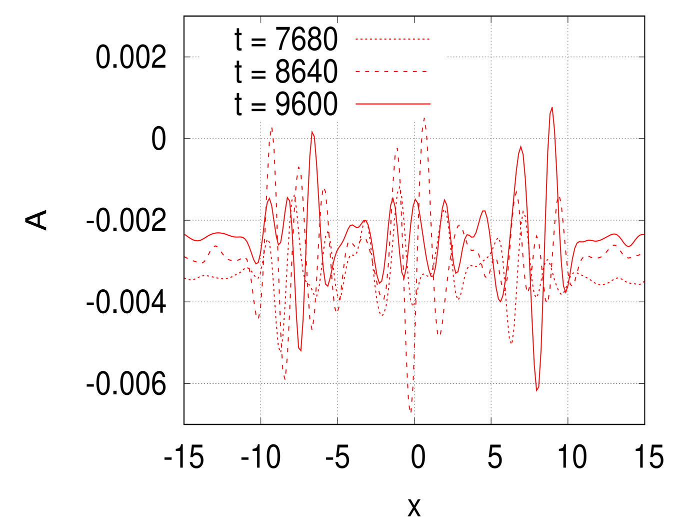
RESULTS



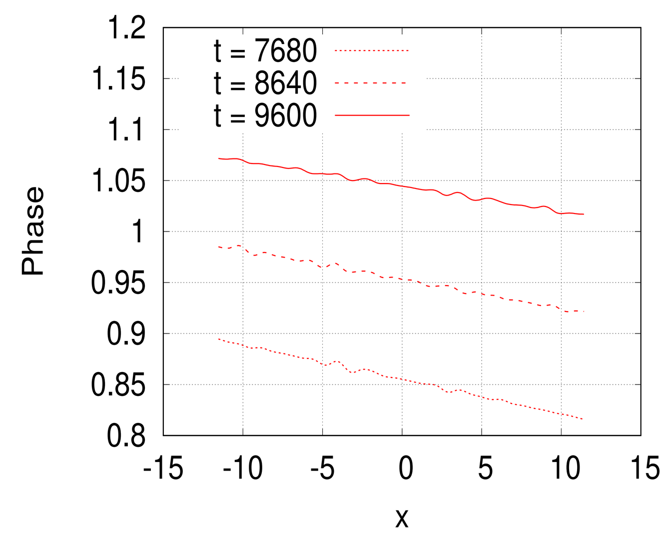
(a) Average $\langle A\rangle(t)$



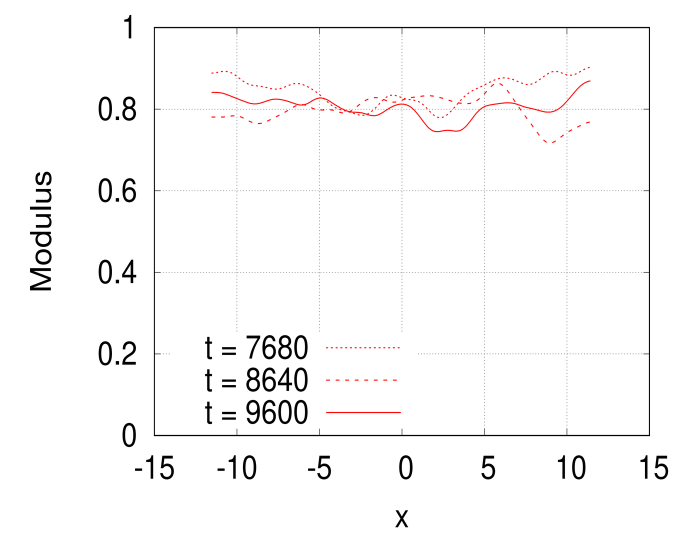
(b) Phase at $t = 40000$



(c) Vector potential $A(x,t)$



(d) Phase $\theta(x,t)$



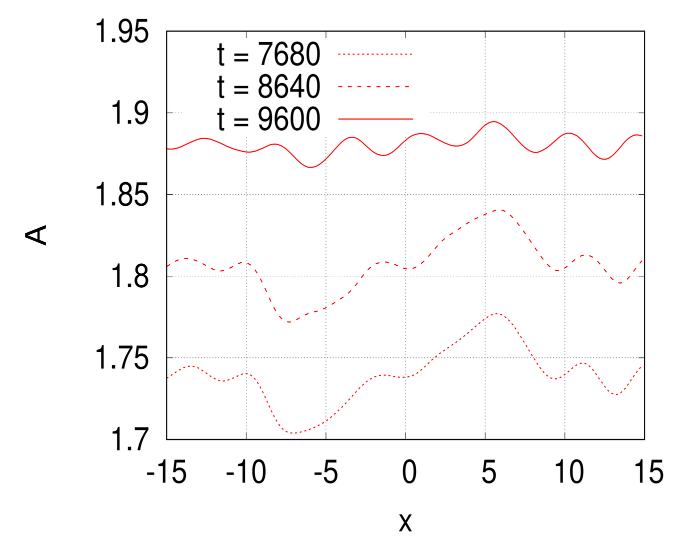
(e) Modulus $\rho(x,t)$

- **Analysis :** $\psi = \rho e^{i\theta}$

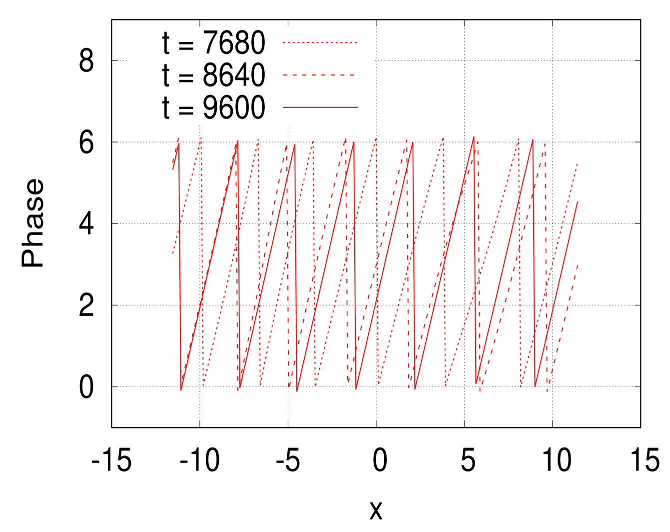
$$A_{tt} - A_{xx} = I\rho^2\left(\frac{\theta_x}{\kappa} - A\right),$$
$$\rho_{tt} - \rho_{xx} = \rho(\theta_t^2 - \theta_x^2) + 2\kappa A\rho\theta_x + \kappa^2\rho(1-\rho^2-A^2),$$
$$\theta_{tt} - \theta_{xx} = \frac{2}{\rho}(\theta_x\rho_x - \theta_t\rho_t) - \kappa A_x - 2\kappa A\frac{\rho_x}{\rho}.$$

→ The interface condition at $x = \pm L$: $\rho_x = 0, \quad \theta_x = \kappa A$.
→ Full solution : $A = A_\infty, \ \rho = 1, \ \theta_x = \kappa A_\infty$.

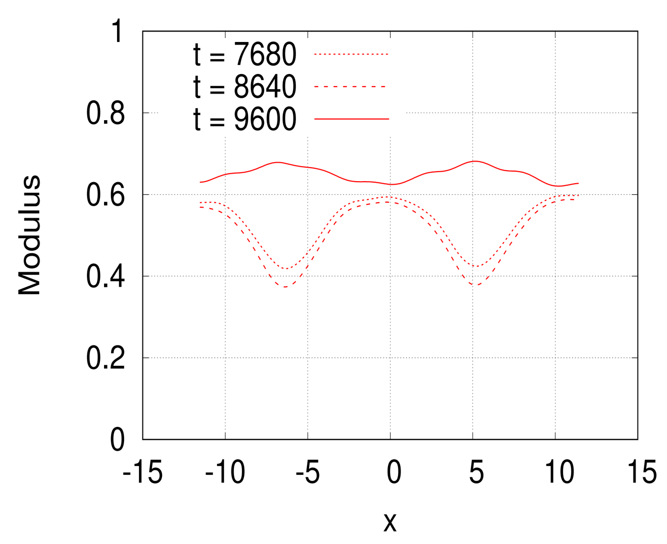
- **Parameters of (a), (b), (c), (d), (e) :** $\kappa = 1, x_d = 0.05, a_0 = 3, w_d = 0.2$.
- **Parameters of (f), (g), (h) :** $\kappa = 1, x_d = 0.01, a_0 = 3, w_d = 0.6$.



(f) Vector potential $A(x,t)$



(g) Phase $\theta(x,t)$



(h) Modulus $\rho(x,t)$

CONCLUSION

Results indicate trapped potential field dependent on defects:

1. The absence of defect leads to a potential close to zero.
2. The presence of defects trapped a non zero potential in the superconducting domain.
3. The density of defects enhances the value of the trapped potential.

A finite element code using the free software *Freefem* is being developped and it leads so far to similar results.